Dynamic Modelling and Control of Differential-Drive Mobile Robot

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***Abstract*—This paper addresses the dynamics and trajectory tracking control of the differential-drive mobile robot (DDMR). The center of mass point and the mid-point on the axis between the driving wheels, as two reference points of generalized coordinate which leads definitely different results, are usually selected to model the dynamics of the DDMR. This paper establishes the kinematic and dynamic models simultaneously on the foundation of the center of mass point of DDMR. Firstly, a formulation for DDMR dynamics is developed based on Lagrangian mechanics, where the Lagrange multipliers are introduced to solve the problem of nonholonomic constraints. Next, a controller combining velocity and torque control is proposed based on backstepping method to solve the trajectory tracking problem, and its asymptotic stability is proven by Lyapunov theory. Finally, the numerical simulation results demonstrate its effectiveness and efficiency.**

Keywords*—*nonholonomic constraint, backstepping control, dynamic modelling, trajectory tracking

I. INTRODUCTION

Wheeled mobile robots (WMR) have received booming interests from the research communities and the general public alike, thanks to their capacity to extend the workspace into unstructured environments where a high degree of autonomy is required [1]. For the engineers and researchers in the field of control engineering, a lot of literatures dealing with WMR control are available. These articles mainly aim at solving the problem of motion under nonholonomic constrains using the kinematic model, while a few refer to the problem integrating the kinematics and the dynamics of the mobile robot.

Differential drive mobile robot (DDMR) is one of the most widely used WMR and the relative kinematic models are proposed by many researchers. Kinematic models of DDMR are simple and valid when mobile robot is travelling with low velocity, low acceleration and light load [2]. However, for the lack of dynamic constraints, the mobile robot cannot immediately change its velocity to the desired value, which leads to delay between navigation computer and robot controller. Therefore, the dynamics of DDMR are necessary to deal with the navigation problem on high-speed scenes. Un-fortunately, few literatures investigate thoroughly the dynamic modelling approach which take into consideration the nonholonomic constraints step by step. The construction of nonlinear dynamical model of DDMR subjected to nonholonomic constraints is difficult for the control engineers who are not concerned with it. Therefore, a detailed procedure of DDMR dynamic modelling needs to be developed.

In addition, few researchers concentrated on the fact that the dynamic models are definitely different when the reference point of generalized coordinate is changed. More specifically, as shown in Fig. 1, the dynamic model based on the center of mass point A is distinguished from the one based on the mid-point on the axis of the driving wheels C. However, a common phenomenon happened in the material that the generalized coordinate of kinematic model based on the center of mass while the dynamic model based on the mid-point on the axis between the driving wheels. Additionally, some papers gave different results for the same DDMR used, which add the confusion to dynamic modelling [3].

There are two main methodologies to DDMR dynamic formation including Lagrangian approach [2, 4] and Newton-Euler approach [5, 6]. And both two methods will lead to the same result finally. In terms of Newton-Euler method, active forces, created by actuators, and constraint forces generated by the interaction between mobile robot wheels and ground have to be taken into account. In fact, Newton-Euler approach exist a few difficulties for calculating these forces. In- stead of forces, Lagrangian mechanics only consider the energies in the system, i.e., the kinetic energy and the potential energy [7].

It is noteworthy that the Lagrangian approach usually formulates the dynamic model of holonomic systems. In terms of nonholonomic systems, the normal practice is to introduce nonholonomic constraints into dynamic equation using Lagrange multipliers [3, 8]. Then, the Lagrange multipliers will be eliminated by some additional simplification operations to reduce the system complexity.

A good dynamic model of DDMR is the base of trajectory tracking problem, which is the main component of navigation problem. Many nonlinear feed- back controllers have been proposed in literature [9-14]. The common idea of these control algorithms is to design velocity control inputs considering only the kinematic model which may lead to unexpected errors caused by the impractical perfect velocity tracking assumption. Consequently, it is more realistic to define torque control inputs for the DDMR. Backstepping control approach offers a useful method for converting the velocity control into torque control. In [15], an adaptive backstepping control scheme based on the virtual decomposition control was presented to solve the non-holonomic mobile manipulator robot tracking control problem. [16] dealt with the problem of balancing and trajectory tracking of Two Wheeled Balancing Mobile Robots with backstepping Sliding Mode Controller. In [17], a backstepping controller was proposed to settle down the trajectory tracking problem of a four-wheel drive differential steering system.

A further kind of approaches have been proposed in [18] and [19]. These approaches take the actuators dynamic into consideration which guarantees that the nonholonomic mobile robot tracks a given trajectory. The complicated controller structure may cause heavy computing burden when applied to high order nonholonomic systems including high order derivative given signal.

