An ICA Algorithm for Separation of Convolutive Mixture of Periodic Signals

Doron Benzvi and Adam Shafir

July 14, 2018

An ICA Algorithm for Separation of Convolutive Mixture of Periodic Signals

***Abstract* -** This paper describes Independent Component Analysis (ICA) based fixed-point algorithm for the blind separation of convolutive mixture of periodical signals. The proposed algorithm extracts independent periodical sources from their mixtures in frequency domain, where they are represented by their sets of harmonics. The individual harmonics are separated by referring to narrow frequency segments of the mixed signals, which include two harmonics each at most. The algorithm offers a simple solution to the permutation problem common to source separation using ICA.

***Keywords* -** Blind signal separation, independent component analysis, convolutive mixture.

1. INTRODUCTION

The goal of Blind Signal Separation (BSS) is to estimate latent sources from their mixed observations without any knowledge of the mixing process. This challenging problem has bagged much research attention due to very wide area of applications.

Mathematically, a BSS problem can be described as the process of estimating R original source signals

$$s(n) = [s_1(n), s_2(n),....., s_R(n)]^T$$

from their M observed mixed signals derived from the sensors:

$$x(n) = [x_1(n), x_2(n),....., x_M(n)]^T$$

In the simplest case the mixing process produces instantaneous mixture, whereby the source signals are linearly combined with constant coefficients to produce the mixtures. A more challenging case is the convolutive mixing. The source signals undergo distortion by different systems generating the mixtures at the sensors:

$$x_j(n) = \sum_{i=1}^{R} \sum_{k=0}^{N-1} h_{ji}(k) s_i(n-k) \qquad j = 1,2,..., M \quad (1)$$

where $h_{ji}(n)$ is an N-point impulse response of the FIR system between source i and sensor j.

The corresponding relation in frequency domain is

$$X_j(f) = \sum_{i=1}^{R} H_{ji}(f) \cdot S_i(f) \qquad j = 1,2,..., M \quad (2)$$

where $H_{ji}(f)$ is the frequency response of the system between source i and sensor j.

The problem of BSS for convolutive mixtures can be solved by extending ICA algorithms developed for instantaneous mixtures. There are mainly two different approaches to solve the convolutive BSS problem: in time domain [2], [3], [4], and in frequency domain [5], [6], [7]. The frequency domain solution is less complicated, involving multiplication rather than convolution. One method, first proposed by Smaragdis [5], converts the frequency domain convolutive mixture into instantaneous mixture for different frequency bins. STFT is applied to the observation signals, and the same frequency bin from successive segments are collected to form new observaton signals. Then, the unmixing matrix is sought for every frequency bin using fixed point ICA algorithm in complex domain. The source signals are restored a frequency at a time.

Consider the case of two sources and two observations (*M=R=2* in eq. 2). The convolutive mixture is formulated as

$$\begin{bmatrix} X_1(f) \\ X_2(f) \end{bmatrix} = \begin{bmatrix} H_{11}(f) & H_{12}(f) \\ H_{21}(f) & H_{22}(f) \end{bmatrix} \begin{bmatrix} S_1(f) \\ S_2(f) \end{bmatrix} \qquad (3)$$

The ICA separates the signals in each frequency bin independently to provide the independent components, the restored signals:

$$\begin{bmatrix} \hat{S}_1(f) \\ \hat{S}_2(f) \end{bmatrix} = \begin{bmatrix} W_{11}(f) & W_{12}(f) \\ W_{21}(f) & W_{22}(f) \end{bmatrix} \begin{bmatrix} X_1(f) \\ X_2(f) \end{bmatrix} \qquad (4)$$

where $W(f)$ is the separating matrix for frequency f.

One major problem with this frequency domain solution lies in the indeterminacy of scaling and permutation. Scaling indetermincay means that the scaling of each frequency bin can be different, leading to spectral deformations of the original source signals. Permutation indeterminacy means that the order of the restored signals may be different for different frequency bins, ending in mixed frequency content of the restored source signals.

