Numerical Modeling of Viscoelastic Pipelines Vibrations Considering External Forces

Kholidaxon Komilova, Qamariddin Zhuvonov, A Tukhtabaev and K Ruzmetov

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Kh.M. Komilova[a1], Q. Zhuvonov[a], A.Tukhtaboev[b], K.Ruzmetov[c]

[a]«Tashkent institute of irrigation and agricultural mechanization engineers» National Research University, Tashkent, Uzbekistan
[b]Namangan Civil Engineering Institute, Namangan, Uzbekistan
[c]Tashkent State Agrarian University, Uzbekistan

Abstract. The vibration process of an extended section of a pipeline conveying medium are studied in the paper using the well-known classical models of laminar flow of an ideal incompressible fluid and the Bernoulli – Euler rod bending. A mathematical model of linear problems on transverse vibrations of a rectilinear viscoelastic pipeline with a medium moving inside it is derived. A computational algorithm has been developed to solve the problems of the dynamics of a viscoelastic pipe conveying fluid flow. To describe the stress-strain dependence, the Boltzmann-Volterra integral model is used. Based on the developed computational algorithm, a package of applied computer programs has been created that allows conducting numerical studies of dynamic strain of a pipeline conveying fluid flow, based on the Winkler base, considering the viscosity properties of the structure material and the pipe bases, axial forces, internal pressure, resistance forces and external forces. When modeling linear problems, a number of new dynamic effects were investigated: it was found that an account of viscoelastic properties of the material and the pipeline base leads to a decrease in amplitude and frequency of pipe vibrations; it was revealed that with an increase in the frequency of external loads, the amplitude and frequency of pipe vibrations increase. It is shown that an increase in external load parameter leads to an increase in vibration amplitude, and to a decrease in vibration frequency.

Keywords: mathematical modeling, computational algorithm, viscoelasticity, pipeline, vibrations, external forces.

1. Introduction

Pipelines are widespread elements of technology used in oil and gas industry, engineering structures, aircrafts and in other sectors of economy. A pipeline with a fluid flowing under pressure is a design element of many systems. They are used in nuclear facilities, in the aircraft industry, in oil and gas industry, in chemical production facilities, in water supply systems in residential buildings, and in a great number of other objects surrounding a person. In the event of high-pressure pipeline destruction, depressurization of a joint or pipe rupture at the point of attachment, the consequences of such accidents can lead to significant material losses and to fatalities and environmental disasters. In view of severe economic and environmental consequences of possible accidents, increased demands are placed on the strength of the structures being developed and on the piping systems that are part of them. In most cases, full-scale testing (up to destructive loads) of pipelines conveying fluids, is not always possible or difficult due to their high cost. Under these conditions, mathematical modeling of the dynamics of pipeline systems with fluid becomes especially relevant (Li Qian et al., 2020a, 2020b).

One of the main problems of pipeline transport systems is their susceptibility to corrosion due to pipe material contact with aggressive media. According to statistics, most accidents occurring in oil- and gas-conveying pipelines are the result of corrosion processes. In connection with this, there is an acute problem of finding alternative ways to modernize oil and gas pipeline systems, especially when transporting aggressive media. An obvious promising and modern direction is the introduction of pipes made of high-strength and corrosion-resistant composite materials.

[1] Corresponding author. E-mail: komilova591@mail.ru

The study of vibration of pipelines made of composite materials is of great theoretical and applied interest. The solution to this problem is an effective application of the theory of viscoelasticity to real processes. Therefore, the methods and problems of viscoelastic pipelines vibration attract great attention of researchers.

In (Yuanhui Wang & Yiming Chen, 2020), the dynamic behavior of viscoelastic pipes conveying fluid was studied. With account for physical and mechanical properties of the pipeline material, a mathematical model has been developed that describes nonlinear variables with integro-differential equations of fractional order under corresponding initial and boundary conditions. A computational algorithm for polynomials is developed. The influence of the parameters on displacement, acceleration, strain and stress was studied. It was shown that dynamic properties are affected by a variable fractional order and a varying fluid velocity.

In (Wasiu A. Oke & Yehia A. Khulief, 2018) a numerical study of the vibration of a composite pipe conveying fluid was carried out. The effect of damaged internal surfaces on the vibration behavior of a composite pipe was studied.

