A Review on How to Survive During an Epidemic, Natural Calamity and All-Hazards

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Abstract— **To survive, protect and secure oneself in disasters like the epidemic which is akin to a situation where one has to trade-off to protect both from asymptomatic infected persons who are difficult carriers to assess and who have attributed to a significant number of Covid-19 cases as well as natural hazardous situations. This is clear because the healthy person (defender) must make decisions about both, for resources which are scarce like that of emergency health services. In this study, we have considered a game when the agents, i.e. the healthy person (defender) and the asymptomatic infected person/infection, move simultaneously. There may be some other realistic situations that may occur, such as when the defender makes a move first; and when the asymptomatically infected/disease progresses first. Finally, we arrive at conditions, where we observe that when all-hazards protection is not amounting to a higher cost, it can combinedly protect both the natural disaster and infection. We also see that, as the cost increases, either or both protection against pure natural disaster protection or protection against specific asymptomatic infection joins in, based on the effectiveness of cost.**

Keywords—All-Hazards, Game Theory, Epidemic, Natural Hazard

I. INTRODUCTION

Few defences are effective against epidemics, especially against asymptomatic infections, or only against natural disaster. For Example, methods like social distancing, including steps of Janta Curfew in India to tackle Covid 19 in India can be effective against the spread of epidemics specifically through asymptomatic infections. Similarly, protecting coastline through wetlands can be used against tsunamis, hurricanes and various other types of natural hazards. Different kinds of investment—say, emergency planning or strengthening would be taken to counter "all-hazards" protection. Here, in this study, we intend to understand how a defender, i.e. a healthy person needs to distribute/ allocate his/her investments between protecting against natural disaster, asymptomatic infective persons and "all-hazards." At first glance, one might expect the unit cost of "all-hazards" protection to be high, in which case protection against epidemic agents like asymptomatic infections and natural disaster on an individual basis may be preferable. However, this will not always be the case; for example, one can imagine that improving wetlands might be so costly in some situations that it would be cheaper to harden buildings instead. Natural disasters are a subcategory of non-intentional attacks.

In contrast, epidemic contagion by asymptomatic infected agents may not be purely non-intentional, nor they may be intentional as these may be challenging to assess. For ease of exposition, this study refers to the trade-off between epidemic based on asymptomatic infective carriers and natural disasters. Still, the results can be well applied to trade-offs between intentional and nonintentional attacks.

In this study, we use contest success functions to adequately represent the interaction between the healthy person, i.e. defender and asymptomatic infected person and the natural disaster. Contest success functions represent the interactions between intelligent agents. Contest success functions in the case of a natural emergency are considered more of a passive threat than in case of asymptomatic infects as in case of epidemics like Covid 19 may be unorthodox. Still, it serves a way to specify the intensity of this threat in a parametric way. Some Assumptions include,1) epidemic like Covid 19 can be spread naturally through various medium, i.e. it can be considered as more of a natural hazard and 2) Covid 19 is spread through corona positive infected persons who may be asymptomatic and may not have been tested.

In Section 2, we present a simple model where we formulate to model the healthy person, i.e. defender's investments as well as his/her utilities. Section 3 analyses the model when the defender and the asymptomatic infected person move simultaneously under different conditions. Section 4 provides a couple of useful propositions, and we conclude in Section 5.

II. THE MODEL

Let us consider the healthy person, i.e. a defender, the person who defends against infected persons, who are asymptotic in an epidemic, i.e. people who do not have any symptoms but are infected, and a natural hazard which can occur with a probability p has been valued at V

Suppose the defender executes an effort e1 at unit cost c1 to protect an inherent risk, and effort e2 at cost c2 to protect against infected asymptotic persons in an epidemic, an effort e3 at cost c3 against all other hazards. The infected asymptomatic person/ infection moves around freely among the population incurs an effort $I \geq 0$ at unit cost C1. The expenditures are ci*ei (i=1,2 and 3), and I*C1 can reflect capital costs and expenses such as labour costs, while the magnitude of the natural disaster is given by a constant N.

One of the assumptions that this paper makes is that the contests between the health person(defender) and asymptotically infected person, takes the form of conflict as well as rent-seeking (Hirshleifer, 1995; Skaperdas, 1996).

If a is a contest success function satisfying the following conditions, for the contest involving natural disaster, the healthy person retains a fraction a of its asset where

$$\partial a/\partial e_1 > 0$$
$$\partial a/\partial e_3 > 0$$
$$\partial a/\partial N < 0$$

For the game involving the asymptotic infected person, the defender maintains an expected fraction b of its asset, and the asymptotic infected person gets the other fraction B = 1 – b, where b represents contest success function as below

$$\partial b/\partial e_2 > 0$$
$$\partial b/\partial e_3 > 0$$
$$\partial b/\partial I < 0$$

The fractions a, b and B are fractions of the value of the person (if a natural disaster or asymptotic infected person/ infection results in partial damage), or probabilities that a natural disaster or an asymptotic infected person ultimately kills the person having a value V.

