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## Abstract

Rolling stock is one of the major assets for a railway transportation company. Hence, their utilization should be as efficiently and effectively as possible. Railway undertakings are facing rolling stock scheduling challenges in different forms - from rather idealized weekly strategic problems to very concrete operational ones. Thus, a vast of optimization models with different features and objectives exist. Thorlacius et al. (2015) provides a comprehensive and valuable collection on technical requirements, models, and methods considered in the scientific literature. We contribute with an update including recent works. The main focus of the paper is to present a classification and elaboration of the major features which our solver R-OPT is able to handle. Moreover, the basic optimization model and algorithmic ingredients of R-OPT are discussed. Finally, we present computational results for a cargo application at SBB CARGO AG and other railway undertakings for passenger traffic in Europe to show the capabilities of R-OPT.

## Keywords

Rolling Stock Rotation Planning, Combinatorial Optimization Model, Freight Transportation

## 1 Introduction

An efficient utilization of rolling stock is a major goal of any railway undertaking. Due to the complexity of operating railway systems, rolling stock schedules must be adapted several times during the years. There are a vast of reasons, which can be ordered by importance as follows timetable changes, railway construction activities, and new rolling stock. For freight railways undertakings the order book and customer demand can vary significantly between seasons. In addition, there are also exceptional circumstances. E.g. an accident between two cargo trains in November 2022 leads to a complete line closure between Berlin and Hanover such that re-routing of all trains is needed for a foreseeable time period. In such a case obviously dispatching, re-scheduling and also re-optimization of all resources are

useful and can support the way back to the 'normal' state of the rolling stock schedules.

Therefore, different departments at railway undertakings have to deal with the construction of rolling stock schedules for various timeframes from strategical planning to adaptations at the day of operations. Being able to find optimal rolling stock schedules gives any railway undertaking a competitive advantage. This is the motivation for the development of optimization models and software in order to support the planners to resolve all their different planning challenges.

The focus of this paper is to shed light on the different problem variants and applications for rolling stock roster optimization and the wide range of features needed in practice. We provide a classification of the most common features of rolling stock planning discussed in the literature with respect to their influence on the model. Moreover we present the optimization module R-OPT which is integrated in the planning software IVU.RAIL and discuss how these features are handled.

The paper is organized as follows: Section 2 presents and classifies key features in rolling stock planning. We shortly present the basic vehicle scheduling model inside of R-OPT in Section 3. Section 4 discusses the algorithmic concept and ingredients of R-OPT. Finally, we will present in Section 5 results of R-OPT for real world scenarios from the freight traffic operator SBB CARGO AG and other railway undertakings for passenger traffic in Europe.

## 2 The Zoo of Features

The basic input of any rolling stock planning problem is a timetable. A timetable is a set of mandatory trips which have to be operated by a valid rolling stock assignment. There are two fundamentally different use cases discussed in the scientific literature that can be handled by the rolling stock optimizer module R-OPT:

1. cyclic planning (mostly for strategic long-term planning) and
2. acyclic planning (mostly for operational short or mid-term planning and dispatching)

In the cyclic case, a repeating time period, typically a planning-week, is assumed. In contrast to that, the acyclic case considers a sequence of concrete calendar days with an initial state of the rolling stock, i.e. it is known where the rolling stock is located and when the next maintenances are due. However, the basic questions to answer are very similar if not the same for both use cases:

- What is the minimal number of vehicles to cover all mandatory trips?
- Which vehicle type is assigned to each mandatory trip?
- In which train configuration is the mandatory trip realized?
- What is the successor trip of each trip?
- Which deadhead trips are required?
- Are vehicles hauling other vehicles?
- Where and when do the required maintenances of each vehicle take place?

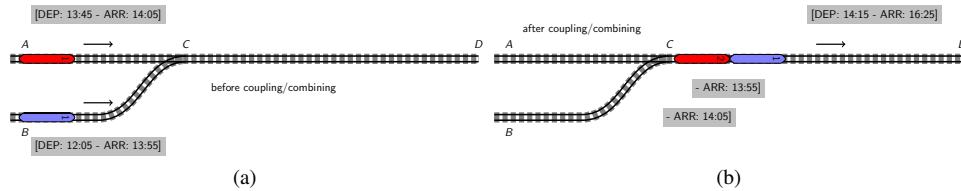


Figure 1: Example for coupling of a double-traction extracted from splitting and combining in Reuther (2017).

