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Adaptive Fuzzy Control for Fractional-Order Nonlinear System with Unknown Dead Zone

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***Abstract*—In this paper, an adaptive fuzzy backstepping dynamic surface control (DSC) scheme is proposed for fractional-order nonlinear systems (FONSs) in strict-feedback form with external disturbances and unknown dead zone. Fuzzy logic systems (FLSs) are utilized to approximate unknown nonlinear functions. By utilizing the DSC to avoid the inherent problem of 'explosion of complexity' in the backstepping technique, at the same time, constructing the dead zone inverse to compensate for the dead zone effect. Finally, the raised method can ensure that all the signals of the fractional-order closed-loop system are bounded, and the tracking error becomes arbitrarily small.**

Keywords—Fractional-order system, backstepping, DSC, unknown dead zone

I. INTRODUCTION

Fractional-order (FO) calculus is an ancient and novel subject. As early as the founding of fractional calculus, some scholars began to consider its meaning. Due to the lack of practical application drive, it develops very slowly. In the long history of nearly 300 years, it has been regarded as a profound pure theoretical problem in the field of pure mathematics. Fractional calculus is abstruse and abstract, and its development has been difficult. It was not until the 1980s that scholars discovered that there are systems in nature and engineering where the order of calculus is not integer. Because of its ability to describe the system model more accurately, fractional calculus has become a powerful tool for studying fractal geometry and fractional dynamics, such as fractional-order uncertain viscoelastic models [1], etc.

With the emergence of FO system, FO control is also developed. However, the research on fractional control is still in its infancy, and there are still many areas that need to be improved. Backstepping control technique has been a powerful method to deal with non-smooth nonlinear system. From then on, adaptive backstepping control technique has received widespread concern. FLSs and neural networks (NNs) have the characteristics of approximating unknown nonlinear continuous functions. The combination of backstepping control with adaptive fuzzy or NNs control are applied to integer-order nonlinear systems (IONSs), and many results have been achieved. For example, the authors in [2] designed an adaptive NNs-based decentralized control approach for uncertain switched interconnected nonlinear system in nonstrict-feedback form with the prescribed performance. However, because FO calculus is more complicated than integer-order (IO) calculus, some results obtained in IONSs may not be directly applied to FONSs. Moreover, in comparison with the IO controller, the FO controller possesses higher design freedom and better robustness and transient performance. So, in recent years, some scholars have begun to turn their eyes from IONSs to FONSs. Fractional calculus has been paid more and more attention. After continuous research, many results have been achieved. For example, in [3], an adaptive fuzzy backstepping control method put forward strict-feedback FONSs with unknown external disturbances. Nevertheless, in the use of backstepping control technique, some nonlinear functions need to be derived repeatedly, which leads to the issue of 'explosion of complexity'. [4] is combined DSC with backstepping control to avoid the issue. Another point that needs to be noted is that in the actual control task, some components expose some subtle problems due to their own objective factors, such as non-smooth nonlinear. Dead zone is a common non-smooth nonlinearity, which exists may severely restrict system properties and even do great damage to system stability. In order to handle the system with unknown dead zone, a method was raised in [5]. A robust adaptive control put forward IONSs with unknown dead zone without constructing the dead zone inverse and it assumes that the slopes of dead zone must be equal without considering unequal situation.

By the aforementioned observations, in the article, the main contribution is as follows. Considering the slopes and the breakpoints of the dead zone are not equal, that is, the asymmetric dead zone, the dead zone inversion method is used to handle this problem. Utilizing the dead zone inverse and the compensation term to compensate for the influence of the dead zone, external disturbances and approximation errors in the FONSs. By bringing in DSC, the calculation is simplified. Finally, it proves that the adaptive fuzzy DSC method can ensure the stability of the FONSs and has good tracking performance.

The article structure is as follows. In Section II, we bring in some preliminaries, formulations and system descriptions. The detailed design steps are described in Section III. Some conclusions are given in Section IV.

II. PROBLEM FORMULATIONS AND PRELIMINARIES

A. System descriptions

Consider the following FONSs in strict-feedback form:

$$\begin{aligned} {}_0^C D_t^\alpha x_1 &= f_1(x_1) + x_2 + d_1(t) \\ {}_0^C D_t^\alpha x_i &= f_i(\bar{x}_i) + x_{i+1} + d_i(t) \qquad 2 \le i \le n-1 \\ {}_0^C D_t^\alpha x_n &= f_n(\bar{x}_n) + D(u) + d_n(t) \\ y &= x_1 \end{aligned} \tag{1}$$

where $0<\alpha<1$ is the system order, $\bar{x}_i=[x_1,x_2,...x_i]^{\mathrm{T}}\in R^i$ is the system state vector, and $y\in R$ is the system output. $f_i(.)$, $(i=1,2,...,n)$ is the smooth unknown nonlinear function. $d_i(t)$, $(i=1,2,...,n)$ is the unknown but bounded external disturbance, $|d_i(t)|\le \bar{d}_i$, $\bar{d}_i$ is an unknown positive constant. Let y_d be a desired signal. y_d , ${}_0^C D_t^{\alpha} y_d$ and ${}_0^C D_t^{2\alpha} y_d$ are smooth, available and bounded. $D(u)\in R$ is an asymmetrical dead zone output. According to [11], its definition as following

$$D(u)=\begin{cases} m_r(u-b_r), & u\ge b_r \\ 0, & b_l<u<b_r \\ m_l(u-b_l), & u\le b_l \end{cases} \tag{2}$$

where b_r and b_l are the breakpoints, m_r and m_l represent the slopes, respectively. The coefficients $m_r>0$, $b_r\ge 0$, $m_l>0$ and $b_l\le 0$ are unknown constants and $b_r\ne|b_l|$, $m_r\ne m_l$.