In (Bahaadini & Saidi, 2018), vibrations of a single-walled carbon nanotube were studied. Based on the Timoshenko theory using the Hamilton principle, equations of motion of nanotubes considering the temperature were obtained. The influence of compressive axial force, rotation speed, the ratio of gravity and mass of fluid on the critical velocity of flutter and divergence were investigated.

In (Xiao-Ying Zhao et al., 2018), vibrations of a viscoelastic pipe conveying fluid at displacement excitation of a harmonic base were studied. In a geometrically nonlinear statement based on the Euler-Bernoulli theory, mathematical models of vibration problem and dynamic stability of a viscoelastic pipe were built under external load changing over time. To describe the processes of viscoelastic material strain, the Kelvin model of the hereditary theory of viscoelasticity was used.

The widespread use of new composite materials in nuclear facilities, in the aircraft industry, in oil and gas industry, in chemical production facilities, as well as in other branches of mechanical engineering, requires further improvement of mechanical models of deformable bodies and the development of calculation methods, taking into account the viscoelastic properties of the material of thin-walled structures. Thus, an undoubted scientific and practical interest presents the construction of mathematical models that allow us to study the dynamic processes of viscoelastic pipelines conveying gas-fluid.

The aim of this work is to develop the existing methods for calculating viscoelastic pipelines vibration; development of mathematical models, computational algorithms and software for solving linear problems on free vibrations of viscoelastic thin-walled pipelines based on the Euler-Bernoulli theory of beams; analysis of the effect of viscoelastic properties of the pipeline material; analysis of the influence of external periodic forces on pipelines vibration.

2. The equation of motion of viscoelastic pipelines conveying fluid

Consider a viscoelastic pipeline in the form of a rod of length L lying on a soil base. Soil in these areas is modeled by a viscoelastic Winkler base with a coefficient of soil response k_1. A steady flow of incompressible fluid having axial velocity U_f and axial force N_0 is conveyed through the pipeline. Placing the origin at the left end in the intersection of pipeline axis, the pipe length is plotted on the abscissa. The displacements of the points of the pipeline axis along z axis are denoted by $w\ (x, t)$.

Based on (Badalov et al., 2007; Khudayarov et al., 2016, 2019a, 2019b, 2019c, 2019d, 2019e, 2020), the equation of motion of pipelines conveying fluid flow based on the Winkler base, shown in Fig. 1, considering the viscosity properties of structures material, axial forces, internal pressure and external forces has the form:

$$EI\left(1-R^*\right)\frac{\partial^4 w}{\partial x^4}+2m_f U_f\,\frac{\partial^2 w}{\partial t\partial x}+(m_f+m_p)\frac{\partial^2 w}{\partial t^2}+$$

$$\left[m_f U_f^2-N_0+A_p P_i\right]\frac{\partial^2 w}{\partial x^2}+k_1\left(1-R_1^*\right)w=(m_f+m_p)f\Omega^2\sin(\Omega t). \qquad (1)$$

Here E –is the modulus of elasticity of the pipe material; I is the moment of inertia of the section; EI and L – are the pipe rigidity and length; w – is the deflection; x is an independent variable, the longitudinal axial coordinate of a pipe; m_f is the mass of fluid per unit length of pipe; m_p is the mass of pipe per unit length; $A_p=\pi r_1^2$ is the cross-sectional area of the pipe; r_1 is the inner radius of the pipe; P_i is the internal pressure; N_0 is the tensile (compressive) force; k_1 is the rigidity of the Winkler base; f and Ω are the amplitude and frequency of changes in external force.

Equation (1) is solved under the following boundary conditions

$$w\left(x,t\right)=\frac{\partial^2 w\left(x,t\right)}{\partial x^2}=0 \text{ at } x=0,\ x=L; \qquad (2)$$

under initial conditions

$$w\left(x,0\right)=\vartheta(x), \quad \dot{w}\left(x,0\right)=\psi(x), \qquad (3)$$

where $\vartheta(x)$, $\psi(x)$ are the given, smooth enough, functions in the field of arguments change.

3. Solution methods

Approximate solution of equation (1) is sought in the form:

$$w\left(x,t\right)=\sum_{n=1}^{N} w_n\left(t\right)\xi_n\left(x\right), \qquad (4)$$

where $w_n\left(t\right)$- are some functions to be determined, and functions $\xi_n\left(x\right)$ are selected so that each term of the sum (4) satisfies the boundary conditions (2).