Using the "common ratio formula (Hausken, 2005; Skaperdas, 1996; Tullock, 1980)"

$$a = \frac{e_1+e_3}{e_1+e_3+N}, b = \frac{e_2+e_3}{e_2+e_3+I}, B = \frac{I}{e_2+e_3+I} \text{----------------------(1)}$$

Though in the current case we have considered a and b separately, the healthy person's (defender's) success of survival p depends mainly on the minimum (a,b)

The defender's (healthy person) U_D and infected Utility function U_{Inf} are given as below

$$U_D = [p * (\frac{e_1+e_3}{e_1+e_3+N}) + (1-p) * (\frac{e_2+e_3}{e_2+e_3+I})] * V - c_1 * e_1 - c_2 * e_2 - c_3 * e_3$$
$$U_{Inf} = -(1-p) * (\frac{I}{e_2+e_3+I}) * V - I * C_I \text{ ----------------(2)}$$

In this paper, we have considered the simultaneous movement of the healthy person(defender) and the asymptotic infected person. We can use even if they do not move simultaneously, as long as the agents/persons moving later are not aware of any earlier actions as in the case of Covid-19 epidemic. But there may be situations where the games may be sequential, i.e. which are also called dynamic. Here, either the asymptotic infected person may make the first move, or the healthy person may make the first move, and the agents moving later may be aware of earlier agent's actions. The mathematical approach will be different, and solutions can be obtained using backward induction.

III. Analysis of the Model

The three first-order conditions for the health person(defender), as well as the first-order condition for the asymptomatic infected person which is unique, is given below.

Please refer Appendix 1, where the Hessian Matrix, which satisfies the second-order conditions for maxima for the defender, U_{Inf} has been proved. Further, the second-order term for the infected person has also been shown to be convex.

$$\frac{\partial U_D}{\partial e_1} = \frac{p*N*V}{(e_1+e_3+N)^2} - c_1 = 0$$
$$\frac{\partial U_D}{\partial e_2} = \frac{(1-p)*I*V}{(e_2+e_3+I)^2} - c_2 = 0$$
$$\frac{\partial U_D}{\partial e_3} = \frac{p*N*V}{(e_1+e_3+N)^2} + \frac{(1-p)*I*V}{(e_2+e_3+I)^2} - c_3 = 0 \text{----------(3)}$$
$$\frac{\partial U_{\inf}}{\partial I} = \frac{-(1-p)*(e_2+e_3)*V}{(e_2+e_3+I)^2} - C_I = 0$$

To solve for the above set of first-order conditions, we need to settle for three unknowns (e1 + e3, e2 + e3 and I)

We have three conditions for which we need to solve for these unknowns

1) $c_1 + c_2 = c_3$
2) $c_1 + c_2 < c_3$
3) $c_1 + c_2 > c_3$

For situation 1) $c_1 + c_2 = c_3$, we can obtain an interior solution, where there are multiple optima and which means that the cost that occurs in all-hazards protection, i.e. the investment is as effective as protection from both asymptomatic infected persons/infections and natural disaster individually. Although equality is very unlikely to hold in practice. Hence an interior solution is doubtful, as unit costs of such sizes are realistic, and therefore we do not consider this condition further.

None withdraws from a simultaneous game as we use ratio contest success functions which imply that all are in equilibrium. (Equilibrium is defined as a solution from which no unilateral deviation of an agent is possible.) Hence, we can never have e2=e3=0 or I=0 for if this were to happen, then the relevant agencies would have to choose off equilibrium behaviour. Hence the five relevant corner solutions are e1=0, e2=0, e3=0, e1=e2=0, e1=e3=0 as illustrated below

Let us consider the condition when 2) $c_1 + c_2 < c_3$

In this case, we see that the sum of the defender's other two-unit costs is less than the more general all-hazards protection. Solving the first, second, and fourth equations in (3)

$$e_1 = \begin{cases} \sqrt{\frac{pN}{c_1/V}} - N when \frac{p}{c_1/V} \geq N \\ 0 otherwise \end{cases}$$
$$e_2 = \sqrt{\frac{(1-p)*C_1*V}{c_2*[C_1-(1-p)*V]}},$$
$$e_3 = 0,$$

$$I = \sqrt{\frac{(1-p)*V*[C_1-(1-p)*V]}{C_1*c_2}}$$

$$This\ exists\ if$$
$$C_1 \geq (1-p)*V \text{ -------------------------------(4)}$$