The inverse of the dead zone nonlinearity can be shown as follows

$$u=D^{-1}(v)=\frac{v+\hat{b}_{r,m}}{\hat{m}_r}\omega+\frac{v+\hat{b}_{l,m}}{\hat{m}_l}(1-\omega) \tag{3}$$

where v is a designed control law. $\hat{b}_{r,m}$, $\hat{b}_{l,m}$, $\hat{m}_r$ and $\hat{m}_l$ are the estimates of the dead zone parameters $m_r b_r$, $m_l b_l$, m_r and m_l , respectively.

Define ω as

$$\omega=\begin{cases} 1 & v\ge 0 \\ 0 & v<0 \end{cases} \tag{4}$$

Substituting (3), (4) into (2), we have

$$D(u)-v=(\frac{v+\hat{b}_{r,m}}{\hat{m}_r}\tilde{m}_r-\tilde{b}_{r,m})\omega+(\frac{v+\hat{b}_{l,m}}{\hat{m}_l}\tilde{m}_l-\tilde{b}_{l,m})(1-\omega)+\delta \tag{5}$$

where $\tilde{b}_{r,m}=b_{r,m}-\hat{b}_{r,m}$, $\tilde{m}_l=m_l-\hat{m}_l$, $\tilde{b}_{l,m}=b_{l,m}-\hat{b}_{l,m}$ and $\tilde{m}_r=m_r-\hat{m}_r$ are the parameter errors. $\delta=-m_r o_r(u-b_r)-m_l o_l(u-b_l)$ satisfies $|\delta|\le\delta^*$ with δ^* being unknown positive constant.

where

$$o_r=\begin{cases} 1 & 0\le u\le b_r \\ 0 & others \end{cases} \text{ and } o_l=\begin{cases} 1 & b_l\le u\le 0 \\ 0 & others \end{cases} \tag{6}$$

B. *Preliminaries*

Definition 1 [6]: Suppose that $F:[t_0,+\infty)\to R$ is a continuously differentiable function, its Caputo fractional order differentiable with order α ($\alpha\in(\omega,\omega-1)$, $\omega\in N^+$) is defined as:

$${}_0^C D_t^{\alpha} F(t)=\frac{1}{\Gamma(\omega-\alpha)}\int_0^t \frac{F^{(\omega)}(\tau)}{(t-\tau)^{\alpha+1-\omega}}d\tau \tag{7}$$

where $\Gamma(\bullet)=\int_0^{+\infty}\tau^{\bullet-1}e^{-\tau}d\tau$ denotes the Euler's Gamma function, satisfying $\Gamma(1)=1$.

Definition 2 [6]: The Mittag-Leffler function with two parameters can be defined as:

$$E_{\alpha,\phi}(\gamma)=\sum_{j=0}^{\infty}\frac{\gamma^j}{\Gamma(j\alpha+\phi)} \tag{8}$$

where α , $\phi>0$ are constants, γ is a complex number.

Lemma 1 [6]: For two real numbers $\alpha\in(0,1)$, $\xi\in(\pi\alpha/2,\min\{\pi,\pi\alpha\})$ and a complex number β , the following equation holds for all integer $n\ge 1$:

$$E_{\alpha,\beta}(\zeta)=-\sum_{j=1}^{n}\frac{1}{\Gamma(\beta-\alpha j)\zeta^j}+\circ(\frac{1}{|\zeta|^{n+1}}) \tag{9}$$

when $|\zeta|\to\infty$, $v\le|\arg(\zeta)|\le\pi$.

Lemma 2 [6]: Let α satisfy $\alpha\in(0,2)$ and β be an arbitrary real number. For an arbitrary positive constant δ such that $\delta\in(\pi\alpha/2,\min\{\pi,\pi\alpha\})$, then one has

$$E_{\alpha,\beta}(\zeta)\le\frac{\lambda}{1+|\zeta|} \tag{10}$$

where $\lambda>0$, $|\zeta|\ge 0$, and $\delta\le|\arg(\zeta)|\le\pi$.

Lemma 3 [7]: Let $x=0$ be an equilibrium point of the fractional-order nonlinear system ${}_0^C D_t^{\alpha} x(t)=f(t,x(t))$, where $f(\cdot)$ is a Lipsichiz continuous. If there exist a Lyapunov function $V(t,x(t))$ and several class-κ functions g_k , $k=1,2,3$, such that inequalities hold,

$$g_1(\|x(t)\|)\le V\le g_2(\|x_2\|),\ {}_0^C D_t^{\alpha} V\le -g_3(\|x(t)\|) \tag{11}$$

thus ${}_0^C D_t^{\alpha} x(t)=f(t,x(t))(\alpha\in(0,1))$ is asymptotically stable.

