Application of Modern Information Technologies for Solving the Problem of Finding a Rational Compromise in the Interaction of Partners Under the Conditions of Absence Analytical Dependences of Their Objective Functions

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Abstract
For a large class of systems analysis tasks, an important issue is the disclosure of uncertainties. This is due to the variety of goals, properties and characteristics of the studied objects. Today, the task of revealing the conflicts uncertainty in the tasks of choosing the goals of plans and plans in the process of partners interaction or competitors opposition or opponents remains relevant. There are methods in systems analysis that allow you to solve these problems in some cases. They are based on the application of mathematical analysis methods and probability theory. However, these methods are applicable only to problems in which the number of partners and arguments in the analytically given objective functions that determine the purpose of their activities coincide. Since in practice, as a rule, such a restriction is not met, it is important to find approaches to solving problems of disclosing the uncertainty of conflicts in the choice of goals and plans in the process of partners interaction or countering competitors or opponents, arbitrary number of partners and arguments of their target functions, but in the absence of analytical dependencies of these functions. The paper formalizes and describes the problem features of finding a rational compromise in the interaction of any number of partners and any number of arguments that determine their target functions, but in the absence of analytical dependencies of these functions, and defines the approach to its solution using modern information technology. The proposed approach is applicable in the case of the component elements independence of the decision of the interacting entities. It provides for the capabilities integration of the previously developed author's method for the disclosure of uncertainty in the problems of interaction and the software product for the reproduction of functional dependencies in the class of additive functions.

Keywords
Mathematical model, uncertainty, objective function, information technologies, algorithm, software.

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1. Introduction

For a large class of formalized system analysis problems, an important issue is the disclosure of uncertainties. The most common in practice are uncertainties of goals, situations, conflicts. The essence of these uncertainties can be assessed from the work [1]. Today, the task of developing methods and algorithms for revealing the uncertainty of conflicts in the tasks of choosing the goals of plans and plans in the process of partners interaction or opposition of competitors or opponents, as well as the development of information technology for their study.

Currently, there are methods in systems analysis that allow you to solve these problems in some cases. They are based on the application of mathematical analysis methods and probability theory [2-3]. However, these methods are applicable only to problems in which the number of partners and the arguments of the given objective functions that determine the purpose of their activities coincide. Since in practice, as a rule, such restrictions and conditions are not always met, the task of finding approaches to solving problems of disclosing the uncertainty of conflicts in the tasks of choosing the goals of plans and plans in the interaction of partners or countering competitors or opponents (hereinafter, tasks) disclosure of conflicts), which would provide the possibility of solving the problem for any number of partners and the arguments of their target functions, especially in the absence of analytical dependencies of the latter.

Thus, the problem that needs to be solved can be formulated as follows.

Number of partners interacts, each of which has its own goal, which is determined by an unknown objective function, which can theoretically be formed on the basis of known empirical data.

Partners in the process of active interaction can exchange information about their actions.

Find values of arguments of unknown objective functions at which objective functions would reach values that would satisfy all partners.

Given the condition of the problem under study, we can conclude that its solution involves, first, finding a solution in the presence of an arbitrary number of partners and the arguments of their objective functions in the presence of analytical dependencies of the latter, and secondly, the establishment of analytical dependencies of objective functions in the case of only a certain set of empirical data that can determine the arguments and values of objective functions.

Regarding the first problem, it should be noted that in this statement the problem is investigated in [4]. In this paper, we formalize the problem of disclosing uncertainty in the interaction of partners, in which the number of arguments of the objective functions is not necessarily equal to the number of partners. The analysis of the existing approach to the solution of the formulated problem in the absence and presence of situational uncertainty for two and any number of partners is also carried out. Based on the application of technical constraints, an approach to solving the problem is proposed and software and algorithmic support for its implementation is formed. This approach is based on the preliminary formation of the area of acceptable solutions (Pareto area) and the subsequent search for a rational solution in this area, can be used to solve the problem of disclosing the uncertainty of conflicts in the absence and presence of situational uncertainty. Software-algorithmic implementation of the author's approach, presented in [4], allows to automate individual stages of problem solving.

