# Presupposition Projection from the Scope of Quantifiers: a Novel Approach

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**Abstract**

We present a modification of the Transparency Theory [5, 6], in the case of presupposition projection from the scope quantifiers, which allows the force of the quantifier to affect the force of the projected presupposition. The modification is based on letting the Transparency constraint operate on *witness sets* [1], and produces predictions that match currently known empirical results [3, 8], while also making novel predictions that will hopefully serve as input to future psycholinguistic experimentation.

## 1 Introduction

How presuppositions project from the scope of quantifiers is a vexed issue. Heim's Dynamic semantics, [4], predicted universal inferences across the board (the '$\triangleright$' symbol below is to be read as 'presupposes'):

(1) Some/Every/At least $n$/No/At most $n$/... students stopped smoking. $\triangleright$ Every student used to smoke.

On the other hand [2] developed a theory where projection is existential across the board:

(2) Some/Every/At least $n$/No/At most $n$/... students stopped smoking. $\triangleright$ Some student used to smoke.

More recently, experimental forays paint a picture where the projected presupposition has universal force when the quantifier is universal, and existential force when the quantifier is existential, [8]. Importantly, the quantifier 'No' appears to behave like a universal, [3]. In cases where the quantifier is neither a full existential nor a full universal, the projected inference doesn't have robust universal force (see [3] for details).

Our aim in this paper is to see how the intuitions behind the Transparency Theory [5, 6] can be modified to produce results that match the aforementioned empirical picture. We argue that this can be achieved by utilising the notion of a *witness set* of a quantifier, leading to the following result:

(3) a. If $D^{\uparrow}$ is an upwards monotone determiner, then $D^{\uparrow}(\textit{students})(\textit{stopped smoking}) \triangleright D^{\uparrow}(\textit{students})(\textit{used to smoke})$

b. If $D^{\downarrow}$ is a downwards monotone determiner, then $D^{\downarrow}(\textit{students})(\textit{stopped smoking}) \triangleright D^{\downarrow}(\textit{students})(\neg(\textit{used to smoke}))$

The rest of the paper is organised as follows: Section 2 introduces the necessary background on the Transparency Theory and witness sets. Section 3 presents our system and applies it to derive the result in (3). Section 4 concludes.

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# 2 Background

## 2.1 The Transparency theory

We give a brief description of the basics of the Transparency theory, [5, 6], to the extent needed to develop our idea about quantifiers. First, suppose we have a propositional language with two kinds of propositional constants $p$ and $p'p$. $p$ is a simple proposition denoting a set of worlds. $p'p$ is to be understood as the conjunction of $p'$ and $p$, the idea being that $p'$ is the presuppositional component, while $p$ the non-presuppositional component. Assume also that all sentences are interpreted against a context $C$, modelled as a set of possible worlds [7].

Transparency Theory imposes a constraint on expressions of the form $p'p$ by requiring the presuppositional part to be uninformative/redundant in $p'p$ (with respect to the words in $C$). Here's the statement of the constraint:

(4) **Transparency:** A sentence $S$ starting with a string $\alpha\ p'p$ is acceptable in a context $C$ if and only if for all $r$, for all $\beta$ it holds that $C \models \alpha\ p'r\ \beta \leftrightarrow \alpha\ r\ \beta$

What this constraint is requiring is that as soon as a presuppositional $p'p$ constituent is encountered in a sentence $S$, we are able to know that in every world in the context $C$, no matter how $S$ ends (for all $\beta$), and no matter the non-presuppositional component of $p'p$ (for all $r$), $S$ is equivalent to the version where the presupposition has been removed; the presupposition adds nothing, no matter the assertion and no matter the conclusion of the sentence.

To get a sense of how this works, consider the simple case of $\neg p'p$. The constraint requires that for all $r$:

(5) $C \models \neg p'r \leftrightarrow \neg r$

We can calculate a condition on the context that is equivalent to the constraint. First, suppose that the constraint holds. Since it holds for all $r$, take $r$ to be some tautology $\top$. Then $\neg p'r$ becomes $\neg p'$ (recall $p'r$ is a conjunction) and the constraint becomes:

(6) $C \models \neg p' \leftrightarrow \neg\top$

The negation of $\top$ is a contradiction, so the right-hand side is false. Since we have assume the bi-conditional to be true, the left-hand side must also be false. For this to be the case, it must hold throughout $C$ that $p'$ is true.