The main advantage of the proposed algorithm described in the following sections is in the simple solution to the permutation problem it offers.

2. BSS FOR CONVOLUTIVE MIXTURE OF PERIODIC SIGNALS

2.1 BSS for convolutive mixtures of sinusoidal signals

Consider the case where the source signals are sinusoidal. Then, the convolutive mixture can be converted into instantaneous mixture, both in time and frequency.

Remember, that when a system with a frequency response $H(f)$ is operating on a complex sinusoidal signal

$$s_i(n) = e^{j2\pi f_i n}$$

the output signal is

$$s_o(n) = H(f_i)e^{j2\pi f_i n}$$

and in frequency domain

$$S_o(f) = H(f_i) \cdot \delta(f - f_i)$$

The same relations are applicable for real sinusoidal signals, if only half of the frequency range, $[0, \pi]$, is considered, and the second half is just the complex conjugate of the first half.

Hence, for the case of two sinusoidal source signals, with frequencies f_1 and f_2, and two sensors, the convolutive mixtures is formulated as:

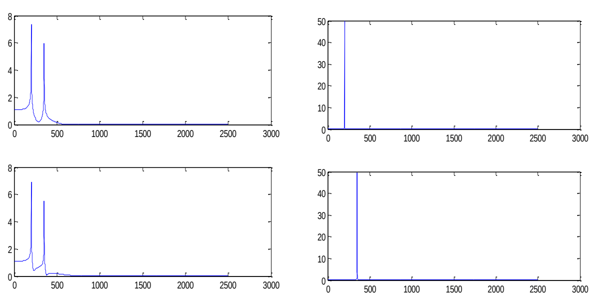
$$\begin{bmatrix} X_1(f) \\ X_2(f) \end{bmatrix} = \begin{bmatrix} H_{11}(f_1) & H_{12}(f_2) \\ H_{21}(f_1) & H_{22}(f_2) \end{bmatrix} \begin{bmatrix} S_1(f) \\ S_2(f) \end{bmatrix} \qquad (5)$$

Practically, since the sinusoidal signals are of a finite length, their transform, $S_1(f)$ and $S_2(f)$ are not exact impulses. The observation signals $X_1(f)$ and $X_2(f)$, include each the two frequencies f_1 and f_2, see Fig. 1.

Applying ICA algorithm for complex signals and coefficients to the observation signals, $X_1(f)$ and $X_2(f)$, results in restoring the separate sinusoidal signals. The particular algorithm used is the kurtosis-based RobustICA, offered by Zarzoso and Comon [8]. The algorithm returns the separated signals, as well as the estimated mixing matrix and the estimated extracting matrix. It turns out that the elements of the returned mixing matrix, *A*, closely resembles the values of the frequency responses involved at the frequencies of the sinusoidal sources. Namely,

$$A \approx \begin{bmatrix} H_{11}(f_1) & H_{12}(f_2) \\ H_{21}(f_1) & H_{22}(f_2) \end{bmatrix} \qquad (6)$$

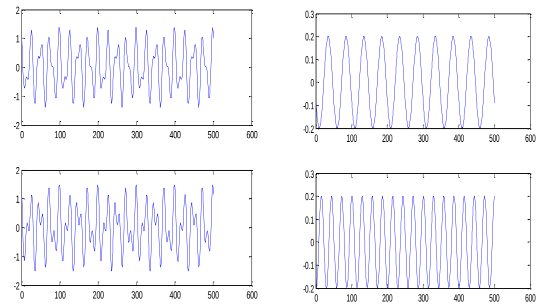
Fig. 1 shows the input sinusoidal source, their convolutive mixtures, and the restored separated signal, both in time and frequency. In the simulation the four convolving systems were randomly selected.