Classical vehicle scheduling, e.g., scheduling for busses, has rules which can be considered individually for a single vehicle, e.g., temporal and spatial consistency of its sequence of trips. In contrast, for rolling stock scheduling most of the complexity of the planning problem arises from the fact that for lots of its features several vehicles have to be considered at once. Apparently, when a passenger trip of the given timetable needs to be covered by a double-traction two vehicles are involved. Figure 1 visualizes this situation. There is a red vehicle operating a trip from station  $A$  to  $C$  arriving at 14:05. Another blue vehicle is running from  $B$  and arrives before the red vehicle at 13:55 in stations  $C$  on the left hand side (a). Then, both are coupled together in  $C$  and operate the trip from  $C$  to  $D$  as a coupled *composition* on the right hand side (b). Note that the number shown on the vehicle corresponds to the position within the composition. Moreover due to the topology at station  $C$ , we have to assume a FIFO principle for the trip to  $D$ . Hence, the blue vehicle must be in front of the coupled composition. In addition, for vehicle scheduling of busses the rotation has a daily structure because they start and end each day at some depot. This is fundamentally different for rolling stock because the sequence of trips to operate between depots can cover several days or even weeks. Thus, in order to check for the maintenance levels, vehicles must be tracked for several days or even weeks. The major requirements in rolling stock planning are:

1. vehicle composition rules,
2. vehicle availability and mix constraints,
3. maintenance constraints,
4. (network) capacity constraints,
5. coupling and decoupling,
6. operational regularity, and
7. a powerful, i.e., richly detailed, objective function.

Due to different planning processes, data availability or organizational responsibilities some requirements are more prioritized than others or some are neglected. Thus, each railway undertaking considers a different set of features with varying level of details. In the following, we will explain the main features:

1. A main characteristic of the majority of railway systems is that vehicles can be combined to form *vehicle compositions* (sometimes also simply called trains). The choice of whether or not a timetable trip can be operated by a certain vehicle composition can depend on technical aspects, e.g., electrification of its underlying route, or on commercial aspects, like the expected passenger demand of a timetabled trip can only be covered by vehicle compositions with sufficient passenger capacity.
2. Not only, but above all the crucial question in a strategic long-term setting is often: What is the minimal number of vehicles to operate a timetable? What are the optimal fleet sizes for the future? In contrast, in the operational case, the quantities of vehicle types are given and must best be respected. There, the main goal is e.g. to minimize the deviation from the standard plan, to minimize the additional coupling operations, or to balance the workload for the rolling stock. The restrictions on the number of vehicle types are formulated as so called *vehicle mix constraints*, that is minimum number and maximum number of assigned vehicles of a certain type.
3. The rolling stock has to be maintained frequently at specific locations. This leads to several maintenance constraints with different technical backgrounds. In the literature cumulative time and distance resources are considered which are classically constrained by upper bounds. Maintenance procedures have to be scheduled within the sequence of each vehicle in order to replenish the resource before the limit is exceeded.
4. Maintenance and also parking activities usually consume scarce infrastructure and crew capacity. This restricts the coupling and decoupling of vehicle compositions as well as the assignment of vehicles to parking tracks. Note that locations with parking or maintenance possibilities are sometimes very limited in the network which can be challenging to finalize rosters at the end of the day.
5. Valid coupling and decoupling of vehicle compositions on a track requires complex shunting dependent on the track layout, which is an optimization problem in its own right, see e.g., Bohlin et al. (2018), Gilg et al. (2018), and Haahr et al. (2017). The station layout implies if a LIFO or FIFO principle must be respected. Deadlocks must be avoided which otherwise would lead to additional resolution times and highly resource consuming shunting activities.
6. Most passenger timetables are based on a periodic pattern that is valid on different days of operation. For example there are the same trips on Monday till Sunday or others exists only on Monday till Friday. Regularity in rolling stock can be related to many aspects. It is preferred to operate repetitive trips on different days with the same vehicle composition. In addition, also the choice of the successor trip should ideally be the same. This is in particular beneficial for the subsequent planning step - duty scheduling.
7. In addition to the complex constraints restricting the space of valid rotations, a planning model should also offer a powerful and controllable objective function. Basically, this means properties which might be desired or unintended by the planners needs to be controllable via configuring the objective function of the model by bonus or penalty parameters. In general our modeling goal is that all relevant properties of