Regarding the second problem, it should be noted that the problem of reproducing functional dependences on an experimentally obtained discrete sample is one of the rather important applied problems. This explains the fact that the theoretical aspects of its solution were given attention in [1]. And the substantiation of algorithm, structure, functional features of the software application of reproduction of functional dependence on discrete sampling was devoted to work [5].

However, despite the presence of approaches to solving individual components of the problem under study, there are no approaches to solving the problem in the above statement. It should be noted that the mechanistic combination of approaches to solving individual problems may not be applicable in the case under study. This is due to the presence of certain features of the physical content of the problem.

The purpose of this work is to formalize and describe the features of the problem of finding a rational compromise in the interaction of any number of partners and any number of arguments that determine their objective functions, but in the absence of analytical dependencies of these functions.

2. Formalization of the studied problem

To achieve this goal, it is considered appropriate: first of all to formalize the research problem; to carry out the analysis of features of statement of a problem which are caused by its physical maintenance; determine approaches to its solution taking into account the peculiarities of the production.

Formalized description of the problem.

Entities may be involved in solving a problem $C_1, C_2, ..., C_n$.

Each of the subjects is assigned its own unique set of tasks, the solution of which is entrusted only to the relevant responsible subject, and a set of tasks, which can be joined by other subjects or combinations thereof, but which are a priority for the responsible subject.

The ability to solve a problem depends on the decision elements that are made by the subjects $C_1, C_2, ..., C_n$. Each of the subjects $C_i \left(i = \overline{1, n} \right)$ makes decision $X_i \left(i = \overline{1, n} \right)$ elements of which are indicators $x_{i1}, x_{i2}, ..., x_{iC_i} \left(i = \overline{1, n} \right)$.

Solving the problem Π is a priority for one of the subjects. Let's denote this subject by $C_в$.

Among the elements of the decision of each subject are those indicators that determine the effectiveness of the subject in the territorial and temporal limits of the urgency of the problem Π (hereinafter, within the problem Π). These elements of the solution apply to the whole range of tasks of the subject, which he solves within the problem Π.

The decisions made by each entity must ensure that their target functions are max $y_i \left(i = \overline{1, n} \right)$.

If the subject makes decisions in the interests of solving the priority task of the subject $C_в$, i.e. the problem Π, the value of its own objective function should not decrease below the threshold level $y_i^* \left(i = \overline{1, n} \right)$.

At the same time, there is another objective function $\bar{y}_в$ for the subject $C_в$, which determines the possibility of solving not all the problems of the subject $C_в$ within the problem Π, but only the problem Π itself. The minimum possible value of the efficiency of solving the problem Π by the subject $C_в$ is denoted by $\bar{y}_в^*$.

Then the solution of the problem Π involves finding such elements of the solution of all subjects $C_1, C_2, ..., C_n$ that would ensure the fulfillment of the criteria

$$y_i \to \max \left(i = \overline{1, n} \right), \quad (1)$$

$$\bar{y}_в \to \max \quad (2)$$

and satisfied the conditions

$$y_i \geq y_i^* \left(i = \overline{1, n} \right), \quad (3)$$

$$\bar{y}_в \geq \bar{y}_в^*. \quad (4)$$

3. Analysis of the features of the problem

The features of the problem are as follows.

1. Entities $C_1, C_2, ..., C_n$ are partners. They can adjust their decisions in the interests of another entity (one or more).

2. The number of indicators-elements of the decision of the subjects $C_1, C_2, ..., C_n$ may be different. That is, the numbers $1C_1, 2C_2, ..., nC_n$ are not necessarily equal.

3. Target functions $y_i \left(i = \overline{1, n} \right)$ subjects $C_1, C_2, ..., C_n$ are not represented as analytical dependencies.