(a) (b)

Fig.1 Separating two sinusoidal source signals from their convolutive mixtures. (a) the observed mixed signals in frequency (b) the restored separated signals

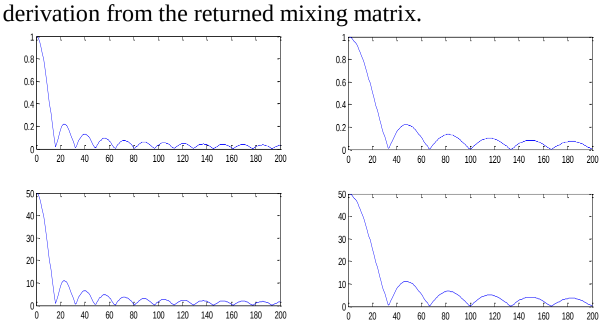
The results in time domain are demonstrated in Fig. 2.



(a) (b)

Fig.2 Separating two sinusoidal source signals from their convolutive mixtures – results in time (a) the observed mixed signals (b) the restored separated signals

By varying the frequencies of the two sinusoidal source signals, the frequency response of the convolving systems can be estimated from the returned mixing matrix A. Fig. 3 shows the frequency response for two of the convolving systems, and their derivation from the returned mixing matrix.



(a) (b)

Fig.3 Estimating the frequency response of the convolving system from the returned mixing matrix A. (a) H_{11} and its estimate (b) H_{12} and its estimate

2.2 BSS for convolutive mixtures of signals composed of a linear combination of sinusoidal components

Once it is understood how two sinusoidal signals can be separated from their convolutive mixtures, the next stage is to try and separate two signals, composed each of several sinusoidal components at different frequencies, from their convolutive mixtures. Suppose

$$s_1(n) = \sum_{i=1}^{L} a_i \cos(2\pi f_i n), \quad s_2(n) = \sum_{k=1}^{L} b_k \cos(2\pi f_k n)$$

where the frequencies in s_1 and s_2 are supposed to be different.

The signals are convolutively mixed by four randomly selected systems, as in eq. (5).

The proposed strategy is to divide the observation signals in frequency, $X_1(f)$ and $X_2(f)$, into consecutive short segments, such that in each segment there are at most two prominent frequencies, presenting two sinusoidal elements. This assumption is not too limiting, since the segment size in frequency may be reduced to meet the assumption.

The ICA algorithm is then applied to each segment separately, and the restored separated signals are constructed by combining the separated signals from the different segments. Fig. 4 shows the results of separating two source signals, each with four sinusoidal components.

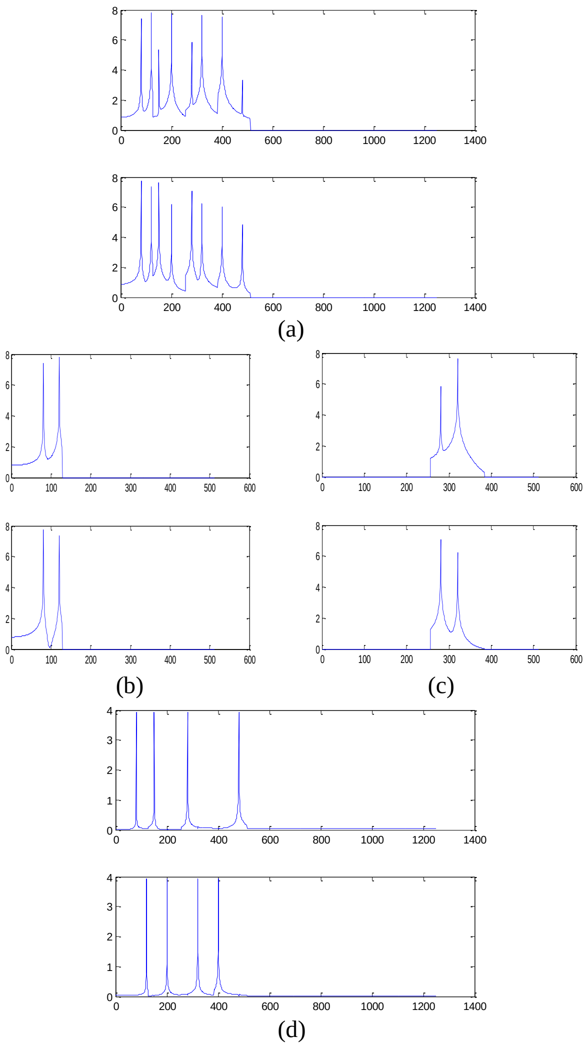


Fig. 4 Separating two source signals, each with a few frequency components, from their convolutive mixtures. (a) the spectrum of the two mixed signals (b)-(c) dividing the two mixed signals into segments with two sinusoidal components each (d) the two separated signals in frequency.