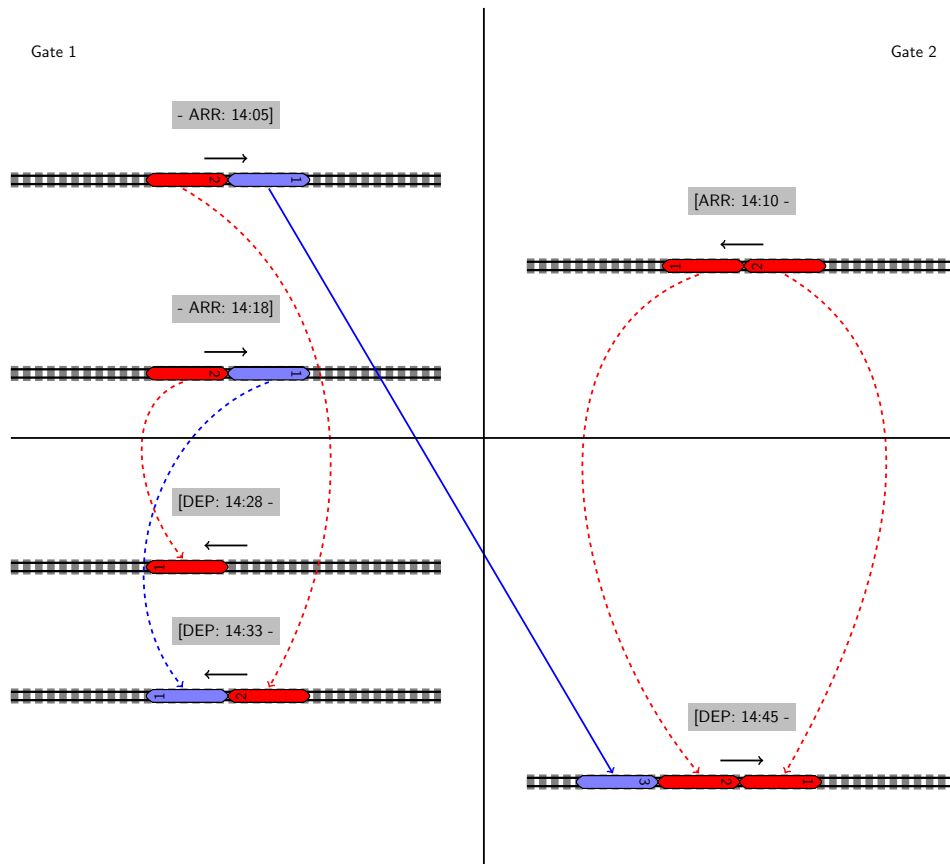


Figure 2: Example for the handling of coupling and decoupling in R-OPT.

the solution are measured and are potentially controllable as a part of the objective function using a weighted sum technique. Specifically, major economic aspects must be reflected like vehicle cost, coupling cost, composition cost per kilometer and hour on a detailed level. For example cost for falling below the required passenger demand of a trip, rewards for regularity, extra costs for exceeding capacity or maintenance intervals and many more are adjustable aspects.

Let us classify these requirements and aspects of rolling stock planning into *horizontal* and *vertical* features. This concept is based on the perspective of a classical Gantt chart that represents the activities of single vehicles. That is, horizontal features are concerning a set of vehicles at once while vertical features are features, that mostly are dependent on a sequence of trips for a single vehicle.

Thus, requirements 1,2,4, 5 and 6 are clearly vertical constraints and only 3 can be classified as horizontal. Figure 2 shows an example how coupling decoupling is handled by R-OPT. Each station is either a deadend station with one gate or a through station with two gates. The latter one is assumed in the example where on the left hand side all events at Gate 1 are shown and on the right hand side all at Gate 2, respectively. Three compositions arrive

at different times shown at the top. The composition arriving at Gate 2 consists of two red vehicles and the two other compositions arrive at Gate 1, respectively. Let us assume that these are all participating vehicles of the coupling schedule at the shown station. Then, the two red vehicles turn around at Gate 2 and where coupled together to a new composition with a blue vehicle for the departure at the bottom. The model in R-OPT assumes that all arriving compositions are decoupled into single vehicles and then re-coupled or combined into the departing compositions. Note that, when assigning arrivals to departures, position and chronological logic must be respected. The arcs between the vehicles in Figure 2 show the assignment from arriving vehicles to departing ones, i.e. going from top to the bottom. In case the vehicle turns around, i.e., that is exactly if it arrives and departs at the same gate, the arc is dashed and solid otherwise, e.g., the blue vehicle arriving at Gate 1 and departing at Gate 2. R-OPT offers the possibility to penalize violations of these logical rules because these might result in undesired coupling conflicts or even deadlocks. The assignment in Figure 2 does not violate any position or chronological rule. However, if we consider the same assignment of vehicles but a swapped order of the two departures at Gate 1 the situation changes, i.e., the joined composition of a blue and a red vehicle departs before the single red vehicle. In that case the blue and red vehicle composition would be blocked by the red vehicle which would lead to a penalty in the objective for this infeasible connection as part of the assignment.

To the best of the authors' knowledge, there is still a lack of standardization of rolling stock circulation planning within the railway industry even though the concepts and motivation of the requirements are very similar. A comprehensive and valuable collection on technical requirements, models and methods considered in the scientific literature can be found in Thorlacius et al. (2015) and Thorlacius (2017). There, a table of features covers the state of research until 2015. We contribute with an update including recent work as well as fix some entries in Table 1. Note that in the original table the technical report version of Borndörfer et al. (2016) is listed there as reference 21 from 2012. We feel free to correct some of the missing entries, i.e., Maintenance by time, Column Generation, and Branch & Price. We also add another column for regularity (6), which is a key feature for rolling stock planning especially for passenger traffic. Features 2, 4, and 5 relate to the column depot planning and its requirements to handle the depot capacity and topology. Seminal works make use of constraints to model train composition, see Fioole et al. (2006). Fundamentally different is the hypergraph formulation developed by Borndörfer et al. (2016) in order to directly represent the joint execution of a trip. Later, Reuther (2017) provides a model extension to integrate not only the position of vehicles within the consists, but also its orientation, i.e., this is relevant in case of combining and splitting vehicles as well as a comfort constraints for the first class. Haahr and Lusby (2017) are the first work which consider the problem of integrating rolling stock scheduling with train unit shunting and depot planning.