Substituting (4) into equation (1) and applying the Bubnov-Galerkin method to this equation, we obtain a system of integro-differential equations (IDE) with respect to coefficients (4):

$$\sum_{n=1}^{N} a_{kn}\ddot{w}_n+2U_f\sqrt{\beta_{fp}}\sum_{n=1}^{N}\gamma_{nk}\dot{w}_n+\sum_{n=1}^{N}c_{kn}\left(1-R^*\right)w_n+$$

$$\left[U_f^2-\overline{N}_0+\overline{P}\right]\sum_{n=1}^{N}b_{kn}w_n+k_w\sum_{n=1}^{N}a_{kn}\left(1-R_1^*\right)w_n=2\gamma_k\omega^2 f\sin(\omega t). \qquad (5)$$

$$w_n(0)=w_{0n}; \quad \dot{w}_n(0)=\dot{w}_{0n}; \quad k=1,2,\dots,N.$$

Introduce the notations

$$\overline{N}_0 = N_0\frac{L^2}{EI}; \quad \beta_{fp} = \frac{m_f}{m_f + m_p}; \; k_w = \frac{k_1 L^4}{EI}; \quad \gamma_p = \frac{A_p L^2}{I}; \; \omega = \overline{\omega}\cdot L^2\left(\frac{m_f + m_p}{EI}\right)^{0,5};$$

$$\overline{P} = \frac{r_1^2 L^2 P_i}{EI};\; a_{kn} = \int_0^1 \xi_n(x)\xi_k(x)dx;\; b_{kn} = \int_0^1 \xi_n''(x)\xi_k(x)dx; \;\; c_{kn} = \int_0^1 \xi_n^{(IV)}(x)\xi_k(x)dx; \;\; \gamma_k = \int_0^1 \xi_k(x)dx.$$

Then, a numerical method (Badalov et al., 2007, 1987; Khudayarov et al., 2019, 2020a, 2020b, 2022) is applied to the system (5) describing linear problems on viscoelastic pipelines vibrations. Based on this method, an algorithm for the numerical solution of system (5) is described. Integrating system (5) twice in t, we can write it in integral form and using a rational transform, the weakly singular features of integral operators R^* and R_1^*. are eliminated. Assuming then $t=t_i$, $t_i=i\Delta t$, $i=1,2,\ldots$ ($\Delta t=const$) and replacing the integrals with some quadrature formulas for calculating $w_n= w_n(t)$, we obtain the following recurrence relation:

$$\begin{aligned}
&\sum_{n=1}^{N} a_{kn} w_{in} + 2U_f\sqrt{\beta_{fp}}\sum_{n=1}^{N} B_i \gamma_{kn} w_{in} = \\
&\sum_{n=1}^{N}\left(\dot{w}_{0n} t_i + w_{0n}\right) a_{kn} - \sum_{j=0}^{i-1} B_j \Bigg\{ 2U_f\sqrt{\beta_{fp}}\sum_{n=1}^{N}\gamma_{kn} w_{jn} + \\
&\left(t_i - t_j\right)\Bigg\langle \sum_{n=1}^{N} c_{kn}\left[w_{jn} - \frac{A}{\alpha}\sum_{s=0}^{j} C_s \exp(-\beta t_s) w_{j-s,n}\right] + \\
&k_w \sum_{n=1}^{N} a_{nk}\left(w_{jn} - \frac{A_1}{\alpha_1}\sum_{s=0}^{j} C_{1s}\exp(-\beta t_s) w_{j-s,n}\right) + \\
&\sum_{n=1}^{N} b_{kn}\left[U_f^2 - \overline{N}_0 + \overline{P}_i\right] w_{jn} + 2\gamma_k \omega^2 f \sin(\omega t_j)\Bigg\rangle\Bigg\}
\end{aligned} \tag{6}$$

$i=1,2,\ldots;$ $\quad n=\overline{1,N};$ $\quad m=\overline{1,L}$; where B_j, C_s and C_{1s} are the numerical coefficients related to the quadrature formulas of the trapezoid (Khudayarov, 2005a, 2005b, 2010, 2019; Abdikarimov and Khudayarov, 2014; Khudayarov and Turaev, 2019a, 2019b, 2020; Khudayarov and Bandurin, 2007; Komilova, 2020).