4. The problem under study may be set in such a way that the decision $X_i \left(i = \overline{1, n} \right)$ may contain indicators $x_{i1}, x_{i2}, ..., x_{iC_i} \left(i = \overline{1, n} \right)$ that relate to different elements of the decision of one entity, or the problem can be set so that decisions

$X_j \left(j = \overline{1,m} \right)$ can contain a set of elements of decisions of all entities $C_1, C_2, ..., C_n$ on individual indicators.

An example of the problem under study may be as follows.

The following subjects $C_1, C_2, ..., C_n$ of the security and defense sector of Ukraine act as subjects:

C_1 - Armed Forces of Ukraine (AFU);

C_2 - State Border Guard Service of Ukraine (SBGSU);

C_3 - National Guard of Ukraine (NGU);

C_4 - Security Service of Ukraine (SSU).

The problem Π is the activity of the sabotage and reconnaissance group (SRG) in the border area.

The solution of problem Π is to localize the actions of the SRG.

The solution $X_i \left(i = \overline{1,n} \right)$ is the following set of indicators:

x_{i1} - quantitative characteristics (for example, the percentage) of the personnel of the active division of the entity $C_i \left(\mathrm{i} = \overline{1,4} \right)$, which operates within the problem Π and may be involved in its solution;

x_{i2} - quantitative characteristics of the entity's personnel $C_i \left(\mathrm{i} = \overline{1,4} \right)$, which may be additionally involved in solving the problem Π;

x_{i3} - quantitative characteristics (for example, the percentage) of technical means (weapons and military equipment) of the active unit of the subject $C_i \left(\mathrm{i} = \overline{1,4} \right)$, which are used within the problem Π and can be involved in its solution;

x_{i4} - quantitative characteristics of technical means (weapons and military equipment) of the subject $C_i \left(\mathrm{i} = \overline{1,4} \right)$, which may be additionally involved in solving the problem Π;

x_{i5} - the possible duration of the involvement of resources (personnel and technical means) of the active unit of the subject $C_i \left(\mathrm{i} = \overline{1,4} \right)$ to solve the problem Π;

x_{i6} - the possible duration of the involvement of additional resources (personnel and technical means) of the subject $C_i \left(\mathrm{i} = \overline{1,4} \right)$ to solve the problem Π.

Functions $y_i \left(i = \overline{1,4} \right)$ are the effectiveness of the functions of the AFU, SBGSU, NGU, SSU, respectively, within the problem Π.

4. The author's approach to solving the problem under study, taking into account the peculiarities of its formulation

The author's approach to solving the research problem in the absence of analytical representation of objective functions involves the implementation of two stages: first, the establishment of analytical dependences for objective functions $y_i \left(i = \overline{1,n} \right)$; secondly, directly the search for a rational compromise, i.e. solving problem (1) - (4).

The method of realization of the second stage in the presence of analytical representation of target functions $y_i \left(i = \overline{1,n} \right)$ is described in detail in work [4].

Thus, the main problem of solving the problem is the implementation of the first stage, ie the establishment of functional dependencies $y_i \left(i = \overline{1,n} \right)$.

It should be noted that the problem of reproducing functional dependences on an experimentally obtained discrete sample is one of the rather important applied tasks, both operations research and system analysis [1, 6-7]. Its complexity is due to the fact that at the initial stage of formation of the concept and design of the operation can be known only incomplete, heterogeneous input information, which can act as empirical data, expert assessments, a priori information about analogues and prototypes, some information about the purpose and quality indicators. standard restrictions and data, etc. It is these data that can determine the arguments of the desired functional dependencies that determine the target functions of the partners in the problem studied in this paper. However, under these conditions, the choice of analytical forms of objective functions, justification of their content and purpose is an informal procedure that can be performed only by the researcher. The peculiarity of the problem is that the required functions should not only be as close as possible to the empirical data on a certain criterion, but also have extreme properties. The specificity of the extreme properties is due to the limited interval of setting the original data and is that the perturbations at the limits of the interval significantly affect the extreme properties of the function. This feature is fundamental and causes a more complex structure of approximation functions than in interpolation problems [8-14]. Another important feature is the need to choose a rational compromise between conflicting requirements: maximizing the level of reliability of the procedure for identifying the desired pattern, which necessitates increasing the complexity of the class of approximation functions, and minimizing the complexity and complexity of the procedure for forming the required dependence. Due to the poor choice of functions, it may happen that the reproduced function will approximate certain output data for most of a given interval, but in general will poorly describe the true functional dependence. This explains the fact that to reproduce the functional dependencies $y_i\left(i=\overline{1,n}\right)$ should use the methods described in [15-17].