Grimm et al. (2019) provides an algorithmic extension to tackle the maintenance constraints by dynamically using cuts. The focus of Hoogervorst et al. (2021) is to quickly provide solutions with a heuristic for the dispatching or re-scheduling case. In contrast to that, the strategic weekly problem is considered by the authors of Gao et al. (2022). The algorithmic Branch & Price approach for different layered networks is very similar to Borndörfer et al. (2016) and to the implementation in R-OPT, respectively. Differences exists basically in the concrete construction of the graph and the handling of the hyperarcs.

	Scope	Topic	Process	Requirements Integration	Solution Method				
	Cyclic	Passenger railway	Composition planning	Train composition order	Commercial (MIP) solver	Heuristic	Column Generation	Branch & Price	Branch & Price & Cut
Borndörfer et al. (2016)	✓	✓	✓	✓	✓	✓	✓	✓	-
Reuther (2017)	✓	✓	✓	✓	✓	✓	✓	✓	-
Haahr and Lusby (2017)	-	✓	✓	✓	✓	✓	✓	✓	✓
Grimm et al. (2019)	✓	✓	✓	✓	✓	✓	✓	✓	✓
Hoogervorst et al. (2021)	-	✓	✓	✓	✓	✓	✓	✓	✓
Gao et al. (2022)	✓	✓	✓	✓	✓	✓	✓	✓	✓
Zhao et al. (2023)	-	✓	✓	✓	✓	✓	✓	✓	✓
this work	✓	✓	✓	✓	✓	✓	✓	✓	-

Table 1: Classification of this work and recent literature on optimization for rolling stock planning based on Thorlacius (2017).



In Reuther (2017) hyperarcs are constructed dynamically and are a fundamental part of the model formulation. In contrast to that Gao et al. (2022) use a graph extension for consists and additional variables and constraints to model coupling and decoupling. Furthermore, their model makes use of maintenance or service paths as proposed in Grimm et al. (2019). Zhao et al. (2023) present a rather exceptional non-linear approach to consider the integrated optimization of train formations and rolling stock schedules with time-dependent demand. The MINLP formulation is solved by a reformulation as a MIP using a commercial solver for a single line of a Chinese subway network.

### 3 The Model inside R-OPT

The core of R-OPT is an optimizer to solve a multi-depot vehicle scheduling problem with compositions. Our paper builds directly up on the notation and definitions presented in Reuther (2017) and Reuther and Schlechte (2018). Given is a set of trips  $T$  to be covered by vehicles of certain types. Set  $F$  denotes the set of valid vehicle types. The main difference to classical vehicle scheduling is that for rolling stock scheduling various trips must be covered by several vehicles which are operated in a consists of vehicles colloquially known as “train”. Reuther (2017) used the term *configuration* for the set of combined vehicles and compositions if in addition the position and orientation of the individual vehicle within is fixed. We restrict ourselves to the presentation of compositions with positions only for the sake of simplicity. Thus, a composition  $f = (f_1, f_2, \dots, f_n) \in F^n$  of size  $n \in \mathbb{N}$  models that at position  $i$  the vehicle group  $f_i$  is used. For each trip  $t$  we are given a set of valid compositions, e.g. the set  $\{(blue), (blue, red), (red, blue), (red, red)\}$ . Thus, in this example we can choose from four compositions with two different sizes and types for this trip. Note that depending on the composition size the absolute position 2 is present or not.

In Chapter 1 of Reuther (2017) a reformulation of the hypergraph model by using standard arcs is presented which is also the base model of R-OPT. This mathematical model can be formulated with the construction of layered digraphs. The technical layer  $N = (T, L)$  consists of nodes  $T$  for each trip  $t$  and arcs  $L$  if the trips can be performed consecutively, i.e., locations and times must fit. To decide whether trip  $t \in T$  can be performed before  $s \in T$  we use a complex turn time rule concept, if no movement of the vehicles is needed, or an even more complicated ruleset to determine if a possible deadhead trip exists to bring a vehicle without passengers from the end location of node A to the start location of node B in time. In that case an arc  $(t, s) \in L$  exists to represent this connection.  $Z(t)$  denotes the set of valid compositions for  $t$ . Further details are considered in the vehicle layer  $D$  where the set of nodes  $V$  are triples  $(t, f, p)$  of trips, vehicle types, and positions. The definition of  $Z(t)$  and  $L$  induce the enumerated representation as  $V$  and  $A$  so that operating the trip  $t$  with vehicle type  $f$  at absolute position  $p$  is modeled.

Table 2 lists the necessary symbols to finally formulate the multi-depot vehicle scheduling problem with compositions. The binary decision variable  $y_t^z$  is one if trip  $t$  is operated with composition  $z$  or zero otherwise. The binary decision variable  $x_a$  is one if arc  $a$  is part of the solution which means a vehicle of type  $f$  is operating trip  $t$  in a composition at position  $o$  before succeeding with trip  $s$  at position  $p$  with  $a = ((t, f, o), (s, f, p))$ . The binary decision variable  $y_l^z$  is one if the trip sequence of  $l$  is operated with composition  $z$  or zero otherwise. The slack variable  $u_a$  is relevant to track compositions changes, i.e., decoupling of a composition into individual vehicles.

