Split Domination Number of Rooted Product of Some Simple Graph

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Abstract:

Kulli and Janakiram first proposed the concept of a split domination number. Let G be a graph with V as the vertex set and S as the subset of V as the vertex set. S is said to be a dominating set when every vertex in V-S is adjacent to at least one vertex of S. The graph has a split dominating set if the induced sub graph is disconnected. In this article, we studied split domination in graph theory for many types of special graphs.

Keywords: Domination number, Split Domination number, Rooted product, Comb graph.

1 Introduction

Graphs are one of the most common representations of both natural and man-made structures. In physical, biological, social, and information systems, graphs can be used to model a wide range of relationships and processes. Graphs can be used to illustrate a wide range of real-world issues. The fields of graph theory, computer engineering, and operations research experienced exponential growth in the late twentieth century and early twenty-first century. Graphs are used in computer science to describe communication networks, data organization, computing devices, computation flow, and so on. A directed graph, for example, can describe a website's link structure, with vertices representing web pages and directed edges representing links from one page to another. Travel, biology, computer chip design, and a variety of other industries can all benefit from a similar approach. As a result, developing algorithms to manage graphs is a hot topic in computer science. As a result, developing algorithms to man- age graphs is a hot topic in computer science. Graph rewrite systems are frequently used to describe and depict graph transformations. Graph databases, which are de- signed for transaction-safe, persistent storing and querying of graph-structured data, are a complement to graph transformation systems that focus on rule-based in-memory graph manipulation. Each edge of a graph can be given a weight, which can be used to extend its structure. Weighted graphs, also known as graphs with weights, are used to illustrate structures in which pairwise links have numerical values. The weights could, for example, indicate the length of each road in a graph representing a road network.

2 Preliminaries

Definition 2.1. Dominating set: Let G be a graph with vertex set V and edge set E. Let S be the subset of vertex set V. If every vertex in <V-S> is adjacent to minimum one vertex of S then S is said to be a dominating set.

Definition 2.2. Dominating number: The size of a smallest dominating set is referred as the dominating number of a graph G denoted by (G).

Definition 2.3. Split Dominating set: If the induced sub graph <V-D> is disconnected, then the dominating set D is called split dominating set.

Definition 2.3. Split Dominating set: The split dominating number (G) of G is the minimum cardinality of the split dominating set.

Definition 2.5.Non split Domination number: A dominating Set D of G is a non-split dominating set, if the induced sub graph <V-D> is connected. Non split domination number $\gamma_{ns}(G)$ is the minimum cardinality of a non-split dominatingset.

Observation 2.6. Let K_n be the complete graph with n vertices then $\gamma_s(K_n \odot K_m) = n$.

Observation 2.7. If K_m be the complete graph with m vertices and P_n be the path graph with n vertices and C_n be the cycle graph with n- vertices then $\gamma_s(P_n \odot K_m) = \gamma_s(C_n \odot K_m) = n$.

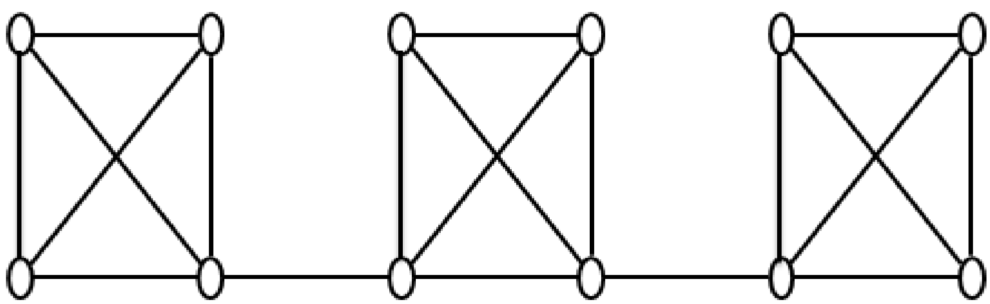

Figure: 1.

Observation 2.8. $\gamma(K_n \odot K_n) = \gamma_s(K_n \odot K_n)$.

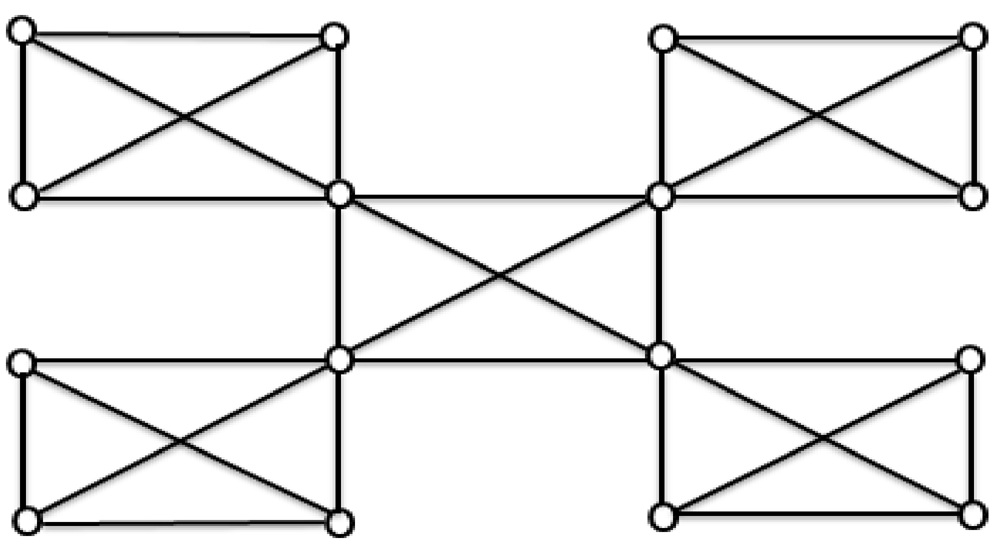

Figure: 2.