And the respective utilities are

$$U_D = [p * (\frac{1}{1 + \frac{N}{\sqrt{\frac{pNV}{c_1}} - N}}) + (1-p) * (\frac{1}{1 + \frac{I * c_2 * [C_1 - (1-p) * V]}{\sqrt{(1-p) * C_1 * V}}})] * V - c_1 * (\sqrt{\frac{pNV}{c_1}} - N) - c_2 * (\frac{\sqrt{(1-p) * C_1 * V}}{I * c_2 * [C_1 - (1-p) * V]})$$

$$when \frac{p}{c_1/V} \geq N$$

$$U_D = \frac{p}{N} + (1-p) * V/I - c_2 * (\frac{\sqrt{(1-p) * C_1 * V}}{I * c_2 * [C_1 - (1-p) * V]})$$

$Otherwise$

$$U_{\text{inf}} = -(1-p) * (\frac{V}{1 + \frac{I * c_2 * [C_1 - (1-p) * V]}{\sqrt{(1-p) * C_1 * V}}}) - C_1 * I$$

In this case, the defender protects against the natural disaster, except when the natural disaster is not sufficiently damaging to justify the cost or investment involved in protection. If the natural disaster is an agent that acts intentionally, N would have been chosen at a level where effort e1 would always be positive in equilibrium. However, since N is an independent variable and exogenously preferred, e1=0 is possible Further e1=e3=0 is also possible however e2=e3=0 is not possible due to the reasons that the asymptomatic infected person's intentions are not clearly known and hence cannot be compared of that with the natural disaster.

We observe that e1 is inverse U shaped in N and that e2 /I = (C1/V)*(c2/V) Thus, the defender doesn't protect from a natural disaster in case threat is too small, but also when risk is massive as it cannot be countered cost-effectively. By contrast, the defender always invests in protection from asymptomatic infected persons, since withdrawal means losing himself when the asymptomatic infected person invests an arbitrarily small effort. Thus, if the healthy person doesn't take actions to defend himself against the asymptomatically infected persons, it increases the threat to lose his life.

Now, consider the condition 3)$c_1 + c_2 > c_3$.

In this case, the sum of the defender's other two-unit costs has a higher cost than a general all hazards cost. This means that either e1=0 or e2 =0 at equilibrium. When e1=0, then e3 is applied against the disaster. For convenience, let s1 = e1 + e3, and s2 = e2 + e3. Then, solving the various equations, i.e. second, third, and fourth equations in (3)

$$c_1 + c_2 > c_3$$

$$e_1 = 0$$

$$s_1 = e_1 + e_3 = \sqrt{\frac{pNV}{c_2 - c_3}} - N$$

$$s_2 = e_2 + e_3 = \frac{(1-p) * V * C_1}{c^2{}_2 * (1 + \frac{C_1}{c_2})^2}$$

$$I = \frac{(1-p)*V}{c_2*(1+\frac{C_1}{c_2})^2} \text{ -----(5)}$$

When s1≤ s2, then equation (5) implies that the defender, i.e. the healthy person invests in all-hazards protection at level s1, and e2 provides the remaining needed defence against the asymptotic infected person. If c2 is sufficiently large, then e2=0. This can occur when c2 < c3, and means that all-hazards protection takes care of both the disaster and the asymptotic infected person.

Though we do not take efforts to analyse this case here, solving it amounts to setting e1=e2=0, and hence we solve the third and fourth equations in (3) concerning e3 and I, which gives a higher than second-order equation. By contrast, when c1 + c2 > c3 but e2 =0, then e3 is applied against asymptotically infected person. Solving the various equations of 3, i.e. the first, third, and fourth equations in (3) gives

$$c_1 + c_2 > c_3$$
$$e_2 = 0$$

$$s_1 = e_1 + e_3 = \sqrt{\frac{pNV}{c_1}} - N$$

$$s_2 = e_2 + e_3 = \frac{(1-p) * C_1 * V}{(c_1 - c_3)^2 * (\frac{c_3 - c_1}{C_1} - 1)^2}$$

$$I = \frac{(1-p)*V}{(c_3-c_1)*(\frac{c_1-c_3}{C_1}+1)^2} \text{------------(6)}$$

When s2 ≤ s1, equation (6) implies that the defender invests in all-hazards protection at level s2, and e1 provides the remaining much-needed defence against the natural disaster. If c1 is relatively large, then we will have effort e1 =0. This can occur when cost c1 < c3, and means that all-hazards protection takes care of both the disaster and the asymptomatic infected person. This is similar to the condition faced before where we obtain a higher-order equation and thus can neglect it.

IV. USEFUL PROPOSITIONS

Proposition 1:

The healthy person, i.e. defender, may have advantages in moving first, than in the simultaneous game because it may be associated with a sufficiently low unit cost of defence, as well as protection of his life as the defender can deter the asymptomatic person altogether.

Proposition 2:

In case of the asymptomatic infected moving first, than in a simultaneous game, the healthy person may be deterred to incur an effort which may result in zero utility and hence a loss of his life.