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In this paper, a new backstepping control rule for determining mobile robot wheels torques is given. It provides a methodology to convert kinematic model control into dynamic model control. This system is composed of two subsystem: velocity control and torque control subsystem. The first designs a feedback velocity control input for the kinematic steering system with the goal of stabilizing the pose error. The other subsystem computes the torque applied to the dynamics to make the mobile robot converge to the velocity delivered by the first subsystem. The $1-\cos\phi$ [20–22] is introduced to find an appropriate Lyapunov function to prove the stability of the control rule.

The remainder is organized as follows. Section 2 introduces the theoretical background of the DDMR, and establishes the kinematic model subject to nonholonomic constraint. Different from [3] and [23], our kinematic equation develops according to the center of mass point. In Section 3 Lagrange formulation is presented and the kinematic steering system presented in Section 2 is used to eliminate Lagrange multiplies. Backstepping controller that takes dynamics into consideration to change velocity control input into torque control input for the actual mobile robot is developed in Section 4. Meanwhile, the stability of the feedback control system is proven by Lyapunov theory. Section 5 conducts a computer simulation for the proposed backstepping controller and the conclusion is given in Section 6.

II. Kinematic Modelling of DDMR

In this section, the theoretical background of nonholonomic constraint is discussed and the kinematic equations of DDMR are presented. In terms of kinematic model, there are two options of the reference point of generalized coordinate: one is point A, the center of mass, and the other is point C, the mid-point on the axis between the driving wheels as shown in Fig. 1. In this paper, we choose point A and the other can be extended as the same way.

A. Coordinatie Transformation

A schematic of a typical DDMR is depicted in Fig. 1. It consists of a chassis, two driving wheels mounted on the same axis, and a front free wheel. The motion and orientation are achieved by two independent actuators.

As illustrated in Fig. 1, two different coordinate frames have been defined for the purpose of describing the position of the DDMR in the environment. The origin of the robot coordinate frame $\{X_R, C, Y_R\}$ is defined to be mid-point C on the axis between the wheels. The center of mass point A of the robot is assumed to be on the axis of symmetry, at a distance d from the point C. Moreover, the inertial coordinate frame is a global frame which is fixed in the environment and denoted as $\{X_I, O, Y_I\}$.

The position of the robot in the inertial frame can be specified by the vector ${}^I\boldsymbol{q} = [x_a, y_a, \phi]^{\mathrm{T}}$, where x_a, y_a denote the coordinates of point A, and φ is the orientation of the frame $\{X_R, C, Y_R\}$ with respect to the $\{X_I, O, Y_I\}$.

Due to the fact that the trajectory of the vehicle is constrained to the horizontal plane, the position of any point on the robot can be defined as ${}^I\boldsymbol{X} = [{}^Ix,\ {}^Iy, 0]^{\mathrm{T}}$ and ${}^I\boldsymbol{R} = [{}^Rx,\ {}^Ry, 0]^{\mathrm{T}}$ in the inertial frame and robot frame.

B. Nonholonomic Constraints

Wheeled vehicles are generally subjected to nonholonomic constraints. A nonholonomic system is subjected to at least one non-integrable constraint which limits the local mobility of the system [24]. The DDMR has two assumptions: no lateral slip constraint and pure rolling constraint.

we can derive

$$\begin{cases} {}^I\dot{x}_{P_R} = {}^I\dot{x}_a - d\dot{\phi}sin\phi + L\dot{\phi}cos\phi \\ {}^I\dot{y}_{P_R} = {}^I\dot{y}_a - d\dot{\phi}cos\phi + L\dot{\phi}sin\phi \end{cases} \tag{1}$$

We can obtain the differential coordinates of left wheel in the same way

$$\begin{cases} {}^I\dot{x}_{P_L} = {}^I\dot{x}_a + d\dot{\phi}sin\phi - L\dot{\phi}cos\phi \\ {}^I\dot{y}_{P_L} = {}^I\dot{y}_a - d\dot{\phi}cos\phi - L\dot{\phi}sin\phi \end{cases} \tag{2}$$

Then the contraints can be written in matrix form:

$$\Lambda(q)\dot{q} = 0 \tag{3}$$

where

$$\boldsymbol{\Lambda}(\boldsymbol{q}) = \begin{bmatrix} -sin\phi & cos\,\phi & -d & 0 & 0 \\ cos\,\phi & sin\phi & L & -R & 0 \\ cos\,\phi & sin\phi & -L & 0 & -R \end{bmatrix} \tag{4}$$

and $\dot{q} = [\dot{x_a}, \dot{y_a}, \dot{\phi}, \dot{\theta_R}, \dot{\theta_L}]^{\mathrm{T}}$