Now suppose that $p'$ is true throughout $C$. This is enough to make the constraint in (5) hold. If the left-hand side $\neg p'r$ is true in some world in $C$ for some arbitrary $r$, then it must be that $r$ is false in that world (since we assumed that $p'$ is true in every world in $C$). But then the right-hand side is also true. If the right-hand side is true in some world for some arbitrary $r$, then $\neg p'r$ is also true. So, the equivalence holds just in case $C \models p'$, i.e. the presupposition $p'$ 'projects' from the scope of the negation.

## 2.2 Projection from the scope of quantifiers

Now suppose we move to more complex language where we also have two types of (one-place) predicate symbols $P$ and $P'P$. $P$ is a normal predicate denoting a set of entities in some world $i$ in $C$, while $P'P$ is a predicate that has $P'$ as its presuppositional component, and denotes the intersection of the sets denoted by $P'$ and $P$. We also have determiner symbols $D_j,\ j \in \mathbb{N}$, and quantificational sentences of the form $D_j(A)(B)$ which are interpreted in accordance with Generalized Quantifier theory, [1]. Following [5], we will assume that all of our determiners obey Conservativity, Extension, and Permutation-invariance. We will refer to string of the form $D(A)$ as *quantifiers* (keeping to the definition of a generalized quantifier as a family of sets).

The case that interests us is that of a presuppositional expression in the scope of a quantifier, i.e. $D(A)(P'P)$. The constraint then becomes (see [5] for the exact details):

(7) For all $R$, it holds that $C \models D(A)(P'R) \leftrightarrow D(A)(R)$

[5] shows that under certain conditions, this constraint is equivalent to requiring that in every world $i \in C$, every entity in $[\![A]\!]^i$ be also in $[\![P']\!]^i$. Hence, a universal presupposition is projected whereby *Every/Some/Most/No/... student stopped smoking* presupposes that *Every student used to smoke*.

Our argument is that we can predict presuppositions that depend on the actual quantifier being used, if we shift the domain of application of the constraint to the *witness sets* that can be used to verify a quantificational sentence.

## 2.3 Witness sets

Barwise & Cooper, [1], famously suggested that when processing a quantificational statement of the form $D(A)(B)$, comprehenders are not necessarily calculating the entire set of properties denoted by $D(A)$ and then checking if $[\![B]\!]$ is in it. Instead they reason by using so-called *witness sets* of $D(A)$. Intuitively a witness set for a quantifier $D(A)$ is a subset of $[\![A]\!]$ that is of the correct quantificational amount for $D$. For instance, a witness set for $Most(stundets)$ is a subset of students that contains most of the students:

(8) a. $[\![students]\!] = \{a,\ b,\ c\}$
b. The set of witness sets for $Most(stundets)$ is $\{\ \{a,\ b,\ c\},\ \{a,\ b\},\ \{b,\ c\},\ \{c,\ a\}\ \}$

[1] define witness sets for quantifiers $D(A)$ that *live on* $A$, so first we give the definition of the *live on* property:

(9) A quantifier $D(A)$ **lives on** $A$ iff for all $X$, $X \in [\![D(A)]\!] \leftrightarrow (X \cap [\![A]\!]) \in [\![D(A)]\!]$

For example *Some*(*man*) lives on *man* since the following holds:

(10) [Some man $VP$] iff [Some man is a man who $VP$]

Note that all conservative determiners produce quantifiers that have the *live on* property: if $D$ is conservative, then for all $X,\ Y,\ D(X)(Y) \leftrightarrow D(X)(X \cap Y)$. For $X = A$ then, it holds that for all $Y$, $D(A)(Y) \leftrightarrow D(A)(A \cap Y)$. Hence, when $D$ is applied to some restrictor argument $A$ it produces a quantifier that lives on $A$. Since we have restricted attention to determiners that satisfy Conservativity, all of our quantifier will have the *live on* property. With this out of the way, here's the definition of *witness set*:

(11) A **witness set** for a quantifier $D(A)$ living on $A$ is any subset $w$ of $[\![A]\!]$ such that $w \in [\![D(A)]\!]$

The reason [1] suggest that one use witness sets to reason about the truth or falsity of a quantificational expression $D(A)(B)$ is the following proposition:

(12) **Proposition 1** (Barwise and Cooper): Let $w$ range over witness sets for a quantifier $D(A)$ living on $A$. Then, for any $B$:

- If $D$ is upwards monotone, $D(A)(B)$ is true iff $\exists w[w \subseteq \llbracket B \rrbracket\ ]$
- If $D$ is upwards monotone, $D(A)(B)$ is true iff $\exists w[\ \llbracket B \rrbracket] \cap \llbracket A \rrbracket \subseteq w\ ]$

This proposition suggests a straightforward procedure for evaluating the truth or falsity of a quantificational statement $D(A)(B)$. If $D$ is upwards monotone, then find a witness set $w$ that is a subset of the property denoted by $B$. If $D$ is downwards monotone, then find a witness set $w$ of $D(A)$ such that $\llbracket A \rrbracket \cap \llbracket B \rrbracket$ is a subset of $w$. If no witness set of the right kind can be found, the sentence is false.

We are going to say that a witness set for a qunatifier $D(A)$ **verifies** a quantificational expression $D(A)(B)$ iff $w$ fulfills the relevant condition from Proposition 1 above (depending on the monotonicity of $D$).

# 3 The current framework

We now combine witness sets with Transparency. Recall that in the case of $D(A)(P'P)$, the original Transparency constraint was:

(13) $\forall P :\ C \models D(A)(P'P) \leftrightarrow D(A)(P)$

Recall also the intuition behind this definition was that a presupposition should not contribute any information: when evaluating the truth of a presuppositional expression (no matter the assertion), this should be equivalent to ignoring the presupposition.

Our twist is the following: instead of requiring that $P'$ be uninformative with respect to $D(A)(P'P)$, we are going to require that $P'$ be uninformative with respect to a witness set that makes $D(A)(P'P)$ true:

(14) **Witness Set Transparency:** Given a quantificational expression $D(A)(P'P)$, it must hold that: $\forall P :\ C \models \exists w\ \ [w\ verifies\ D(A)(P'P)\ \leftrightarrow\ w\ verifies\ D(A)(P)]$

Parallel to the original Transparency intuition, this is saying that when in some world $i \in C$ you are picking a witness set to verify a presuppositional quantificational expression (no matter the assertive component), this should be equivalent to using this witness set to verify the same quantificational expression without the presupposition.

Before moving on, a note on the object-language: it will be helpful below to have access to a predicate-level tautology ($P_\top$), so we assume it as part of our language. The interpretation of $P_\top$ in some world $i$ is just the domain of entities in $i$.

We are now going to apply our **Witness Set Transparency** to the case of upwards and downwards monotone determiners, in order to isolate necessary and sufficient conditions under which the **Witness Set Transparency** constraint holds for such determiners.

## 3.1 Upwards

Suppose that $D$ is an upwards monotone determiner. We claim that **Witness Set Transparency** holds for $D(A)(P'P)$ if and only if in every world in $C$, there is a witness set $w$ for $D(A)$ such that $w$ verifies $D(A)(P')$.

To see this first suppose that **Witness Set Transparency** holds for $D(A)(P'P)$. This means that for all $P$ in every world in $C$, there exists a witness set $w$ for $D(A)$ such that, $w$ verifies $D(A)(P'P)$ iff $w$ verifies $D(A)(P)$. Since this holds for all worlds in $C$, take some arbitrary world $i$.

Since $D$ is upwards monotone, $w$ verifies $D(A)(P'P)$ iff $w \subseteq [\![P]\!]^i$ (by Proposition 1). Since this verification holds for all $P$, it must hold for $P = P_\top$. Thus it holds:

(15) $w \subseteq [\![P'P_\top]\!]^i \leftrightarrow w \subseteq [\![P_\top]\!]^i$

Since $[\![P'P_\top]\!]^i = [\![P']\!]^i \cap [\![P_\top]\!]^i$, this is just equal to $[\![P']\!]^i$. Thus we have:

(16) $w \subseteq [\![P']\!]^i \leftrightarrow w \subseteq [\![P_\top]\!]^i$

Trivially, it holds that $w \subseteq [\![P_\top]\!]^i$ (since $P_\top$ is just the set of all entities in $i$). Since we are assuming the bi-conditional in (16) to be true, this means that $w \subseteq [\![P']\!]^i$ is true, and by Proposition 1, this means that $D(A)(P')$ is true in $i$. Since $i$ was arbitrary, this reasoning goes through for every world in $C$.