Lemma 4 [8]: For all $\varpi>0$ and $S\in R$, the following inequality will hold:

$$0 \le |S| - \frac{S^2}{\sqrt{S^2+\varpi^2}} < \varpi \tag{12}$$

Lemma 5 [9]: Let $x(t) \in R$ be a smooth function. For all $t \ge t_0$, it satisfies

$$\frac{1}{2}\,{}_0^C D_t^\alpha (x^{\mathrm T}(t)x(t)) \le x^{\mathrm T}(t)\,{}_0^C D_t^\alpha x(t) \tag{13}$$

Lemma 6 [10]: Let $f(x)$ be a continuous function defined on a compact set Ω. Then for any constant $\varepsilon > 0$, there exists an FLS such as

$$\sup_{x\in\Omega} |f(x) - \theta^{\mathrm T}\varphi(x)| \le \varepsilon \tag{14}$$

From Lemma 6, we can use FLS to approximate unknown function $f_i(\bar{x}_i)$ as

$$\hat{f}_i(\bar{x}_i \mid \theta_i) = \theta_i^{\mathrm T}\varphi_i(\bar{x}_i) \tag{15}$$

Due to [10], define the optimal parameter vector θ_i^* as:

$$\theta_i^* = \arg\min_{\theta_i\in\Omega_i}[\sup_{\bar{x}_i\in U} |\hat{f}_i(\bar{x}_i \mid \theta_i) - f_i(\bar{x}_i)|] \tag{16}$$

where U and Ω_i are compact sets for $\bar{x}_i$ and θ_i, respectively. The minimum approximation errors ε_i are defined as

$$\varepsilon_i = f_i(\bar{x}_i) - \hat{f}_i(\bar{x}_i \mid \theta_i^*) \tag{17}$$

where ε_i satisfies $|\varepsilon_i| \le \varepsilon_i^*$, ε_i^* is an unknown positive constant.

III. Adaptive Fuzzy Control Design and Stability Analysis

A. *Adaptive Fuzzy Control Design*

Consider the coordinate transformation as follows

$$S_1 = x_1 - y_d,\ S_i = x_i - \xi_{i-1},\ \rho_{i-1} = \xi_{i-1} - \alpha_{i-1} \tag{18}$$

where S_1 is the tracking error, S_i, $i = 2,...,n$ is the surface error, ξ_i is new intermediate variable which can be gained by making the intermediate control function α_i through the fractional-order dynamic surface filter, ρ_i is the fractional-order dynamic surface filter output error.

Step1: From (1), (17) and (18), the derivative of S_1 is

$${}_0^C D_t^\alpha S_1 = S_2 + \xi_1 + d_1 - {}_0^C D_t^\alpha y_d + \tilde{\theta}_1^{\mathrm T}\varphi_1(x_1) + \theta_1^{\mathrm T}\varphi_1(x_1) + \varepsilon_1 \tag{19}$$

where θ_1 is the estimation of θ_1^*, $\tilde{\theta}_1 = \theta_1^* - \theta_1$ is the parameter error.

Consider the following Lyapunov function:

$$V_1 = \frac{1}{2}S_1^2 + \frac{1}{2\gamma_1}\tilde{\theta}_1^{\mathrm T}\tilde{\theta}_1 + \frac{1}{2\bar{\gamma}_1}\tilde{\Delta}_1^2 \tag{20}$$

where $\gamma_1 > 0$ and $\bar{\gamma}_1 > 0$ are design constants, $\Delta_1 = \varepsilon_1^* + \bar{d}_1$ and $\hat{\Delta}_1$ is the estimation of Δ_1, $\tilde{\Delta}_1 = \Delta_1 - \hat{\Delta}_1$ is the parameter error.

From (19) and (20), the time derivative of V_1 is

$$\begin{aligned}{}_0^C D_t^\alpha V_1 \le{}& S_1(S_2 + \xi_1 - {}_0^C D_t^\alpha y_d + \tilde{\theta}_1^{\mathrm T}\varphi_1(x_1) + \theta_1^{\mathrm T}\varphi_1(x_1)) \\ &+ |S_1|\Delta_1 - \frac{1}{\gamma_1}\tilde{\theta}_1^{\mathrm T}\,{}_0^C D_t^\alpha \theta_1 - \frac{1}{\bar{\gamma}_1}\tilde{\Delta}_1\,{}_0^C D_t^\alpha \hat{\Delta}_1\end{aligned} \tag{21}$$

Through Lemma 4, we can get

$$|S_1|\Delta_1 \le \Delta_1\varpi + \Delta_1 S_1^2 \Big/ \sqrt{S_1^2 + \varpi^2} \tag{22}$$

Design the adaptation laws and the intermediate control function α_1 as follow

$$\begin{cases} {}_0^C D_t^\alpha \theta_1 = \gamma_1 S_1 \varphi_1(x_1) - \sigma_1\theta_1 \\ {}_0^C D_t^\alpha \hat{\Delta}_1 = \bar{\gamma}_1 S_1^2 \Big/ \sqrt{S_1^2+\varpi^2} - \bar{\sigma}_1\hat{\Delta}_1 \end{cases} \tag{23}$$

$$\alpha_1 = -c_1 S_1 - \theta_1^{\mathrm T}\varphi_1(x_1) - \hat{\Delta}_1 S_1 \Big/ \sqrt{S_1^2+\varpi^2} + {}_0^C D_t^\alpha y_d \tag{24}$$

where σ_1 and $\bar{\sigma}_1$ are positive design constants. $\hat{\Delta}_1 S_1 \Big/ \sqrt{S_1^2+\varpi^2}$ is a compensation term used to compensate for ε_1^* and $\bar{d}_1$, as well as in the following steps.