C. Forward Kinematic

The forward kinematic of mobile robot can be written in this form

$${}^I\dot{q} = \begin{bmatrix} {}^I\dot{x}_a \\ {}^I\dot{y}_a \\ \dot{\phi} \end{bmatrix} = \boldsymbol{S}(\boldsymbol{q})\boldsymbol{v} \tag{5}$$

where

$$\boldsymbol{S}(\boldsymbol{q}) = \begin{bmatrix} cos\,\phi & -dsin\phi \\ sin\phi & d\,cos\,\phi \\ 0 & 1 \end{bmatrix}, \boldsymbol{v} = \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{6}$$

and $|v| \le V_{max}, |\omega| \le W_{max}$ and W_{max} represent the maximum linear and angular velocities of the mobile robot respectively. System (5) is also named the steering system.

III. Dynamic Modelling of DDMR

Actually, the inputs of kinematic model are velocity commands while the inputs of a real mobile robot are forces or torques. In other words, the dynamics of a system are ignored while dealing with the control problem of DDMR. Naturally, It is important to derive the dynamic model and explore its characteristics for control purpose.

A mobile robot subjected to m constraints, which has a n-dimensional configuration space $\mathcal{C}$ with generalized coordinates $(q_1, q_2, \cdots, q_n)$, can be described by [25, 26]

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} + F(\dot{q}) + G(q) + \tau_d = \boldsymbol{B}(\boldsymbol{q})\boldsymbol{\tau} + \boldsymbol{\Lambda}^{\mathrm{T}}(\boldsymbol{q})\boldsymbol{\lambda} \tag{7}$$

where $M(q) \in \Re^{n\times n}$ is a symmetric, positive definite inertia matrix, $V(q,\dot{q}) \in \Re^{n\times n}$ is the centripetal and coriolis matrix, $F(\dot{q}) \in \Re^{n\times 1}$ denotes the surface friction, $G(q) \in \Re^{n\times 1}$ is the gravitational vector, τ_d denotes bounded unknown disturbances including un-structured unmodeled dynamics, $B(q) \in \Re^{n\times r}$ is the input transformation matrix, $\boldsymbol{\tau} \in \Re^{n\times 1}$ is the input vector, $\Lambda \in \Re^{m\times n}$ is the matrix associated with the constraints, and $\lambda \in \Re^{m\times 1}$ is the vector of constraint forces.

For systems with holonomic constraints, all constraints are integrable into geometrical constraints. If the constraints are nonholonomic, this approach does not work. There is no general method to handle the nonholonomic problems. The dependent equations can be eliminated by the method of Lagrange multipliers only for those special nonholonomic constraints given in differential form [27]. For constraints in the form of equalities, the Lagrange equation of the first kind can be written in the following form:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial q_v}\right)-\frac{\partial T}{\partial q_v}=Q_v+\sum_{l=1}^{s}\lambda_l a_{lv} \quad (8)$$

for $v=1,2,\cdots,n$. Where n denotes the dimension of coordinates, s represents the number of constraints, T is kinetic energies of the DDMR, Q_v equals the generalized force in the direction of q_v, λ means the Lagrange multipliers vector, and a_{lv} denotes the coefficient of corresponding constraint equation.

The motion equations of the DDMR are given by

$$\begin{cases} m\ddot{x}_a+2m_w d\ddot{\phi}\sin\phi+2m_w d\dot{\phi}^2\cos\phi=\boldsymbol{\Lambda}^{\mathrm{T}}(\mathbf{1})\boldsymbol{\lambda} \\ m\ddot{y}_a-2m_w d\ddot{\phi}\cos\phi+2m_w d\dot{\phi}^2\sin\phi=\boldsymbol{\Lambda}^{\mathrm{T}}(\mathbf{2})\boldsymbol{\lambda} \\ I\ddot{\phi}+2m_w d\ddot{x}_a\sin\phi-2m_w d\ddot{y}_a\cos\phi=\boldsymbol{\Lambda}^{\mathrm{T}}(\mathbf{3})\boldsymbol{\lambda} \\ I_w\ddot{\theta}_R=\tau_R+\boldsymbol{\Lambda}^{\mathrm{T}}(\mathbf{4})\boldsymbol{\lambda} \\ I_w\ddot{\theta}_L=\tau_L+\boldsymbol{\Lambda}^{\mathrm{T}}(5)\boldsymbol{\lambda} \end{cases} \quad (9)$$

where $\boldsymbol{\Lambda}^{\mathrm{T}}(\boldsymbol{i})$ means the ith row of $\boldsymbol{\Lambda}^{\mathrm{T}}$ introduced in (4).