Symbol	Description
$T$	set of trips
$F$	set of vehicle types
$Z(t)$	set of valid compositions for node/trip $t$
$L$	set of arcs related to consecutive succession of trips with the same vehicle (deadheads, turns)
$N = (T, L)$	directed event-activity scheduling graph
$V$	set of vehicle nodes with $v = (t, f, p) \in V$ with trip $t$ , vehicle type $f$ and position $p \in \mathbb{N}$ within the composition
$A$	set of standard vehicle arcs $a = (u, v)$ with $u, v \in V$
$D = (V, A)$	directed vehicle event-activity scheduling graph with vehicle types and positions
$\delta_{out}(v)$	$\subseteq A$ all arcs starting from $v \in V$
$\delta_{in}(v)$	$\subseteq A$ all arcs ending in $v \in V$
$x_a$	binary variable, if arc $a$ is used
$y_t^z$	binary variable, if node $t$ is assigned to composition $z$
$y_l^z$	binary variable, if arc $l$ is assigned to composition $z$
$u_a$	slack variable to repair position $t$ at node $u$ and position $h$ at node $v$ for arc $a = uv$ and vehicle $f$
$c_a$	operational cost of vehicle $f(a)$ for the connection of the trips covered by $a$
$c_t^z$	operational cost of composition $z$ and trip $t$
$c_l^z$	operational cost of composition $z$ and the connection of trips covered by $l$
$d_a$	coupling and decoupling cost of vehicle $f(a)$

Table 2: All symbols for the vehicle scheduling model with compositions.

$$\min \sum_{a \in A} c_a x_a + d_a u_a + \sum_{t \in T} \sum_{z \in Z(t)} c_t^z y_t^z + \sum_{l \in L} \sum_{z \in Z(l)} c_l^z y_l^z \quad (1)$$

$$\text{s.t.} \quad \sum_{a \in \delta_{out}(v)} x_a - \sum_{a \in \delta_{in}(v)} x_a = 0 \quad \forall v \in V \quad (2)$$

$$\sum_{a \in \delta_{out}(v)} x_a - \sum_{z \in Z(t, f)} y_t^z = 0 \quad \forall v = (t, f, p) \in V \quad (3)$$

$$\sum_{z \in Z(t)} y_t^z = 1 \quad \forall t \in T \quad (4)$$

$$u_a - x_a + \sum_{z \in Z(l)} y_l^z = 0 \quad \forall a \in A \quad (5)$$

$$x_a \in \{0, 1\} \quad \forall a \in A \quad (6)$$

$$y_t^z \in \{0, 1\} \quad \forall t \in T, z \in Z(t) \quad (7)$$

$$y_l^z \in \{0, 1\} \quad \forall l \in L, z \in Z(l) \quad (8)$$

$$u_a \in \{0, 1\} \quad \forall a \in A \quad (9)$$

The objective in rolling stock scheduling is manifold. Formula (1) states in a generic way the focus on minimizing the cost of the schedule. Cost of operating a trip  $t$  with a given composition  $z$  is given by  $c_t^z$ . Cost of connecting two trips with  $l$  and the given composition  $z$  is denoted by  $c_l^z$ , respectively. Cost which can be directly mapped on a vehicle using an arc  $a$ , e.g., cost for deadhead kilometer if the trip at the tail of  $l$  ends in another location than

the trip at the head of  $l$  starts, are denoted by  $c_a$ . Moreover,  $d_a$  defines the cost for coupling or decoupling a composition in its individual vehicles.

Equality (2) models flow conservation for all nodes of  $D$ . Constraints (3) couples the chosen composition  $z$  at  $t$  with the sum of all outflow arc variables  $x$  from node  $v$ . Note that the set  $Z(t, f)$  are all compositions of  $Z(t)$  containing  $f$ , and  $Z(l)$  are all valid composition for  $l$ , respectively. The set partitioning constraints (4) ensure that each node, i.e., trip,  $t$  is assigned to exactly one composition  $z$  from set  $Z(t)$ . Constraints (5) couple the chosen composition  $z$  on arc  $l$  with the sum of all vehicle arc variables  $x$ . In case that all contained vehicles of composition  $Z(l)$  are chosen correctly with respect to the given ordering of  $z$ , then  $y_l^z$  is set to one. Otherwise it must be zero and all auxiliary slack variables  $u_a$  are active to count the number of vehicles which are part of the decoupling activity. Finally, all variables domains are defined in (6)-(9).

## 4 The Algorithmic Approach

R-OPT is a standalone solver without using any commercial modeling language or optimization solver. The entire code is implemented in C/C++ and based on various components, e.g., a network simplex implementation for the minimum-cost flow problem denoted as MCF, see Löbel (1998), a bundle solver, see Borndörfer et al. (2008), to approximately solve linear programming formulations, and a rapid branching heuristic for integer programs, see Borndörfer et al. (2012). In a nutshell, the algorithmic approach implemented in R-OPT consists of the following steps:

1. Solve the base model presented in Section 3 with a Lagrangean relaxation approach using the bundle method and the MCF solver with column generation for the arcs.
2. Find an optimal solution of the vehicle scheduling model with at most one maintenance rule, i.e., with the most restrictive or important one.
3. Extend the schedule iteratively by an heuristic until all maintenance rules are fulfilled.
4. Improve the schedule iteratively by the method discussed in Borndörfer et al. (2015) which is based on the acyclic TSP heuristic of Kanellakis and Papadimitriou (1980).