Substituting (22)-(24), adding and subtracting $S_1\alpha_1$ into (21), we can get

$${}_0^C D_t^\alpha V_1 \le -c_1 S_1^2 + S_1(S_2 + \xi_1 - \alpha_1) + \Delta_1\varpi + \frac{\sigma_1}{\gamma_1}\tilde{\theta}_1^{\mathrm T}\theta_1 + \frac{\bar{\sigma}_1}{\bar{\gamma}_1}\tilde{\Delta}_1\hat{\Delta}_1 \tag{25}$$

Let the intermediate control function α_1 pass through a fractional-order dynamic surface filter with a time constant τ_1 to get ξ_1.

$$\tau_1\,{}_0^C D_t^\alpha \xi_1 = -\rho_1 - \frac{\tau_1 \hat{M}_1^2 \rho_1}{\sqrt{\hat{M}_1^2\rho_1^2 + \varpi^2}} - \tau_1 S_1,\ \xi_1(0) = \alpha_1(0) \tag{26}$$

Then we have

$$ {}_0^C D_t^\alpha \rho_1 = -\frac{\rho_1}{\tau_1} - \frac{\hat{M}_1^2 \rho_1}{\sqrt{\hat{M}_1^2 \rho_1^2 + \varpi^2}} - S_1 + G_1(\cdot) \tag{27} $$

where $G_1(\cdot)$ is a continuous function. From the existing results [4], there is an unknown positive constant M_1 such that $|G_1(\cdot)| \le M_1$ in a given compact set Ψ_1. $\hat{M}_1$ is the estimation of M_1, $\tilde{M}_1 = M_1 - \hat{M}_1$ is the parameter error.

Step i: From (1), (17) and (18), the derivative of S_i is

$$ \begin{aligned} {}_0^C D_t^\alpha S_i &= S_{i+1} + \tilde{\theta}_i^{\mathrm{T}} \varphi_i(\bar{x}_i) + \xi_i + \theta_i^{\mathrm{T}} \varphi_i(\bar{x}_i) + \varepsilon_i + d_i \\ &\quad + \frac{\rho_{i-1}}{\tau_{i-1}} + \frac{\hat{M}_{i-1}^2 \rho_{i-1}}{\sqrt{\hat{M}_{i-1}^2 \rho_{i-1}^2 + \varpi^2}} + S_{i-1} \end{aligned} \tag{28} $$

where θ_i is the estimation of θ_i^*, $\tilde{\theta}_i = \theta_i^* - \theta_i$ is the parameter error.

Consider the following Lyapunov function:

$$ V_i = V_{i-1} + \frac{1}{2} S_i^2 + \frac{1}{2\gamma_i} \tilde{\theta}_i^{\mathrm{T}} \tilde{\theta}_i + \frac{1}{2\bar{\gamma}_i} \tilde{\Delta}_i^2 + \frac{1}{2\bar{\bar{\gamma}}_{i-1}} \tilde{M}_{i-1}^2 + \frac{1}{2} \rho_{i-1}^2 \tag{29} $$

where $\bar{\bar{\gamma}}_{i-1}$, $\bar{\gamma}_i$ and γ_i are positive design constants, $\Delta_i = \varepsilon_i^* + \bar{d}_i$ and $\hat{\Delta}_i$ is the estimation of Δ_i, $\tilde{\Delta}_i = \Delta_i - \hat{\Delta}_i$ is the parameter error.

From (28) and (29), the time derivative of V_i is

$$ \begin{aligned} {}_0^C D_t^\alpha V_i &\le {}_0^C D_t^\alpha V_{i-1} + S_i (S_{i+1} + \tilde{\theta}_i^{\mathrm{T}} \varphi_i(\bar{x}_i) + \xi_i + \theta_i^{\mathrm{T}} \varphi_i(\bar{x}_i) \\ &\quad + \frac{\rho_{i-1}}{\tau_{i-1}} + \frac{\hat{M}_{i-1}^2 \rho_{i-1}}{\sqrt{\hat{M}_{i-1}^2 \rho_{i-1}^2 + \varpi^2}} + S_{i-1}) + |S_i| \Delta_i \\ &\quad + \rho_{i-1} (-\frac{\rho_{i-1}}{\tau_{i-1}} - \frac{\hat{M}_{i-1}^2 \rho_{i-1}}{\sqrt{\hat{M}_{i-1}^2 \rho_{i-1}^2 + \varpi^2}} - S_{i-1}) + \rho_{i-1} G_{i-1}(\cdot) \\ &\quad - \frac{1}{\gamma_i} \tilde{\theta}_i^{\mathrm{T}} \, {}_0^C D_t^\alpha \theta_i - \frac{1}{\bar{\gamma}_i} \tilde{\Delta}_i \, {}_0^C D_t^\alpha \hat{\Delta}_i - \frac{1}{\bar{\bar{\gamma}}_{i-1}} \tilde{M}_{i-1} \, {}_0^C D_t^\alpha \hat{M}_{i-1} \end{aligned} \tag{30} $$