In each segment the algorithm separates the two sinusoidal components from the mixed signals. The permutation problem arises in the need to assign in each segment one frequency component to S_1 and the other to S_2.

2.3 The Solution to the Permutation Problem

It was found out, that if in a segment with two frequency components the frequency from S_1 is smaller than that from S_2, then the order of the returned mixing matrix A is as in eq. (6). Alternatively, if the frequency from S_1 is larger than that from S_2, then the columns of the returned mixing matrix are reversed, namely,

$$A \approx \begin{bmatrix} H_{12}(f_2) & H_{11}(f_1) \\ H_{22}(f_2) & H_{21}(f_1) \end{bmatrix}$$

In both cases the sinusoidal components are separated. Still, the permutation problem arises as to whether to assign the separated component with the smaller frequency to S_1 or to S_2.

The solution found relies on the continuity of the frequency response of the convolving systems. It entails that the mixing matrices of consecutive segments should be similar. Namely, if in two consecutive segments the smaller frequency comes from the same source signal, the returned mixing matrices for these segments should be similar. Then, the smaller frequency in the two consecutive segments are assigned to the same source. Alternatively, if in one segment the smaller frequency belongs to one source and in the next segment it belongs to the second source, then the returned mixing matrix for the two segment differ significantly (due to the columns inversion). In that case, the smaller frequency in the second segment is assigned to the opposite source to that of the first segment. The results presented in Fig. 3-5 demonstrate that the permutation problem is resolved, assigning the sinusoidal components to the correct source signal.

2.4 BSS for convolutive mixtures of multiple periodic signals

Previous sections have dealt with the separation of two convolutively mixed periodic source signals. The expansion of the method to the case of multiple periodic source signals has been examined. When all source signals are periodic signals, they are properly separated from their convolutive mixture regardless of how many they are. Moreover, the permutation problem is resolved, similarly to the two-signals case: there is a strict correlation between the order of frequencies and the order of the columns in the coefficients matrix returned. For example, in case of 3 sinusoidal source signals with frequencies $f_1 > f_2 > f_3$, the columns of the mixing matrix A returned are arranged with the coefficients of the 3rd signal first.

Fig. 5 displays the results of applying the method to 3 sinusoidal source signals.