The use of software and algorithmic software for the implementation of these methods, which is given in [5], allows you to automate the process of finding functions $y_i\left(i=\overline{1,n}\right)$. However, it should be noted that the application of such an approach has some difficulties.

To assess them, we analyze the initial data for the reproduction of functional dependencies, which are given in table. 1.

If all the data in the table 1 method of reproduction of functional dependencies, which is algorithmized in [5], is applied directly. If the table. 1 there are no individual data or their whole blocks, it is considered appropriate to pre-fill the table. 1, taking into account the minimum possible $\min x_{ij}$, maximum possible $\max x_{ij}$, or most probable x_{ij} values $\bar{x}_{ij}$ $\left(i=\overline{1,n},\ j=\overline{1,C_i}\right)$, that characterize the elements of the decision of the subjects $C_1, C_2, ..., C_n$. Filling in the table. 1 data should be carried out taking into account the physical content of the elements of the decision, their physically possible values and the forecast of the possibility of implementation in a particular case. After filling in the data of table. 1 then you should use the software product developed in [5]. It should be noted that the proposed approach is based on the reproduction of functional dependence in the additive form. The required functions $y_i\left(i=\overline{1,n}\right)$ are represented as a superposition of functions from variables $X_i\left(i=\overline{1,n}\right)$.

This approach is quite possible when the components of the vectors $X_i\left(i=\overline{1,n}\right)$ are independent. If the components of vectors $X_i\left(i=\overline{1,n}\right)$ are dependent, and this is possible in a number of applied problems, then the application of this approach can lead to large deviations of the obtained dependences from the real multifactor regularities, because the effects of vector components $X_i\left(i=\overline{1,n}\right)$ on the properties of functions $y_i\left(i=\overline{1,n}\right)$ will not be taken into account.

Therefore, in this case, when forming the structure of the models should take into account not only the influence on the functions $y_i\left(i=\overline{1,n}\right)$ of a certain group of factors $X_i\left(i=\overline{1,n}\right)$, but also the mutual influence of the components of different vectors $X_i\left(i=\overline{1,n}\right)$. This explains the need for the formation of a hierarchical multilevel system of models in the class of multiplicative functions.

Table 1

Initial data for reproducing analytical dependencies $y_i\left(i=\overline{1,n}\right)$

Empirical data set number (experiment)	X_1				…	X_n				y_1	y_2	…	$y_в$	$\bar{y}_в$	…	y_n
	x_{11}	x_{12}	…	x_{1C_1}	…	x_{n1}	x_{n2}	…	x_{nC_n}							
1 2 … q_0																
$\min x_{ij}$ $\left(i=\overline{1,n},\ j=\overline{1,C_i}\right)$	The minimum possible values x_{ij} for each column from a physical point of view									The minimum allowable limit level of values $y_i^*\left(i=\overline{1,n}\right)$						
$\max x_{ij}$ $\left(i=\overline{1,n},\ j=\overline{1,C_i}\right)$	The maximum possible values x_{ij} for each column from a physical point of view									y_1^*	y_2^*	…	$y_в^*$	$\bar{y}_в^*$	…	y_n^*
$\bar{x}_{ij}$ $\left(i=\overline{1,n},\ j=\overline{1,C_i}\right)$	The most probable values x_{ij} for each column ю															

5. Conclusions

Based on the results of the study, it can be concluded that the proposed approach to solving the problem of finding a rational compromise with the interaction of any number of partners and any number of arguments that determine their target functions, but in the absence of analytical dependencies of these functions component of the decision elements $X_i\left(i=\overline{1,n}\right)$ of the subjects $C_1, C_2, ..., C_n$. It provides a combination of capabilities of the previously developed author's method and software product for the reproduction of functional dependencies in the class of additive functions.