Through Lemma 4, we can get

$$ \begin{cases} |S_i| \Delta_i \le \Delta_i S_i^2 \big/ \sqrt{S_i^2 + \varpi^2} + \Delta_i \varpi \\ \rho_{i-1} G_{i-1}(\cdot) \le |\rho_{i-1}| M_{i-1} \\ \qquad \le \dfrac{\hat{M}_{i-1}^2 \rho_{i-1}^2}{\sqrt{\hat{M}_{i-1}^2 \rho_{i-1}^2 + \varpi^2}} + \varpi + |\rho_{i-1}| \tilde{M}_{i-1} \end{cases} \tag{31} $$

Design the adaptation laws and the intermediate control function α_i are as follow

$$ \begin{aligned} \alpha_i &= -c_i S_i - 2S_{i-1} - \theta_i^{\mathrm{T}} \varphi_i(\bar{x}_i) - \hat{\Delta}_i S_i \big/ \sqrt{S_i^2 + \varpi^2} \\ &\quad - \rho_{i-1} / \tau_{i-1} - \hat{M}_{i-1}^2 \rho_{i-1} \big/ \sqrt{\hat{M}_{i-1}^2 \rho_{i-1}^2 + \varpi^2} \end{aligned} \tag{32} $$

$$ \begin{cases} {}_0^C D_t^\alpha \theta_i = S_i \gamma_i \varphi_i(\bar{x}_i) - \sigma_i \theta_i \\ {}_0^C D_t^\alpha \hat{\Delta}_i = \bar{\gamma}_i S_i^2 \big/ \sqrt{S_i^2 + \varpi^2} - \bar{\sigma}_i \hat{\Delta}_i \\ {}_0^C D_t^\alpha \hat{M}_{i-1} = -\bar{\bar{\sigma}}_i \hat{M}_{i-1} + \bar{\bar{\gamma}}_i |\rho_{i-1}| \end{cases} \tag{33} $$

where σ_i, $\bar{\sigma}_i$ and $\bar{\bar{\sigma}}_{i-1}$ are positive design constants.

Substituting (31)-(33) into (30), adding and subtracting $S_i \alpha_i$, we can get

$$ \begin{aligned} {}_0^C D_t^\alpha V_i &\le -\sum_{j=1}^{i} c_j S_j^2 + \sum_{j=1}^{i} \frac{\sigma_j}{\gamma_j} \tilde{\theta}_j^{\mathrm{T}} \theta_j + (i-1)\varpi + \sum_{j=1}^{i} \frac{\bar{\sigma}_j}{\bar{\gamma}_j} \tilde{\Delta}_j \hat{\Delta}_j \\ &\quad + \sum_{j=1}^{i-1} \frac{\bar{\bar{\sigma}}_j}{\bar{\bar{\gamma}}_j} \tilde{M}_j \hat{M}_j - \sum_{j=1}^{i-1} \frac{\rho_j^2}{\tau_j} + S_i (S_{i+1} + \xi_i - \alpha_i) + \sum_{j=1}^{i} \Delta_j \varpi \end{aligned} \tag{34} $$

Let the intermediate control function α_i pass through a fractional-order dynamic surface filter with time constant τ_i to get ξ_i.

$$ \tau_i \, {}_0^C D_t^\alpha \xi_i = -\rho_i - \frac{\tau_i \hat{M}_i^2 \rho_i}{\sqrt{\hat{M}_i^2 \rho_i^2 + \varpi^2}} - \tau_i S_i, \ \xi_i(0) = \alpha_i(0) \tag{35} $$

Then we have

$$ {}_0^C D_t^\alpha \rho_i = -\frac{\rho_i}{\tau_i} - \frac{\hat{M}_i^2 \rho_i}{\sqrt{\hat{M}_i^2 \rho_i^2 + \varpi^2}} - S_i + G_i(\cdot) \tag{36} $$

where $G_i(\cdot)$ is a continuous function. There is an unknown positive constant M_i such that $|G_i(\cdot)| \le M_i$ in a given compact set Ψ, $\Psi = \Psi_1 \vee \Psi_2 \vee \ldots \vee \Psi_{n-1}$. $\hat{M}_i$ is the estimation of M_i, $\tilde{M}_i = M_i - \hat{M}_i$ is the parameter error.

Step n: From (1), (17) and (18), the derivative of S_n is

$$ \begin{aligned} {}_0^C D_t^\alpha S_n &= (\frac{v + \hat{b}_{r,m}}{\hat{m}_r} \tilde{m}_r - \tilde{b}_{r,m}) \omega + \varepsilon_n + d_n + \tilde{\theta}_n^{\mathrm{T}} \varphi_n(\bar{x}_n) \\ &\quad + (\frac{v + \hat{b}_{l,m}}{\hat{m}_l} \tilde{m}_l - \tilde{b}_{l,m})(1 - \omega) + S_{n-1} + \delta + v \\ &\quad + \frac{\hat{M}_{n-1}^2 \rho_{n-1}}{\sqrt{\hat{M}_{n-1}^2 \rho_{n-1}^2 + \varpi^2}} + \theta_n^{\mathrm{T}} \varphi_n(\bar{x}_n) + \frac{\rho_{n-1}}{\tau_{n-1}} \end{aligned} \tag{37} $$

where θ_n is the estimation of θ_n^*, $\tilde{\theta}_n = \theta_n^* - \theta_n$ is the parameter error.