In Step 1 we solve the Lagrangean relaxation resulting when equations (4) are relaxed and violations are turned into the objective function. The remaining problem then decomposes in a set of min-cost-flow problems per vehicle type. Thus, in each iteration of the bundle algorithm, we solve these problems in parallel with the MCF-solver described in Löbel (1998). To consider one maintenance condition in Step 2 we multiply each node  $v$  by its potential resource consumptions  $R$  to construct a resource expanded graph. That is a node  $v$  in the original graph gives rise to a set of nodes  $v_r$  for all  $r \in R$  in the resource expanded graph. Then, nodes are connected by arcs such that the resource consumption is tracked correctly. That is if an arc in the original graph connects nodes  $u$  and  $v$  consuming  $p$  units of the considered resource, then in the resource expanded graph all nodes  $u_r$  are connected with nodes  $v_{r+p}$ . The bundle method produces a primal solution of the LP-relaxation of the model. To find an integral solution, that is a relaxed rolling stock schedule, we use the rapid branching heuristic, which was introduced in Borndörfer et al. (2008) for integrated duty and vehicle scheduling. In Borndörfer et al. (2012) a generalized concept was given as well as the demonstration to use this heuristic for different applications, e.g., rolling stock

scheduling. This heuristic is a branch-and-bound heuristic that fixes in each iteration not only a single variable, but a set of variables that are close to one in the LP-relaxation. To enlarge the numbers of close-to-one variables a dynamic perturbation of the objective function is used such that the optimal solution does not change too much but the number of near integral variables increases significantly.

After Step 2 we have a rolling stock schedule, i.e. cycles or paths depending on the model type, which might still violate the relaxed maintenance constraints or coupling activities. Hence, Step 3 and 4 are realized by a Variable-Depth Neighborhood Search (VDNS) heuristic that aims to repair violations and to find good solutions by local search. Note that violations result in high cost and the first priority of the objective in Step 3 and 4 are feasibility. Ahuja et al. (2002) and Pisinger and Ropke (2010) provide a comprehensive survey on VDNS and its applications. R-OPT uses chained two- and three-node exchanges in cycles to find improved solutions of the problem. It turned out to be crucial to implement very efficient checks of the feasibility of the resulting solutions and their costs to keep running times of the algorithm reasonably low.

## 5 Computational Results & Application at SBB CARGO AG

Finally, we present results of R-OPT for scenarios from SBB CARGO AG and other railway undertakings. The economic motivation, planning challenges, and benefits from algorithmic decision support at SBB CARGO AG are topic of Gerber et al. (2022). The focus in this paper is on the computational aspects and capabilities of R-OPT. We present results for the following instances. The scenario C\_RE\_484\_2023 considers a strategic weekly scenario with all trips of 2023 for the fleet named RE\_484. During the year 2021 a novel vehicle type, i.e. RE\_484, was introduced step by step. At the final stage the available fleet consisted of 17 new vehicles. The challenge at SBB CARGO AG was then to identify the optimal portion of mandatory trips which were formerly covered by vehicles of other types, i.e., RE\_620 and RE\_420 in order to optimize the usage of the RE\_484 locomotives. One

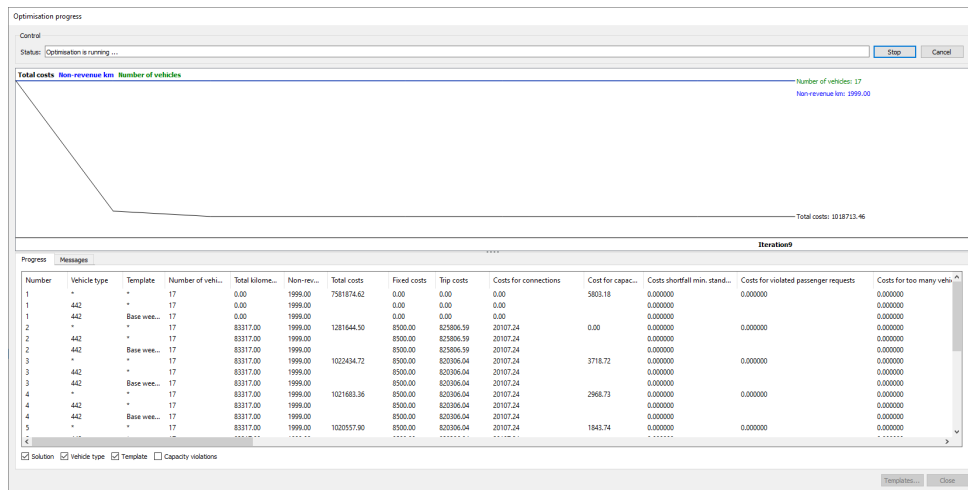


Figure 3: Monitoring view of the optimization R-OPT inside IVU.RAIL.