Thus, the obtained (9) can be represented in the general form given by (7) as

$$\overline{\mathrm{M}}(\mathrm{q})\dot{\mathrm{v}}+\overline{\mathrm{V}}(\mathrm{q},\dot{\mathrm{q}})\mathrm{v}=\overline{\mathrm{B}}(\mathrm{q})\tau \quad (10)$$

where $\bar{M}(q)=\hat{M}(q)T,\ \bar{V}(q,\dot{q})=\hat{V}(q,\dot{q})T$, $\bar{B}(q)=\hat{B}(q), \boldsymbol{v}=[v,\omega]^{\mathrm{T}}=T\beta, T=\left[\frac{1}{R},\frac{L}{R};\frac{1}{R},-\frac{L}{R}\right]$ and

$$\widehat{\boldsymbol{M}}(\boldsymbol{q})=\begin{bmatrix} I_w+R^2\left(\frac{m}{4}+\Delta\right) & R^2\left(\frac{m}{4}-\Delta\right) \\ R^2\left(\frac{m}{4}-\Delta\right) & I_w+R^2\left(\frac{m}{4}+\Delta\right)\end{bmatrix}$$

$$\widehat{\boldsymbol{V}}(\boldsymbol{q},\dot{\boldsymbol{q}})=\begin{bmatrix} 0 & \frac{mdR^2\dot{\phi}}{2L}-\frac{m_w dR^2\dot{\phi}}{L} \\ -\frac{mdR^2\dot{\phi}}{2L}+\frac{m_w dR^2\dot{\phi}}{L} & 0\end{bmatrix}$$

$$\hat{B}(q,\dot{q})=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Delta=\frac{I}{4L^2}+\frac{md^2}{4L^2}-\frac{m_w d^2}{L^2}$$

IV. Trajectory Tracking Control Based on Backstepping Method

Most controllers consider only the kinematic model (6), ignoring the actual vehicle dynamics (10). In this article we design a controller to deal with the kinematic model and dynamic model simultaneously.

Now, the (10) can be rewritten as

$$\tau=\overline{\boldsymbol{B}}^{-1}[\bar{M}(q)u+\bar{V}(q,\dot{q})v] \quad (11)$$

where $\boldsymbol{u}$ is an auxiliary input. Then the dynamic control problem can be transformed into the kinematic control problem

$$\begin{cases}\dot{\boldsymbol{q}}=\boldsymbol{S}(\boldsymbol{q})\boldsymbol{v}\\ \dot{\boldsymbol{v}}=\boldsymbol{u}\end{cases} \quad (12)$$

which represents the state-space of the nonholonomic mobile robot in the type of chain of integrator.

A. Tracking Trajectory

The velocity control will take no account of the parameters of the actual mobile robot. In this section, such a velocity control $\boldsymbol{v}(t)$ is converted into a torque control $\tau(t)$.

The Fig. 2 presents the general structure of the trajectory tracking control system based on the system (6) and (10). The purpose of this control system is to derive a suitable control input obtained by virtual control input $v_d(t)$. Then the (2) is used to compute $\boldsymbol{\tau}(t)$ given $\boldsymbol{u}(t)$.

For the purpose of tracking a reference trajectory, two poses are used: the reference pose $\boldsymbol{q_r}=(x_r,y_r,\phi_r)^{\mathrm{T}}$ and the current pose $\boldsymbol{q_c}=(x_c,y_c,\phi_c)^{\mathrm{T}}$. Now, a reference trajectory can be posed as $\dot{x}_r=v_r\cos\phi_r, \dot{y}_r=v_r\sin\phi_r, \dot{\phi}_r=\omega_r$.

The trajectory tracking problem is to find a velocity control $\boldsymbol{v_d}(t)$ such that $\lim_{t\to\infty}(q_r-q_c)=0$. Then the torque input $\boldsymbol{\tau}(t)$ is computed such that $\boldsymbol{v}\to\boldsymbol{v_d}$ as $t\to\infty$.

Using the backstepping approach [28], one can synthesize the control law forcing system (12) to follow the desired trajectories. Under guide of the backstep- ping control approach, we build the control law by the following two steps:

Step 1: For the first step, we consider the kinematic steering system:

$$\dot{\boldsymbol{q}}=\overline{\boldsymbol{S}}(\boldsymbol{q})\boldsymbol{v} \quad (13)$$

An error pose $\boldsymbol{q_e}$ is introduced, which represents the difference between $\boldsymbol{q_r}$ and $\boldsymbol{q_c}$. The goal of this step is to design a velocity control input to stabilize the pose error.