Consider the following Lyapunov function:

$$ \begin{aligned} V_n &= V_{n-1} + \frac{1}{2} S_n^2 + \frac{1}{2\gamma_n} \tilde{\theta}_n^{\mathrm{T}} \tilde{\theta}_n + \frac{1}{2\bar{\gamma}_n} \tilde{\Delta}_n^2 + \frac{1}{2\bar{\bar{\gamma}}_{n-1}} \tilde{M}_{n-1}^2 \\ &\quad + \frac{1}{2} \rho_{n-1}^2 + \frac{1}{2\chi_1} \tilde{m}_r^2 + \frac{1}{2\chi_2} \tilde{m}_l^2 + \frac{1}{2\chi_3} \tilde{b}_{r,m}^2 + \frac{1}{2\chi_4} \tilde{b}_{l,m}^2 \end{aligned} \tag{38} $$

where χ_1, χ_2, χ_3, χ_4, $\gamma_n, \bar{\gamma}_n$, and $\bar{\bar{\gamma}}_{n-1}$ are positive design constants. $\Delta_n = \varepsilon_n^* + \bar{d}_n + \delta^*$ and $\hat{\Delta}_n$ is the estimation of Δ_n, $\tilde{\Delta}_n = \Delta_n - \hat{\Delta}_n$ is the parameter error.

From (37) and (38), the time derivative of V_n is

$$
\begin{aligned}
{}_0^C D_t^\alpha V_n \le\ & {}_0^C D_t^\alpha V_{n-1} + S_n(v + (\frac{v+\hat{b}_{r,m}}{\hat{m}_r}\tilde{m}_r - \tilde{b}_{r,m})\omega + \theta_n^{\mathrm{T}}\varphi_n(\bar{x}_n) \\
& +(\frac{v+\hat{b}_{l,m}}{\hat{m}_l}\tilde{m}_l - \tilde{b}_{l,m})(1-\omega) + \tilde{\theta}_n^{\mathrm{T}}\varphi_n(\bar{x}_n) + \frac{\rho_{n-1}}{\tau_{n-1}} \\
& + \frac{\hat{M}_{n-1}^2\rho_{n-1}}{\sqrt{\hat{M}_{n-1}^2\rho_{n-1}^2+\varpi^2}} + S_{n-1}) + |S_n|\Delta_n + \rho_{n-1}(-\frac{\rho_{n-1}}{\tau_{n-1}} \\
& - \frac{\hat{M}_{n-1}^2\rho_{n-1}}{\sqrt{\hat{M}_{n-1}^2\rho_{n-1}^2+\varpi^2}} - S_{n-1}) - \frac{1}{\gamma_n}\tilde{\theta}_n^{\mathrm{T}}\, {}_0^C D_t^\alpha \theta_n + |\rho_{n-1}| M_{n-1} \\
& - \frac{1}{\bar{\gamma}_n}\tilde{\Delta}_n\, {}_0^C D_t^\alpha \hat{\Delta}_n - \frac{1}{\bar{\bar{\gamma}}_{n-1}}\tilde{M}_{n-1}\, {}_0^C D_t^\alpha \hat{M}_{n-1} - \frac{1}{\chi_1}\tilde{m}_r\, {}_0^C D_t^\alpha \hat{m}_r \\
& - \frac{1}{\chi_2}\tilde{m}_l\, {}_0^C D_t^\alpha \hat{m}_l - \frac{1}{\chi_3}\tilde{b}_{r,m}\, {}_0^C D_t^\alpha \hat{b}_{r,m} - \frac{1}{\chi_4}\tilde{b}_{l,m}\, {}_0^C D_t^\alpha \hat{b}_{l,m}
\end{aligned}
\tag{39}
$$

Through Lemma 4, we can get

$$
\begin{cases}
|S_n|\Delta_n \le \Delta_n\varpi + \Delta_n S_n^2 \Big/ \sqrt{S_n^2+\varpi^2} \\
|\rho_{n-1}| M_{n-1} \le \dfrac{\hat{M}_{n-1}^2\rho_{n-1}^2}{\sqrt{\hat{M}_{n-1}^2\rho_{n-1}^2+\varpi^2}} + \varpi + |\rho_{n-1}|\tilde{M}_{n-1}
\end{cases}
\tag{40}
$$

Choose the adaptation laws and design actual control law v as follow

$$
\begin{cases}
{}_0^C D_t^\alpha \theta_n = S_n\gamma_n\varphi_n(\bar{x}_n) - \sigma_n\theta_n \\
{}_0^C D_t^\alpha \hat{\Delta}_n = \bar{\gamma}_n S_n^2 \Big/ \sqrt{S_n^2+\varpi^2} - \bar{\sigma}_n\hat{\Delta}_n \\
{}_0^C D_t^\alpha \hat{M}_{n-1} = -\bar{\bar{\sigma}}_{n-1}\hat{M}_{n-1} + \bar{\bar{\gamma}}_{n-1}|\rho_{n-1}| \\
{}_0^C D_t^\alpha \hat{m}_r = \chi_1\omega S_n(v+\hat{b}_{r,m})\big/\hat{m}_r - \zeta_1\hat{m}_r \\
{}_0^C D_t^\alpha \hat{m}_l = S_n\chi_2(1-\omega)(v+\hat{b}_{l,m})\big/\hat{m}_l - \zeta_2\hat{m}_l \\
{}_0^C D_t^\alpha \hat{b}_{r,m} = -S_n\chi_3\omega - \zeta_3\hat{b}_{r,m} \\
{}_0^C D_t^\alpha \hat{b}_{l,m} = -S_n\chi_4(1-\omega) - \zeta_4\hat{b}_{l,m}
\end{cases}
\tag{41}
$$