instance	number of trips	number of vehicle types	number of compositions	number of maintenance rules	none trivial compositions/coupling	capacities	regularity	number of vehicles	time
1	306	1	1	2	-	✓	✓	5	600
2	319	1	1	3	-	-	✓	12	600
3	146	1	1	3	-	✓	✓	4	600
4	382	2	2	3	-	✓	✓	11	1500
5	150	2	2	4	-	✓	✓	6	500
6	1094	5	5	2	-	-	-	34	4000
7	3182	10	10	2	-	-	✓	82	57600
9	1898	8	8	2	-	-	✓	54	30000
10	2315	7	7	0	-	-	✓	59	33400
C_RE_484_2023	671	1	1	0	-	✓	-	11	17
C_RE_620_RE_420_2023	1959	2	2	0	✓	-	-	69	830

Table 3: Results of R-OPT for test cases.

crucial restriction of that process was the necessary qualification of the locomotive driver for that new vehicle type. Thus, the number of depots where training is required should be minimized in that process in addition to the efficient usage of the locomotives. This gives rise to a selection of depots and trips by geographical arguments and a first reasonable subset to be considered for the optimization. In an iterative manner by adapting the input subset of trips R-OPT, amongst other tools, was used to optimally utilized the new fleet. Figure 3 shows the monitoring view of an optimization of R-OPT inside IVU.RAIL. Several KPIs of the current best solution are available live as a progress information such that the planners can stop the optimization during the run, e.g., if the solution quality already fulfills desired requirements.

The subsequent planning step - duty scheduling - was also performed by the optimization module DS-OPT of IVU.RAIL to simulate and analyze the effect on the crew resources. The foundations of the algorithmic approach used in DS-OPT to solve crew scheduling problems at SBB CARGO AG can be found in Grötschel et al. (2003). Thanks to the mentioned decision support tools the go-live of the new fleet was tackled and planned at an early stage and a smooth implementation of this process was realized.

The larger scenario C\_RE\_620\_RE\_420\_2023 is a current example from SBB CARGO AG for the timetable change from 2022 to 2023. This is based on the complete set of

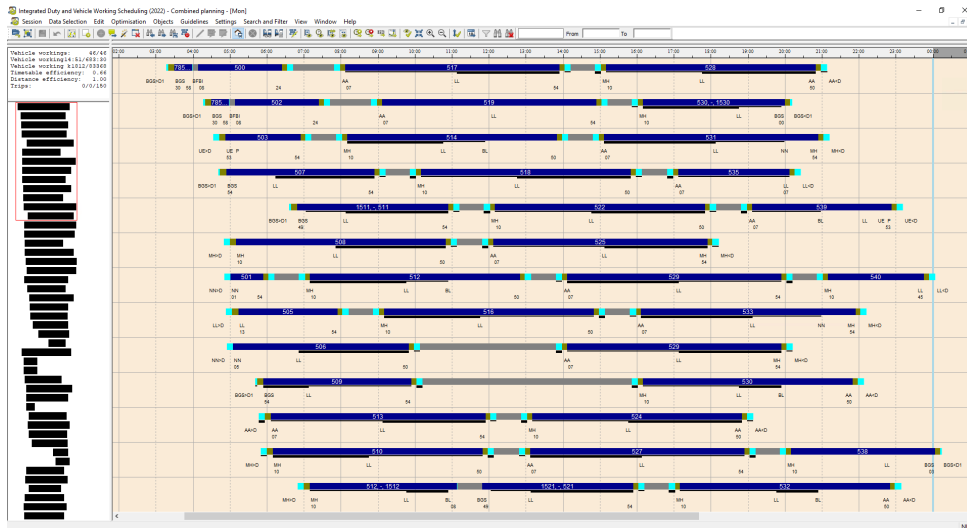


Figure 4: Detailed view of the vehicle working schedule in IVU.RAIL.

trips which are classified as *regularly scheduled* for a weekly cyclic horizon. The crucial decision which of the two vehicle types, i.e. RE\_620 or RE\_420, should operate which trip was optimized by R-OPT. This leads to an optimized rolling stock schedule for both fleets for the regular scheduled traffic. In total, R-OPT needed 69 vehicles of the two different types to cover all the 1959 regular trips.

All other instances are anonymous from other projects with various railway undertakings in Europe. The instances 1-10 are strategic scenarios of a large European railway undertaking providing regional passenger trains. The different feature sets and varying sizes demonstrate the versatile application of R-OPT.