For the convenience of control, the error pose $\boldsymbol{q_e}$ should be denoted in the robot coordinate frame [29].

$$\boldsymbol{q_e}=\begin{bmatrix}x_e\\ y_e\\ \phi_e\end{bmatrix}=\boldsymbol{R^{-1}}(\phi_c)(\boldsymbol{q_r}-\boldsymbol{q_c})=\begin{bmatrix}\cos\phi_c & \sin\phi_c & 0\\ -\sin\phi_c & \cos\phi_c & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_r-x_c\\ y_r-y_c\\ \phi_r-\phi_c\end{bmatrix} \quad (14)$$

By using (22), the derivative of (14) is

$$\dot{\boldsymbol{q}}_{\boldsymbol{e}}=\begin{bmatrix}\dot{x}_e\\ \dot{y}_e\\ \dot{\phi}_e\end{bmatrix}=\begin{bmatrix}\omega_c y_e-v_c+v_r\cos\phi_e\\ -x_e\omega_c+v_r\sin\phi_e\\ \omega_r-\omega_c\end{bmatrix} \quad (15)$$

Then the first scalar function V_1 is proposed as a Lyapunov function

$$\boldsymbol{V_1}=\frac{1}{2}\boldsymbol{x_e^2}+\frac{1}{2}\boldsymbol{y_e^2}+(\mathbf{1}-\mathbf{cos}\,\boldsymbol{\phi_e}) \quad (16)$$

where $V_1\geq 0$, and $V_1=0$ only if $\boldsymbol{q_e}=\mathbf{0}$. Furthermore, by using (15), we can derive

$$\dot{V}_1=x_e\dot{x}_e+y_e\dot{y}_e+\dot{\phi}_e\sin\phi_e=x_e(-v_c+v_r\cos\phi_e)+y_e v_r\sin\phi_e+\sin\phi_e(\omega_r-\omega_c)=x_e(-v_c+v_r\cos\phi_e)+\sin\phi_e(\omega_r-\omega_c+y_e v_r) \quad (17)$$

The stabilization of $\boldsymbol{q_e}$ can be obtained by introducing a first virtual control input $\boldsymbol{v_d}$:

$$\boldsymbol{v_d}=\begin{bmatrix}v_d\\ w_d\end{bmatrix}=\begin{bmatrix}k_1x_e+v_r\cos\phi_e\\ k_2\sin\phi_e+\omega_r+y_e v_r\end{bmatrix} \quad (18)$$

where k_1 and k_2 are positive constant controller gains.

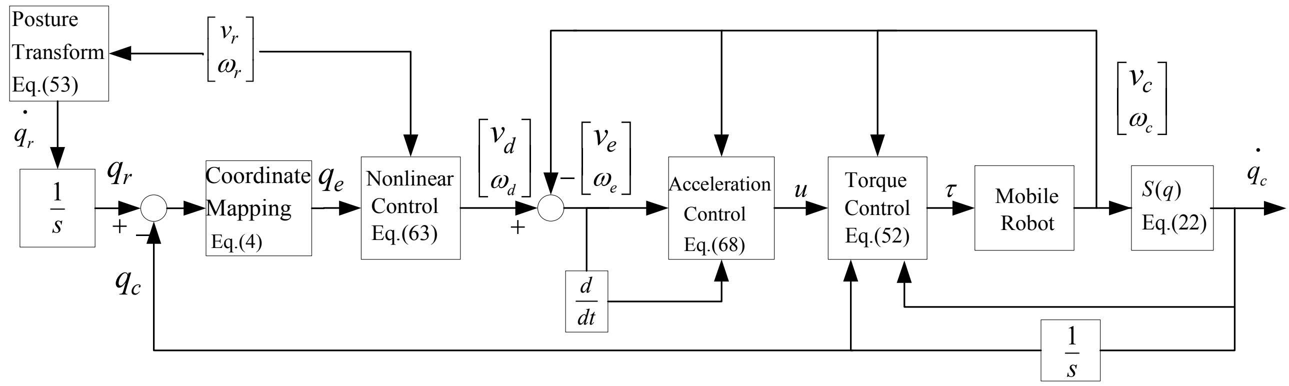

Fig. 1. Trajectory tracking control structure.