For the other direction, assume that that $D(A)(P')$ holds in some arbitrary $C$-world $i$. We need to show that **Witness Set Transparency** holds. Given that $D(A)(P')$ holds in $i$, Proposition 1 tells us that there is a witness set $w$ for $D(A)$ such that $w \subseteq [\![P']\!]^i$. Consider this $w$; we are going to show that **Witness Set Transparency** holds for it, i.e. that:

(17) $\forall P : w$ verifies $D(A)(P'P)$ in $i \leftrightarrow w$ verifies $D(A)(P)$ in $i$

Take an arbitrary $P$, and suppose that $w$ verifies $D(A)(P'P)$ in $i$. Then $w \subseteq [\![P'P]\!]^i$. But then all the elements in $w$ are also in $[\![P]\!]^i$, hence $w \subseteq [\![P]\!]^i$, and $w$ verifies $D(A)(P)$ in $i$.

Now suppose that $w$ verifies $D(A)(P)$ in $i$. Then $w \subseteq [\![P]\!]^i$. But by assumption $w \subseteq [\![P']\!]^i$. So, all the elements of $w$ are elements of both $[\![P']\!]^i$ and $[\![P]\!]^i$m which means that $w \subseteq [\![P'P]\!]^i$. But then $w$ verifies $D(A)(P'P)$ in $i$. Since $i$ was arbitrary, this reasoning goes through for all worlds in $C$, which means that **Witness Set Transparency** holds.

## 3.2 Downwards

Suppose that $D$ is a downwards monotone determiner, and that we have a presuppositional quantificational expression $D(A)(P'P)$. We claim that **Witness Set Transparency** holds if and only if in every $C$-world, there is a witness set $w$ for $D(A)$ such that $w$ verifies $D(A)(\neg P')$.

To see this, first suppose that **Witness Set Transparency** holds for $D(A)(P'P)$. This means that for every $P$ and every world in $C$, there exists a witness set $w$ for $D(A)$ such that $w$ verifies $D(A)(P'P)$ iff $w$ verifies $D(A)(P)$.

Consider an arbitrary world $i \in C$. Since $D$ is downwards monotone, the relevant $w$ in $i$ verifies $D(A)(P'P)$ iff $([\![A]\!]^i \cap [\![P]\!]^i) \subseteq w$ (by Proposition 1). Since this verification holds for all $P$, it must hold for $P = \neg P'$. Thus it holds:

(18) $([\![A]\!]^i \cap [\![P'(\neg P')]\!]^i) \subseteq w \leftrightarrow ([\![A]\!]^i \cap [\![\neg P']\!]^i) \subseteq w$

$[\![P'(\neg P')]\!]^i$ equals the empty set. Hence $([\![A]\!]^i \cap [\![P'(\neg P')]\!]^i)$ equals the empty set, and as the empty set is trivially a subset of any set, $([\![A]\!]^i \cap [\![P'(\neg P')]\!]^i) \subseteq w$ is true. Since we are assuming that the bi-conditional in (18) holds, this means that $([\![A]\!]^i \cap [\![\neg P']\!]^i) \subseteq w$ must hold. But then by Proposition 1, $w$ verifies $D(A)(\neg P')$ in $i$. Since $i$ was arbitrary, it holds throughout $C$ that $D(A)(\neg P')$.

For the other direction, take some arbitrary $C$-world $i$ and assume that there is $w$ such that $w$ verifies $D(A)(\neg P')$ in $i$. We need to show that **Witness Set Transparency** holds, i.e. that there is exists a witness set $w'$ in $i$ such that the following holds:

(19) $\forall P : w'$ verifies $D(A)(P'P)$ in $i \leftrightarrow w'$ verifies $D(A)(P)$ in $i$