Table 3 shows the key numbers and features of the considered instances. For the calculation we used 8 parallel threads on a machine with 128 GB RAM and an Intel(R) Xeon(R) W-2265 CPU (3.50 GHz). From Column 2 to 5 the number of trips, vehicle types, compositions and considered maintenance rules are given. We mark the instance if at least one none

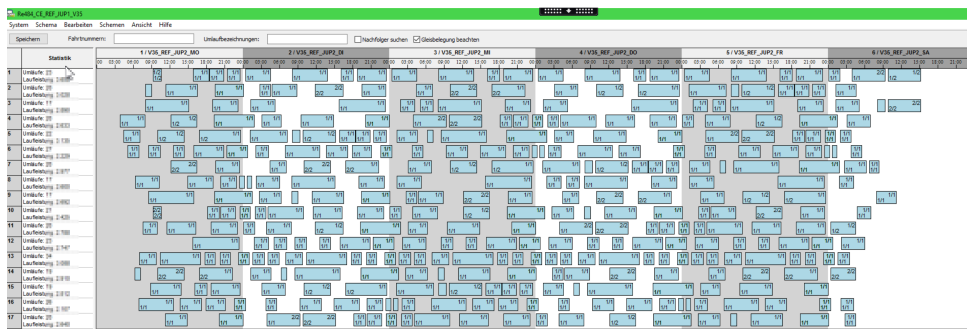


Figure 5: Rolling stock roster solution for C\_RE.484\_2023 shown in IVU.RAIL.

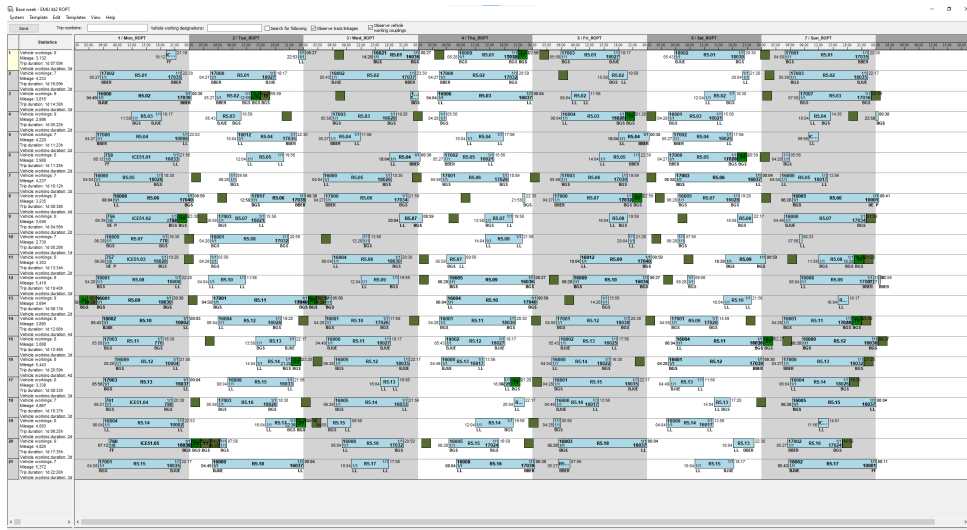


Figure 6: Rolling stock roster solution with maintenance shown in IVU.RAIL.

trivial composition, i.e., one composition with size larger than 1, exists. Note that if this is the case coupling and decoupling must be respected. In addition, we mark if the instance considers resource capacities and regularity. The last two columns denote the number of needed vehicles in the found solution and the necessary runtime of R-OPT. Note that this is only the runtime of R-OPT for the optimization of the mathematical model without collecting and constructing the necessary input data from the database. This step can also take some time in case of a large network with many available deadhead trips. The scenarios 1-10 and the two in detail discussed real world scenarios from SBB CARGO AG demonstrate the wide range of features and use-cases which R-OPT is able to handle.

Figure 4 shows some vehicle workings for one specific day in detail in a classical Gantt chart with time, space, and train information. An exemplary rolling stock cyclic roster solution is shown in Figure 6. One can see the order of vehicle workings (blue) and maintenance services (green) calculated by R-OPT for a sample week (Monday to Friday) with 26 vehicles. Each vehicle working consists of a sequence of trips and starts and ends in a specific depot. At some of these depots maintenance tasks can be scheduled. Figure 5 shows the vehicle workings from Monday to Saturday for the solution of scenario C\_RE\_484\_2023. Note that detailed train information, which are obviously available in IVU.RAIL, are made unrecognizable. The benefits of enabling optimization at SBB CARGO AG are manifold. Direct and measurable are a reduction of planning time, a reduction of empty journey kilometers, and in general an increase of the resource utilization. Furthermore, it allows for data based simulations of what if scenarios in case of construction sites, modification of depot capacities or fleet changes.

## 6 Conclusion

In this article, we provide a comprehensive collection and classification of technical requirements considered in the scientific literature for rolling stock rotation planning. We present

the commercial rolling stock rotation optimizer R-OPT which is able to handle the majority of the discussed features. Moreover, the basic optimization model and algorithmic ingredients of R-OPT are briefly collated. Finally, results from the successful deployment at SBB CARGO AG and computations for railway undertakings for passenger traffic in Europe demonstrate the capabilities of R-OPT. Recent algorithmic developments and feature space extensions of the mathematical models increased and will further increase the acceptance of decision support systems and in particular of optimization modules in the field of rolling stock rotation planning. Promising research directions for rolling stock rotation planning are further integration of aspects related to neighboring planning tasks, i.e., timetabling and track and depot allocation aspects and crew scheduling considerations.

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