The (17) is then $\dot{V}_1 = -k_1 x_e^2 - k_2 \sin^2 \phi_e \leq 0$, and $\dot{V}_1 = 0$ only if $\boldsymbol{q_e} = \boldsymbol{0}$. That means the control input $\boldsymbol{v_d}$ will make the pose error converge to zero.

Step 2: For the second step we consider the following virtual system

$$\dot{\boldsymbol{v}} = \boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{19}$$

Let the second tracking error be

$$\boldsymbol{v_e} = \boldsymbol{v_d} - \boldsymbol{v_c} = \begin{bmatrix} e_4 \\ e_5 \end{bmatrix} = \begin{bmatrix} k_1 x_e + v_r \cos\phi_e - v_c \\ k_2 \sin\phi_e + \omega_r + y_e v_r - \omega_c \end{bmatrix} \tag{20}$$

and its time derivative is

$$\dot{\boldsymbol{v}}_e = \begin{bmatrix} \dot{e}_4 \\ \dot{e}_5 \end{bmatrix} = \begin{bmatrix} k_1 \dot{x}_e + v_r \overset{\cdot}{\cos}\phi_e - v_r \phi_e \sin\phi_e - \dot{v}_c \\ k_2 \phi_e \overset{\cdot}{\cos}\phi_e + \dot{\omega}_r + \dot{y}_e v_r + y_e \dot{v}_r - \dot{\omega}_c \end{bmatrix} \tag{21}$$

Furthermore, it is easy to be concluded from (20)

$$\boldsymbol{v_c} = \begin{bmatrix} v_c \\ \omega_c \end{bmatrix} = \begin{bmatrix} k_1 x_e + v_r \cos\phi_e - e_4 \\ k_2 \sin\phi_e + \omega_r + y_e v_r - e_5 \end{bmatrix} \tag{22}$$

and then substituting this result in (17), we can obtain

$$\boldsymbol{\dot{V}_1 = x_e(-k_1 x_e + e_4) + \sin\phi_e\,(-k_2 \sin\phi_e + e_5)} \tag{23}$$

Consider the following second Lyapunov function candidate:

$$\boldsymbol{V_2 = V_1 + \tfrac{1}{2} e_4^2 + \tfrac{1}{2} e_5^2} \tag{24}$$

Clearly $V_2 \geq 0$ and $V_2 = 0$ only if $\boldsymbol{q_e} = \boldsymbol{0}$ and $v_e = 0$. Furthermore, the differential of V_2 can be derived from (19), (21) and (23)

$$\dot{V}_2 = \dot{V}_1 + e_4 \dot{e}_4 + e_5 \dot{e}_5 = x_e(-k_1 x_e + e_4) + \sin\phi_e\,(-k_2 \sin\phi_e + e_5) + e_4\big(k_1 \dot{x}_e + v_r \overset{\cdot}{\cos}\phi_e - v_r \phi_e \sin\phi_e - u_1\big) + e_5\big(k_2 \phi_e \overset{\cdot}{\cos}\phi_e + \dot{\omega}_r + \dot{y}_e v_r + y_e \dot{v}_r - u_2\big) = -k_1 x_e^2 - k_2 \sin^2\phi_e + e_4\big(x_e + k_1 \dot{x}_e + v_r \overset{\cdot}{\cos}\phi_e - v_r \phi_e \sin\phi_e - u_1\big) + e_5\big(\sin\phi_e + k_2 \phi_e \overset{\cdot}{\cos}\phi_e + \dot{\omega}_r + \dot{y}_e v_r + y_e \dot{v}_r - u_2\big) \tag{25}$$

Nonlinear feedback acceleration control input is proposed as

$$\boldsymbol{u} = \begin{bmatrix} x_e + k_1 \dot{x}_e + v_r \overset{\cdot}{\cos}\phi_e - v_r \phi_e \sin\phi_e + k_3 e_4 \\ \sin\phi_e + k_2 \phi_e \overset{\cdot}{\cos}\phi_e + \dot{\omega}_r + \dot{y}_e v_r + y_e \dot{v}_r + k_4 e_5 \end{bmatrix} \tag{26}$$

where k_3 and k_4 are positive constant control gains. Using the (15), we can obtain

$$\begin{cases} u_1 = x_e + k_1(\omega_c y_e - v_c + v_r \cos\phi_e) + v_r \overset{\cdot}{\cos}\phi_e \\ \qquad -v_r(\omega_r - \omega_c)\sin\phi_e + k_3 e_4 \\ u_2 = \sin\phi_e + k_2(\omega_r - \omega_c)\cos\phi_e + \dot{\omega}_r \\ \qquad +(-x_e \omega_c + v_r \sin\phi_e) v_r + y_e \dot{v}_r + k_4 e_5 \end{cases} \tag{27}$$

Then substitute (27) in (25)

$$\dot{V}_2 = -k_1 x_e^2 - k_2 \sin^2 \varphi_e - k_3 e_4^2 - k_4 e_5^2 \tag{28}$$

Obviously $\dot{V}_2 \leq 0$ and $\dot{V}_2 = 0$ only if $\boldsymbol{q_e} = \boldsymbol{0}$ and $v_e = 0$. Therefore, the equilibrium point $e = 0$ is uniformly asymptotically stable, where $e = [q_e, v_e]^{\mathrm{T}}$.