Observation 2.9. $\gamma(\mathrm{P_n} \odot \mathrm{K_n}) = \gamma_s(\mathrm{P_n} \odot \mathrm{K_m})$.

Observation 2.10. $\gamma(\mathrm{C_n} \odot \mathrm{K_m}) = \gamma_s(\mathrm{C_n} \odot \mathrm{K}_m)$.

3. Main Results:

Algorithm:

Initially, Set $S = \varphi$

Step: 1.

Include all the isolated vertices in S and remove all such vertices from the graph.

Step: 2.

Let a path corresponding to the diameter d of the resulting graph, exist between $v_1 \ and\ v_n$.

Step: 3.

Let $N[v_1] = \{v_{11}, v_{12}, \ v_{13}, \ldots . v_{1k}\}$. Choose v_{1p} such that $|N(v_{1p})| = \max\{|N(v_{1i})|: i = 1,2,3, \ldots .. k\}$. If there are more than one such node select the one which has maximum distance from v_n. Similarly, let $N[v_n] = \{v_{n1}, v_{n2}, \ v_{n3}, \ldots . v_{nl}\}$. Choose v_{nq} such that $|N(v_{nq})| = \max\{|N(v_{ni})|: i = 1,2,3, \ldots .. l\}$. If there are more than one such vertex select the one which has maximum distance from v_1.

Step: 4.

If $d(v_{1p}, v_{nq}) > 3$, then include both the nodes in S. Otherwise, compare the cardinalities of $N(v_{1p})$ with $N(v_{nq})$. If $|N(v_{1p})| > |N(v_{nq})|$, then include v_{1p} in s, else include v_{nq} in S. Let the included vertices be $\alpha_1, \ \alpha_2 \ or \ \alpha_1$ accordingly.

Step: 5.

Let $M = N[\alpha_1] \cup [\alpha_2] \ or \ N[\alpha_1]$ as the case may be delete all the edges having one end at either $\alpha_1 \ or \ \alpha_2$. Also delete all the edges having both ends in M. In this process, if any vertex becomes isolated, delete it from the graph.

Step: 6.

If the degree of any vertex in M reduces to 1, then delete it from the graph.

Step: 7.

Repeat the steps 1-7, until the graph becomes an empty graph.

Theorem 3.1. If $C_{m,m}$ be the comb graph with n vertices then rooted product of Split domination number is $\gamma_s(C_{m,m} \odot \mathrm{C}_{\mathrm{m,m}}) = \left\lceil \frac{n^2}{2} \right\rceil$.

Proof: Let G be a comb graph with n vertices and $v_1, v_2, v_3, \ldots \ldots .., v_n$ be the set of vertices in path graph of caterpillar. Let $u, u_2, u_3, \ldots \ldots .., u_n$ be the set of vertices in the leaves connected by the path graph. The rooted product of two comb graph is n times of first graph connected by second graph.

The number of vertices in rooted product graph was n^2. The split domination number of the product of two graph is then by the preposition $\frac{n^2}{2}$. Hence the rooted product of split domination number of two comb graph is $\left\lceil \frac{n^2}{2} \right\rceil$.

Theorem 3.2. If P_m and P_n be the path graph with m, n vertices respectively then $\gamma_s(P_m \odot \mathrm{P_n}) =$ m $\left\lceil \frac{n}{3} \right\rceil$.

Proof: Let $v_1, v_2, v_3, \ldots \ldots .., v_n$ be the set of vertices path graph with m vertices u, $u_2, u_3, \ldots \ldots .., u_n$ be the set of vertices with path graph with n vertices. The split domination of path graph with n-vertices is $\left\lceil \frac{n}{3} \right\rceil$. The rooted product of path graph with m, n vertices are m times of P_m. So the split domination number of two graph is m $\left\lceil \frac{n}{3} \right\rceil$.

Theorem 3.3. If W_n and W_m be the two wheel graph with n, m vertices respectively then $\gamma_s(W_n \odot \mathrm{W}_m) =$ n.

Proof: Let $u, u_2, u_3, \ldots \ldots .., u_n$ be set of vertices of wheel graph with m vertices v_1, $v_2, v_3, \ldots \ldots .., v_n$ be the set of vertices of second wheel graph. By the definition of rooted product n times of wheel graphs are joined by corresponding vertices of root graph. So the split domination number of the rooted product of wheel graph is enough to n vertices.

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