Prospects for further research include the development of information technology, which would provide full automation of the proposed approach, development of software applications to solve the problem in a multiplicative form and testing the approach to a specific application problem.

6. Aknowledgements

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7. References

[1] M. Z. Zghurovskyi, N. D. Pankratova, Fundamentals of systems analysis. – K.: Publishing group VNV, 2007. – 544 p.

[2] O. V. Borovyk, L. V. Borovyk, L. O. Matokhniuk, Higher Mathematics: Differential calculus of a function of one and many variables. Part 2: textbook. – Khmelnytskyi: publishing house NABGSU, 2016. – 640 p.

[3] V. E. Hmurman, Theory of Probability and Mathematical Statistics. – M.: High school, 1972. – 304 p.
[4] O. V. Borovyk, D. O. Borovyk, Adequate knowledge of the scientific-methodical apparatus for the development of non-significance in the tasks of interaction // Library of Science Practitioners of Military Institute of Kyiv Taras Shevchenko National University. Issue № 55. – К: MIKNU, 2021. – P. 20-40.
[5] O. V. Borovyk, L. M. Traskovetska, Substantiation of the structure and functional features of the software application of the Chebyshev approximation problem as a component of the problem of reproduction of functional regularities by discrete sampling // Collection of scientific works № 57. Series: Military and technical sciences. – Khmelnytskyi: Publishing house. NABGSU, 2012. – P. 86-91.
[6] E. S. Venttsel, Operations research. – M.: Soviet radio, 1972. – 552 p.
[7] V. A. Abchuk, F. A. Matveichuk, L. P. Tomashevskyi, Handbook of operations research / Under the general editorship of F.A. Matveichuk – M.: Voenizdat, 1979. – 368 p.
[8] Greene, H. William, (2002). Econometric analysis (5th ed.). New Jersey: Prentice Hall.
[9] Least squares method // Higher mathematics in examples and problems: Textbook. 2nd edition. - Kyiv: Center for Educational Literature / Klepko V.Iu., Holets V.L.. – 2009. – 594 p.
[10] N. S. Bobrovnycha, Ye. H. Borysevych, Econometrics: textbook / N. S. Bobrovnycha, Ye. H. Borysevych – Odessa: O. S. Popov ONAC, 2010. - 180 p.
[11] G. G. Judge, and ets. The Theory and Practice of Econometrics edition / Judge, G. G. New York: John Wiley, 1985. – P. 3-120.
[12] Computational mathematics. Interpolation and approximation of tabular data: a textbook / L. V. Krylyk, I. V. Bohach, M. O. Prokopova. – Vinnytsia: VNTU, 2013. – 111 p.
[13] Continuous and smooth minimax spline approximation / P. S. Malachivskyi, V. V. Skopetskyi ; [resp. ed. I.V. Sergienko]; NAS of Ukraine, V. M. Hlushkov Institute of Cybernetics, Center of Mat. modeling Inst of applied. probl. Mechanics and mathematics named after Ya. S. Pidstryhach. - К.: Scient. opinion, 2013. – 271 p.
[14] B. P. Dovhyi, A. V. Loveikyn, E. S. Vakal, Yu. E. Vakal, Spline functions and their use. Textbook of Taras Shevchenko Kyiv National University. – К: KNU, 2016.
[15] A. O. Kalenchuk-Porkhanova, L. P. Vakal, Construction of the best uniform approximations of functions of many variables // Computer tools, networks and systems. − 2007. − № 6. P. 141−148.
[16] K. Lantsosh, Practical methods of applied analysis. – M.: Physics and math. lit., 1961. − 524 p.
[17] E. Ya. Remez, Fundamentals of numerical methods of Chebyshev approximation. − К.: Scient. opinion, 1969. - 623 p.