Therefore, by using (18) and (27), the whole system (12) is asymptotically stable according to the following control law:

TABLE I. SYSTEM PARAMETER

Parameter	Value	Unit
Mass of DDMR(m)	11	kg
Mass of base (m_a)	10	kg
Mass of driving wheel (m_w)	0.5	kg
Auxiliary moment of inertial	0.4136	kgm^2
Moment of inertial of each driving wheel with an actuator about the wheel diameter (I_m)	0.001	kgm^2
Moment of inertial of each driving wheel with a motor about the wheel axis (I_w)	0.001	kgm^2
Moment of inertia of the DDMR (without wheels and actuators) about the vertical axis through point A (I_a)	0.4	kgm^2
Half length of axis between the driving wheels (L)	0.1	m
Distance from A to C (d)	0.04	m
Radius of the driving wheels (R)	0.025	m
Velocity controller gain (K_1)	5	Non
Velocity controller gain (K_2)	10	Non
Torque controller gain (K_3)	10	Non
Torque controller gain (K_4)	10	Non

$$\begin{cases} v_d = k_1 x_e + v_r \cos\phi_e \\ \omega_d = k_2 \sin\phi_e + \omega_r + y_e v_r \\ u_1 = x_e + k_1(\omega_c y_e - v_c + v_r \cos\phi_e) + v_r \dot{\cos}\phi_e \\ \qquad -v_r(\omega_r - \omega_c)\sin\phi_e + k_3 e_4 \\ u_2 = \sin\phi_e + k_2(\omega_r - \omega_c)\cos\phi_e + \dot{\omega}_r \\ \qquad +(-x_e\omega_c + v_r \sin\phi_e)v_r + y_e \dot{v}_r + k_4 e_5 \end{cases} \tag{29}$$

V. Simulation Results

In order to verify the effectiveness and efficiency of the proposed backstepping control law, a computer simulation is conducted in MATLAB R2016b/Simulink on a personal computer equipped with an Inter Core i5 processor (3.30 GHz CPU and 8GB RAM) in the environment of Windows 7 OS with the dynamic model of DDMR. Before that, the relevant physical and design parameters are shown in Table 1.

The proposed control law (22) involves the knowledge of $\dot{v}_e$. In order to reduce the computing time originated from analytical derivation difficulties, we estimate them by using the finite difference time approximation $\dot{v}_e = \frac{\Delta v_e}{\Delta t}$, where $\Delta \boldsymbol{v_e}$ represents the change in velocity error and Δt shows the change in time since the previous time step.

This article takes no account of trajectory planning and the reference trajectory satisfies the nonholonomic constraints is given by $v_r = 2$, $\omega_r = 2 + \sin t$, $\dot{x}_r = v_r \cos\phi_r$, $\dot{y}_r = v_r \sin\phi_r$, $\dot{\phi}_r = \omega_r$, and the initial pose $[x_0, y_0, \phi_0] = [0,0,0]$.

Such a situation is depicted in Fig. 3, the curves of reference trajectory and tracking trajectory indicate that the proposed backstepping control law achieves a competitive result. Focus on Fig. 4, one can observe that the position (x, y) and orientation (ϕ) of mobile robot will be in accordance with the reference position (x_r, y_r) and orientation (ϕ_r) in 1.8 seconds.

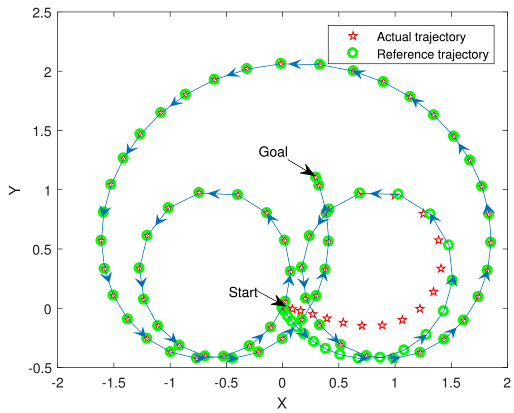

Fig. 2. Mobile robot reference and actual trajectories.