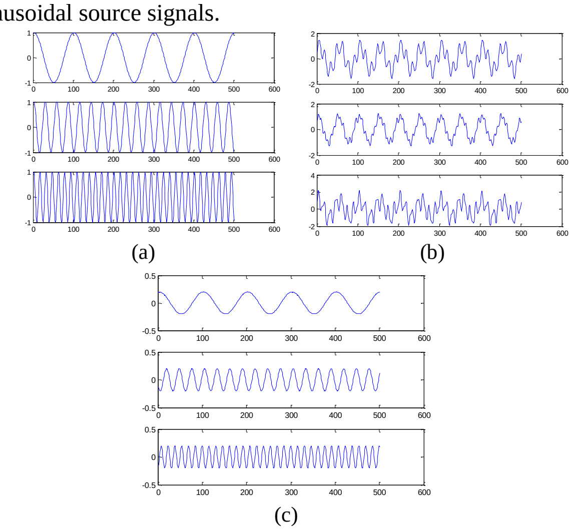


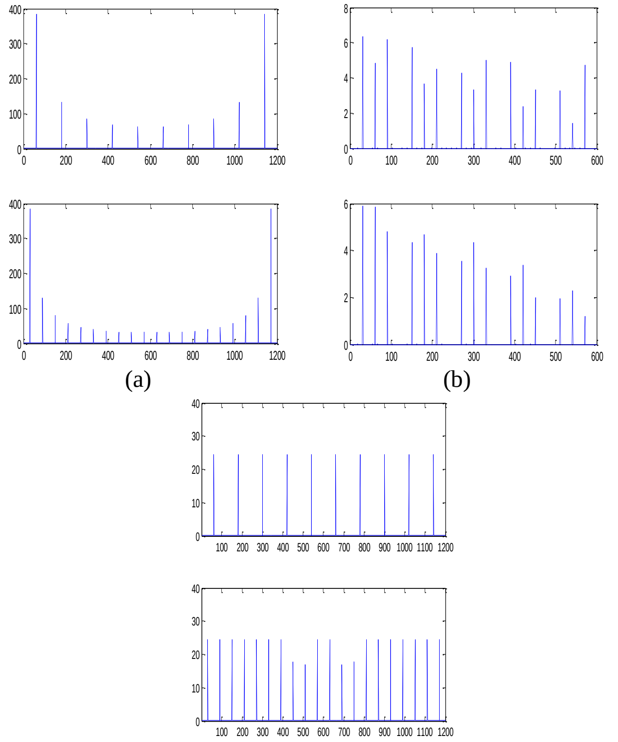
Fig. 5 Separating three sinusoidal source signals from their convolutive mixtures. (a) The original three source signals. (b) The three mixed signals (c) The three separated restored signals.

3. EXPERIMENTAL RESULTS

The algorithm presented has been applied to two more cases: the convolutive mixture of two periodic square wave source signals with different periods, and to the convolutive mixture of two musical notes played by string instruments.

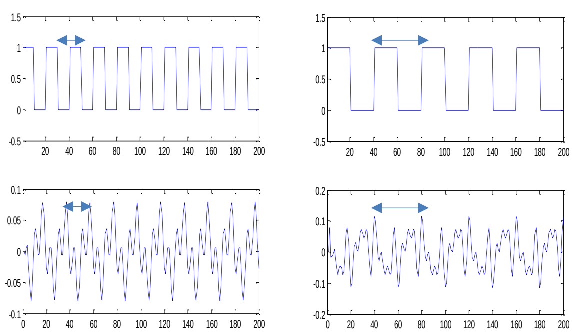
Fig. 6-7 shows the results obtained for BSS of a convolutive mixture of two square-wave source signals with different periods. Fig. 6 shows the results in frequency domain, and Fig. 7 shows the results in time.

It can be seen that the scaling problem of ICA is not resolved, and the amplitude of the different harmonics is not accurate.



(a) (b) (c)

Fig. 6 Separating two square wave source signals from their convolutive mixtures. (a) The original spectrum of the two source signals. (b) the spectrum of the two mixed signals (c) the spectrum of the two separated signals.

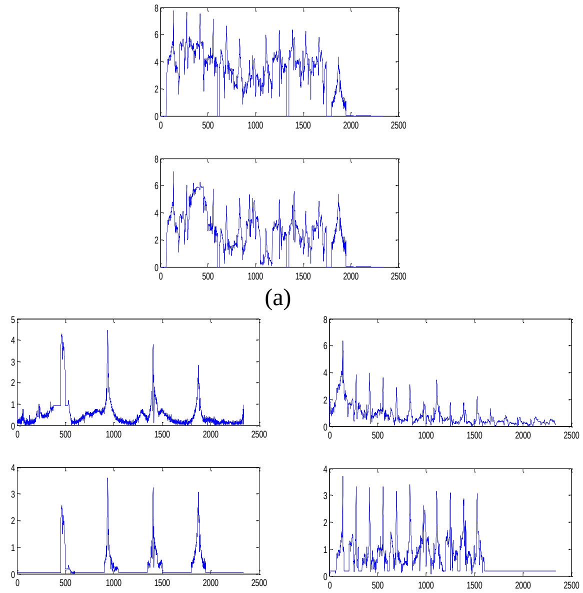


(a) (b)

Fig. 7 Separating two square wave source signals from their convolutive mixtures – results in time (a) The original square wave signal s_1 and the restored signal (b) The original square wave signal s_2 and the restored signal

It can be seen that the scaling problem of ICA is not resolved, and the amplitude of the different harmonics are not accurately restored.