Take an arbitrary $P$. Consider the witness set $w$ such that $w$ verifies $D(A)(\neg P')$ in $i$ (given to us by assumption), and assume that $w$ verifies $D(A)(P'P)$ in $i$. We argue that $w$ also verifies $D(A)(P)$ in $i$. Since we assumed that $w$ verifies $D(A)(P'P)$ in $i$, then $([\![A]\!]^i \cap [\![P'P]\!]^i) \subseteq w$. So $w$ contains all the elements that are in $[\![A]\!]^i$ and $[\![P']\!]^i$ and $[\![P]\!]^i$. But by assumption it also holds that $([\![A]\!]^i \cap [\![\neg P']\!]^i) \subseteq w$, so $w$ contains all the $[\![A]\!]^i$ elements that are in $[\![\neg P']\!]^i$. Therefore, the following holds:

(20) $([\![A]\!]^i \cap [\![P']\!]^i \cap [\![P]\!]) \cup ([\![A]\!]^i \cap [\![\neg P']\!]^i) \subseteq w$

We claim that $([\![A]\!]^i \cap [\![P]\!]^i) \subseteq ([\![A]\!]^i \cap [\![P']\!]^i \cap [\![P]\!]^i) \cup ([\![A]\!]^i \cap [\![\neg P']\!]^i)$.

To see this, consider an element from $([\![A]\!]^i \cap [\![P]\!]^i)$. This element will be a member of either $[\![P']\!]^i$ or $[\![\neg P']\!]^i$. If it is an element of $[\![P']\!]^i$, then it is an element of $([\![A]\!]^i \cap [\![P']\!]^i \cap [\![P]\!]^i)$ and hence is in $([\![A]\!]^i \cap [\![P']\!]^i \cap [\![P]\!]) \cup ([\![A]\!]^i \cap [\![\neg P']\!]^i)$. If it is an element of $[\![\neg P']\!]^i$, then it is in $([\![A]\!]^i \cap [\![\neg P']\!]^i)$, and hence in $([\![A]\!]^i \cap [\![P']\!]^i \cap [\![P]\!]^i) \cup ([\![A]\!]^i \cap [\![\neg P']\!]^i)$. Either way the element is a member of $([\![A]\!]^i \cap [\![P']\!]^i \cap [\![P]\!]^i) \cup ([\![A]\!]^i \cap [\![\neg P']\!]^i)$. But then it holds that:

(21) $([\![A]\!]^i \cap [\![P]\!]^i) \subseteq ([\![A]\!]^i \cap [\![P']\!]^i \cap [\![P]\!]) \cup ([\![A]\!]^i \cap [\![\neg P']\!]^i) \subseteq w$

By Proposition 1, $w$ then verifies $D(A)(P)$ in $i$.

Now assume that $w$ verifies $D(A)(P)$ in $i$. We show that $w$ verifies $D(A)(P'P)$ in $i$.

Since $w$ verifies $D(A)(P)$ in $i$, it holds that $([\![A]\!]^i \cap [\![P]\!]^i) \subseteq w$. For $w$ to also verify $D(A)(P'P)$ in $i$, it needs to hold that $([\![A]\!]^i \cap [\![P]\!]^i \cap [\![P']\!]^i) \subseteq w$. But since $([\![A]\!]^i \cap [\![P]\!]^i \cap [\![P']\!]^i) \subseteq ([\![A]\!]^i \cap [\![P]\!]^i)$, it also holds that:

(22) $([\![A]\!]^i \cap [\![P]\!]^i \cap [\![P']\!]^i) \subseteq ([\![A]\!]^i \cap [\![P]\!]^i) \subseteq w$

Thus, $w$ verifies $D(A)(P'P)$ in $i$. Since $i$ was arbitrary, this holds for all worlds in $C$, and hence **Witness Set Transparency** holds.

# 4 Conclusion

Summing up, our predictions stand as follows:

(23) For an upwards monotone determiner:

a. Some/All/Most/At least $n$/... of my students stopped smoking. $\triangleright$ Some/All/-Most/At least $n$/... of my students used to smoke.

(24) For downwards monotone determiner:

a. None/Few/At most $n$/Less than $n$/... of my students stopped smoking. $\triangleright$ None/Few/At most $n$/Less than $n$/... of my students didn't use to smoke.

Importantly, universal quantifiers (including 'No/None') project universal presuppositions, while existential quantifiers project existential presuppositions. Other quantifiers project presuppositions that do not rise to full universal.

Our approach then is strongly predictive, and at least some of its predictions match the currently known empirical landscape. Of course, further empirical clarification is required to evaluate the full set of predictions, and it is to be hoped that this will happen in the future.

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