$$
\begin{aligned}
v =\ & -c_n S_n - 2S_{n-1} - \theta_n^{\mathrm{T}}\varphi_n(\bar{x}_n) - \hat{\Delta}_n S_n \Big/ \sqrt{S_n^2+\varpi^2} \\
& - \rho_{n-1}/\tau_{n-1} - \hat{M}_{n-1}^2\rho_{n-1} \Big/ \sqrt{\hat{M}_{n-1}^2\rho_{n-1}^2+\varpi^2}
\end{aligned}
\tag{42}
$$

where σ_n, $\zeta_4, \bar{\sigma}_n$, ζ_3 $\bar{\bar{\sigma}}_{n-1}$, ζ_2 and ζ_1 are positive design constants. $\hat{\Delta}_n S_n \Big/ \sqrt{S_n^2+\varpi^2}$ is a compensation term used to compensate for the bound of ε_n, d_n and δ.

Substituting (40)-(42) into (39), we can gain

$$
\begin{aligned}
{}_0^C D_t^\alpha V_n \le\ & -\sum_{j=1}^n c_j S_j^2 + \sum_{j=1}^n \frac{\sigma_j}{\gamma_j}\tilde{\theta}_j^{\mathrm{T}}\theta_j + \sum_{j=1}^n \frac{\bar{\sigma}_j}{\bar{\gamma}_j}\tilde{\Delta}_j\hat{\Delta}_j - \sum_{j=1}^{n-1}\frac{\rho_j^2}{\tau_j} \\
& + \sum_{j=1}^{n-1}\frac{\bar{\bar{\sigma}}_j}{\bar{\bar{\gamma}}_j}\tilde{M}_j\hat{M}_j + \sum_{j=1}^n \Delta_j\varpi + (n-1)\varpi \\
& + \frac{\zeta_1}{\chi_1}\tilde{m}_r\hat{m}_r + \frac{\zeta_2}{\chi_2}\tilde{m}_l\hat{m}_l + \frac{\zeta_3}{\chi_3}\tilde{b}_{r,m}\hat{b}_{r,m} + \frac{\zeta_4}{\chi_4}\tilde{b}_{l,m}\hat{b}_{l,m}
\end{aligned}
\tag{43}
$$

By utilizing Young's inequality, we get

$$
\begin{cases}
\tilde{\theta}_j^{\mathrm{T}}\theta_j \le \|\theta_j^*\|^2/2 - \tilde{\theta}_j^{\mathrm{T}}\tilde{\theta}_j/2 \\
\tilde{\Delta}_j\hat{\Delta}_j \le -\tilde{\Delta}_j^2/2 + \Delta_j^2/2 \\
\tilde{M}_j\hat{M}_j \le -\tilde{M}_j^2/2 + M_j^2/2 \\
\tilde{m}_r\hat{m}_r \le -\tilde{m}_r^2/2 + m_r^2/2 \\
\tilde{m}_l\hat{m}_l \le -\tilde{m}_l^2/2 + m_l^2/2 \\
\tilde{b}_{r,m}\hat{b}_{r,m} \le -\tilde{b}_{r,m}^2/2 + b_{r,m}^2/2 \\
\tilde{b}_{l,m}\hat{b}_{l,m} \le -\tilde{b}_{l,m}^2/2 + b_{l,m}^2/2
\end{cases}
\tag{44}
$$

Taking (44) into (43), we have

$$
\begin{aligned}
{}_0^C D_t^\alpha V_n \le\ & -\sum_{j=1}^n c_j S_j^2 - \sum_{j=1}^n \frac{\sigma_j}{2\gamma_j}\tilde{\theta}_j^{\mathrm{T}}\tilde{\theta}_j - \sum_{j=1}^n \frac{\bar{\sigma}_j}{2\bar{\gamma}_j}\tilde{\Delta}_j^2 \\
& - \sum_{j=1}^{n-1}\frac{\rho_j^2}{\tau_j} - \sum_{j=1}^{n-1}\frac{\bar{\bar{\sigma}}_j}{2\bar{\bar{\gamma}}_j}\tilde{M}_j^2 - \frac{\zeta_1}{2\chi_1}\tilde{m}_r^2 \\
& - \frac{\zeta_2}{2\chi_2}\tilde{m}_l^2 - \frac{\zeta_3}{2\chi_3}\tilde{b}_{r,m}^2 - \frac{\zeta_4}{2\chi_4}\tilde{b}_{l,m}^2 + \eta \\
\le\ & -cV_n + \eta
\end{aligned}
\tag{45}
$$

where $c = \min\{2c_j, 2/\tau_j, \zeta_1, \zeta_2, \zeta_3, \zeta_4, \sigma_j, \bar{\sigma}_j, \bar{\bar{\sigma}}_j\}$, $j = 1...n$,
$\eta = \sum_{j=1}^n \Delta_j\varpi + (n-1)\varpi + \sum_{j=1}^n \bar{\sigma}_j\Delta_j^2/2\bar{\gamma}_j + \zeta_1 m_r^2/2\chi_1$
$+\sum_{j=1}^{n-1}\bar{\bar{\sigma}}_j M_j^2/2\bar{\bar{\gamma}}_j + \sum_{j=1}^n \sigma_j\|\theta_j^*\|^2/2\gamma_j + \zeta_2 m_l^2/2\chi_2$
$+\zeta_3 b_{r,m}^2/2\chi_3 + \zeta_4 b_{l,m}^2/2\chi_4$.