IV. CONCLUSIONS

In this paper, we have considered two threats, threats from natural disaster and threats from an asymptomatic infected person in an epidemic, from which a healthy person, i.e. a defender can protect through three different kinds of investments. These protective defences are against the disaster created by nature only, against asymptomatic infected person/ infection only, or against all types of hazards. The healthy person, i.e., defender makes trade-offs between these three investments, under the assumption that how freely the asymptomatic person moves around in an epidemic, a fixed probability of occurrence a natural disaster of the magnitude that is independent, and the value of the healthy person, i.e. defender, one seeks to protect. We have considered the most probable realistic situation when the agents move simultaneously though in future we can model when the defender moves first or last. When an all-hazards type of protection is relatively cheap, it jointly protects against both the natural disaster as well as against asymptomatic infected person, with no need for either protection for a purely natural disaster or absolute asymptomatic infected/ infection protection. As the cost of all-hazards protection increases above a particular level, either pure protection against natural disaster or pure protection against an infected person (but not both) joins in as contrast to all-hazards protection. As the unit cost of all-hazards protection increases further and higher, it eventually reaches a point, at which point protection against all-hazards falls to zero. However, one can also conclude that the history of large-scale natural disasters is much longer or at least equal to the history of asymptomatic infection. Though this work may indicate that the expenditures increase linearly in the investments, future research can verify the non-linear dependence of spending on the investments

APPENDIX 1

$$\frac{\partial^2 U_{\text{inf}}}{\partial I^2} = \frac{2*(1-p)*(e_2+e_3)*V}{(e_2+e_3+I)^3} > 0$$

i.e

satisfiesconvexity

Hence the function Uinf has a minimum

If e1= x, e2=y and e3=z, c1=a, c2=b, c3= c for convenience, calculating the hessian of the utility function of the defender we have

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$$Hessian\left(\left[\frac{(x+z)}{(x+z+N)}\cdot p+\frac{(y+z)}{(y+z+I)}\cdot(1-p)\right]\cdot V - a\cdot x - b\cdot y - c\cdot z, [x, y, z]\right)$$

Whose result is given below

$$\left[\left[\left[-\frac{2p}{(x+z+N)^2}+\frac{2(x+z)p}{(x+z+N)^3}\right]V+[0], 0, \left[-\frac{2p}{(x+z+N)^2}+\frac{2(x+z)p}{(x+z+N)^3}\right]V+[0]\right], \left[0, \left[-\frac{2(1-p)}{(y+z+I)^2}+\frac{2(y+z)(1-p)}{(y+z+I)^3}\right]V+[0], \left[-\frac{2(1-p)}{(y+z+I)^2}+\frac{2(y+z)(1-p)}{(y+z+I)^3}\right]V+[0]\right], \left[\left[-\frac{2p}{(x+z+N)^2}+\frac{2(x+z)p}{(x+z+N)^3}\right]V+[0], \left[-\frac{2(1-p)}{(y+z+I)^2}+\frac{2(y+z)(1-p)}{(y+z+I)^3}\right]V+[0], \left[-\frac{2p}{(x+z+N)^2}+\frac{2(x+z)p}{(x+z+N)^3}-\frac{2(1-p)}{(y+z+I)^2}+\frac{2(y+z)(1-p)}{(y+z+I)^3}\right]V+[0]\right]\right]$$

To show that above Hessian H is negative semi-definite, it is sufficient to show
the following three conditions:

$$1)|H_{11}| <= 0$$

$$2)\begin{vmatrix} H_{11} H_{12} \\ H_{21} H_{22} \end{vmatrix} \geq 0$$

$$3)\begin{vmatrix} H_{11} H_{12} H_{13} \\ H_{21} H_{22} H_{23} \\ H_{31} H_{32} H_{33} \end{vmatrix} \equiv Hand|H| \leq 0|$$

From 1) We have H11 as

$$\frac{-2*p*N*V}{e_1+e_3+N}$$

Which is negative
From 2 we have

$$\begin{vmatrix} H_{11} H_{12} \\ H_{21} H_{22} \end{vmatrix} = [\frac{2*p*N*V}{(e_1+e_3+N)^3}] * \frac{[(1-p)*\frac{C_i}{V}]^{1/2}}{4*(e_2+e_3)^{3/2}}$$

Which is positive
Finally, from 3 we have

$$\begin{vmatrix} H_{11} H_{12} H_{13} \\ H_{21} H_{22} H_{23} \\ H_{31} H_{32} H_{33} \end{vmatrix} = -\frac{p^2*N^2*V^3}{(e_1+e_3+N)^6} * [\frac{((1-p)*C_i/V)^{1/2}}{(e_2+e_3)^{3/2}}]$$

Which is negative
Hence the Hessian is negative semi indefinite.

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