The responses of linear velocity and angular velocity are given in Fig. 5 and 6 respectively, with a good tracking performance of the desired linear and angular velocity. More specifically, the velocities present short regulating period, small overshoot and slight oscillation. As for Fig. 7, one can notice that the obtained control input u decreases from a high level to an acceptable and physically realizable level for an instant. And Fig. 8 indicates the torques applied to the left and right wheels. The torque of the right wheel rises from 0 to 2.116 Nm in 0.1s and then recovers rapidly to a normal level while the torque of the left encounters a sharp reduction from 6.726 Nm to a reasonable level at the beginning.

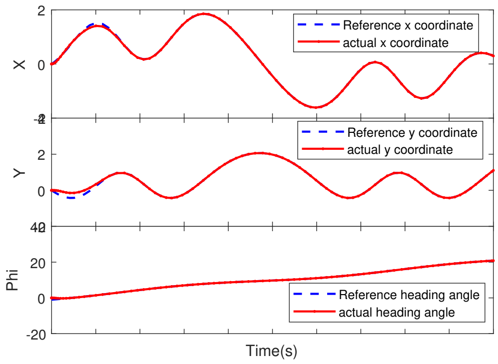

Fig. 3. Mobile robot reference and actual coordinates.

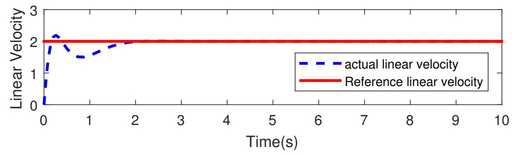

Fig. 4. Reference and actual linear velocities.

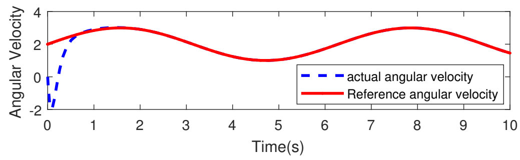

Fig. 5. Reference and actual angular velocities.

Fig. 9 demonstrates that the maximum tracking error of the position(x, y) and orientation ϕ are (-0.097, -0.278) m and 1 rad respectively. However, the tracking errors will converge

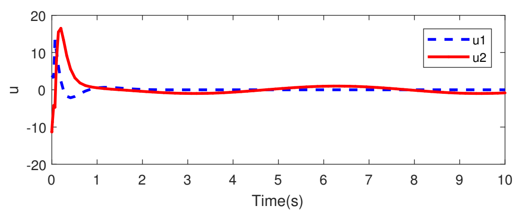

Fig. 6. Acceleration control inputs.

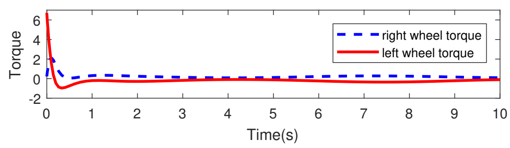

Fig. 7. Applied torque: right and left wheels.

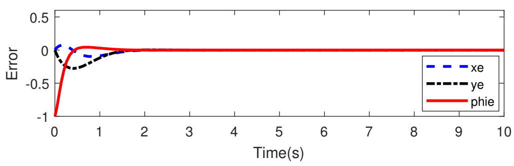

Fig. 8. Trajectory tracking errors.

to zero after just 2 seconds. Overall, these outcomes verify the validity of backstepping controller evidently and indicate that the backstepping control method is appropriate for the DDMR.

VI. Conclusion

The primary objective of this work is to develop a framework of dynamic modelling of the DDMR, in which we establish the dynamics equation using Lagrange method. The phenomenon with few concentrations that two reference points, i.e. the center of mass point and mid-point on the axis between the driving wheels, will lead to different dynamic results, has been discussed. This framework offers an effective tool to the researchers or students in the field of control who focus on designing controller rather than dynamic modelling.

A secondary objective is to propose a new control algorithm of trajectory tracking problem, where the back-stepping control method is used to convert the dynamic control problem into the kinematic control problem. At same time, a numerical simulation is conducted to check the stability of the proposed control algorithm while a nonholonomic mobile robot tracks a reference trajectory.

In this paper, the complete knowledge of the dynamics of the DDMR is assumed. In other words, the disturbances and unmodeled dynamics are not concerned. To cope with these issues, the robust and adaptive control algorithms can be introduced based on some iterative learning methods. In the future work, we plan to implement these ideas. We also want to introduce neural networks or support vector machine to handle with unmodeled disturbances and/or dynamics.

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