B. Stability Analysis

Theorem 1: For fractional-order strict-feedback nonlinear system (1), the actual controller (42), the intermediate control function (23) and (32) and the parameter adaptation laws (24), (33) and (41), guarantee that all signals in the closed-loop system are bounded, and the system output can track the desired signal.

There exists a positive time-varying parameter $\Phi(t)$, combining with (45), we can get

$$
{}_0^C D_t^\alpha V_n + \Phi(t) = -cV_n + \eta \tag{46}
$$

Taking Laplace transform on (46) yields

$$
V_n(s) = \frac{s^{\alpha-1}V_n(0)}{s^\alpha + c} + \frac{s^{\alpha-(\alpha+1)}\eta}{(s^\alpha + c)} - \frac{\Phi(s)}{s^\alpha + c} \tag{47}
$$

Using the inverse Laplace transform on (47), we can obtain

$$V_n(t) = E_{\alpha,1}(-ct^{\alpha})V_n(0) + t^{\alpha}E_{\alpha,\alpha+1}\left(-ct^{\alpha}\right)\eta - \Phi(t) * t^{\alpha-1}E_{\alpha,\alpha}(-ct^{\alpha}) \quad (48)$$

where * is the convolution operator. We consider the last term of (48), we know $\Phi(t)$ and $t^{\alpha-1}E_{\alpha,\alpha}(-ct^{\alpha})$ are not negative functions, so $\Phi(t)*t^{\alpha-1}E_{\alpha,\alpha}(-ct^{\alpha}) \geq 0$. Then we can get

$$V_n(t) \leq E_{\alpha,1}(-ct^{\alpha})V_n(0) + t^{\alpha}E_{\alpha,\alpha+1}\left(-ct^{\alpha}\right)\eta \quad (49)$$

It worth noting that $\arg(-ct^{\alpha}) = -\pi$, $|-ct^{\alpha}| \geq 0$ for all $t \geq 0$ and $\alpha \in (0,2)$. From Lemma 2, there must be a positive constant λ such that

$$|E_{\alpha,1}(-ct^{\alpha})| \leq \lambda / 1 + ct^{\alpha} \quad (50)$$

with $t \to \infty$, we can get

$$\lim_{t\to\infty} E_{\alpha,1}(-ct^{\alpha})V_n(0) = 0 \quad (51)$$

Thus, there must exist a time constant $t_1 > 0$, for arbitrary $t > t_1$ and every $\upsilon_1 > 0$, we can obtain

$$E_{\alpha,1}(-ct^{\alpha})V_n(0) \leq \upsilon_1 \quad (52)$$

Moreover, we use Lemma1 and let $m = 1$, we gain

$$E_{\alpha,\alpha+1}(-ct^{\alpha}) = 1/(\Gamma(1)ct^{\alpha}) + \circ(1/|ct^{\alpha}|^2) \quad (53)$$

According to $\Gamma(1) = 1$, we can regain (53)

$$t^{\alpha}E_{\alpha,\alpha+1}\left(-ct^{\alpha}\right)\eta \leq \eta/c + t^{\alpha}\eta \circ (1/|ct^{\alpha}|^2) \quad (54)$$

There exists a time constant $t_2 > 0$, for arbitrary $t > t_2$ and every $\upsilon_2 > 0$ yields

$$t^{\alpha}\eta \circ (1/|ct^{\alpha}|^2) \leq \upsilon_2 \quad (55)$$

Besides, for every $t_3 > 0$, we appropriately adjust the design parameters to get $\eta/c \leq \upsilon_3$. Therefore, we have

$$t^{\alpha}E_{\alpha,\alpha+1}\left(-ct^{\alpha}\right)\eta \leq \upsilon_2 + \upsilon_3 \quad (56)$$

Invoking (52) and (56), we get

$$V_n \leq \upsilon_1 + \upsilon_2 + \upsilon_3 \quad (57)$$

From the above analysis, once the inequalities (45) and (57) are held, combined with definition of $V_n(t)$ and Lemma 3, we can come to a conclusion that all signals of the fractional-order closed-loop system (1) keep bounded and the tracking error $|S_1| \leq \sqrt{2(\upsilon_1 + \upsilon_2 + \upsilon_3)}$ can converge to a small neighborhood of the origin, for $t > \max\{t_1, t_2\}$ .This completes the proof.

IV. Conclusions

In this article, an adaptive fuzzy fractional-order DSC method has designed for FONSs in strict-feedback form with unknown dead zone and external disturbances. The FONSs under consideration contains asymmetric dead zone and external disturbances. FLSs are used to model unknown nonlinear functions. Utilizing DSC to simplify the calculation. The dead zone inverse and compensation terms are added to compensate for the influence of the dead zone, approximation errors and external disturbances. Finally, the designed scheme can ensure the fractional-order system is stable and has good tracking performance.

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