Fig.8-9 show the results obtained for BSS of a convolutive mixture of two musical notes source signals. The peaks in frequency of the musical note signal are wide due to two reasons: the signals are semi-periodic, since their amplitude decays with time. Secondly, the note is of a finite length. Because of these wide peaks, a peak may be broken erroneously into two consecutive segments, if a fix segmentation size is used. This problem may be avoided if segmentation is dictated by the peaks, namely, a segment is chosen around the peaks rather than choosing segments of fixed frequency range.



(a)

(b) (c)

Fig. 8 Separating two musical notes source signals from their convolutive mixtures. (a) The spectrum of the mixed signals (b) the spectrum of the source signal s_1 and the restored signal (c) The spectrum of source signal s_2 and the restored signal

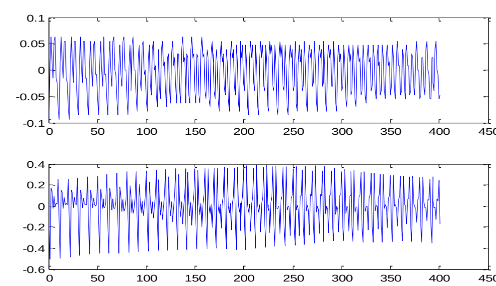


Fig. 9 Separating two musical notes source signals from their convolutive mixtures – results in time: The original musical note signal s_1 (top) and the restored signal (bottom)

4. CONCLUSIONS

In this paper we proposed a method to blindly separate periodic signals from their convolutive mixtures. It relies on the kurtosis-based fixed-point ICA algorithm operating on the mixed signals in frequency domain. Mainly, the spectrum of the mixed signals is subdivided into segments, and each segment is dealt with separately to restore the source signals. The main advantage of the proposed method is in the simple solution it provides to the ICA permutation problem. We validated the performance of our method on artificially generated periodic signals, as well as on real periodic signals of musical notes. The results indicate that the proposed method is able to separate the mixed signals into close estimates of the source signals. Future endeavors are to expand the method to work on mixtures of more than two source signals, and to try and solve the unresolved scaling problem, still harming the results.

5.REFERNCES

[1] Hyvärinen, A., Karhnen J.,Oja E., "*Independent Component Analysis*", John Wiley& Sons, New York, 2001.

[2] Bell, T. Sejnowski. *"An Information-maximization approach to blind separation and Blind Deconvolution*". Neural Computation, 7:1129-1159, 1995.

[3] T.-W. Lee, M. Girolami and R. Lambert. *"Blind separation of delayed and convolved sources"*. In Advances in Neural Information Processing Systems, volume 9, pp. 758-764, MIT Press, 1997.

[4] K. Torkkola. "*Blind separation of convolved sources based on information maximization"*. In Proc. IEEE Workshop on Neural Networks and Signal Processing (NNSP'96), Kyoto, Japan, pp. 423-432, 1996.

[5] P. Smaragdis. "*Blind separation of convolved mixtures in frequency do-main"*. Neurocomputing, 22:21-34, 1998.

[6] N. Mitianoudis, M. Davis. "*Audio Source Separation of Convolutive mixtures"*. IEEE Trans. on Speech and Audio Processing. 11(5):489 , 2003

[7] R. Prasad, H. Saruwatari, K. Shikano, *"An ICA Algorithm for the Separation of Convolutive Mixture of Speech Signals"*, International journal for Electrical and Communication Engineering, Vol.2 No, 11, 2008

[8] V. Zarzoso and P. Comon, *"Robust Independent Component Analysis by Iterative Maximization of the Kurtosis Contrast with Algebraic Optimal Step Size"*, IEEE Transactions on Neural Networks, vol. 21, no. 2, pp. 248-261, Feb. 2010.