Due to the proposed approach, the factor $t_i - t_j$ at $j = i$ in the algorithm for the numerical solution of the problem in formula (6) takes a zero value, i.e. the last term of the sum is zero. Therefore, the summation is done from zero to i -1 ($j=\overline{0, i-1}$).

4. The discussion of results

Based on the developed computational algorithm (6), computer programs were created. The results of calculations are reflected in the graphs in Figs. 1 - 6. When calculating the deflection value by formula (4), the first five harmonics were held (N = 5). Calculations showed that a further increase in the number of terms insignificantly affects the amplitude of pipe vibrations.

4.1. The effect of rheological parameters

The effect of viscoelastic properties of the material on oscillatory process of a viscoelastic pipe when a fluid flow passes through it is investigated. Figure 1 shows the time dependence of the displacements of the midpoint of an elastic (A = 0, curve 1) and viscoelastic (A = 0.1, curve 2) pipe with an elastic base A_1 = 0, under unsteady-state transverse loads. Figure 1 shows that at the initial point of time, the solutions of elastic and viscoelastic problems differ little from each other, and over time this difference becomes significant. As follows from the graphs, the viscosity parameter A reduces the maximum deflection value. In the presented case, vibration amplitudes of a viscoelastic pipe are noticeably less, then that of an elastic pipe (A = 0). The following parameters were selected: α=0.25; β=0.05; A_1=0; β_{fp}=0.32; ω=2.5; k_w=10; $\overline{N}_o = 4$; $\overline{P} = 5$; f=0.004; $U_f = 0.05$.

Calculations were performed for various values of the singular parameter α. Figure 2 shows the plots of deflection function for a viscoelastic pipeline at various values of parameter α = 0.2 (curve 1); α = 0.5 (curve 2) and at a constant flow rate $U_f = 2$. With increasing values of this parameter, the amplitude and frequency of vibrations increase. In calculations, the following parameter values were taken: A=0.02; β=0.05; A_1=0; α_1=0.25; β_1=0.05; β_{fp}=0.32; ω=2.5; k_w=10; $\overline{N}_o = 4$; $\overline{P} = 5$; f=0.004.

Figure 3 illustrates the effect of the base viscosity parameter on the amplitude and frequency of vibrations of a viscoelastic pipeline conveying fluid flow. Three values of the base parameters were studied: A_1= 0 (curve 1); A_1= 0.1 (curve 2); A_1= 0.2 (curve 3). It can be seen from the figure that a change in rheological parameter of base viscosity has a significant effect on the process of viscoelastic pipe vibrations. With increasing viscosity parameter A_1, the system vibrations begin to take on a damped form. The vibration amplitudes decrease and the vibration phase shifts to the right.

4.2. The effect of external loads

Figure 4 shows the calculation results for the case f = 0.004. The series of curves in Fig. 5 describes the trajectory of motion $w(t)$ of the central point of the pipe under unsteady load of a frequency: ω = 2.5 (curve 1); ω = 5 (curve 2); ω = 7.5 (curve 3). Judging by the data presented, the nature of external load frequency is a significant factor affecting the pipe behavior during the oscillatory process. With an increase in frequency of unsteady external loads, the amplitude and frequency of pipe vibrations increase. As we see, at ω = 7.5 (curve 3), starting from t = 12, a harmonic vibrational movement is observed, the pipe experiences high-frequency vibrations of a large amplitude.

The graphs in Fig. 5 indicate a significant effect of the amplitude value of unsteady external load on the behavior of a viscoelastic pipeline. As follows from the graphs in Fig. 5, the influence of the amplitude value of the disturbing external load (f = 0.001 (curve 1); f = 0.005 (curve 2); f = 0.007 (curve 3)) is expressed in a certain increase in the maximum amplitude of deflections. As can be seen, with an increase in external load parameter the amplitude of vibrations increases. The following parameters were used in calculations: A=0.02; α=0.2; β=0.05; A_1=0.1; α_1=0.2; β_1=0.05; β_{fp}=0.32; ω=2.5; k_w=10; $\overline{N}_o = 4$; $\overline{P} = 5$; $U_f = 9.5$.

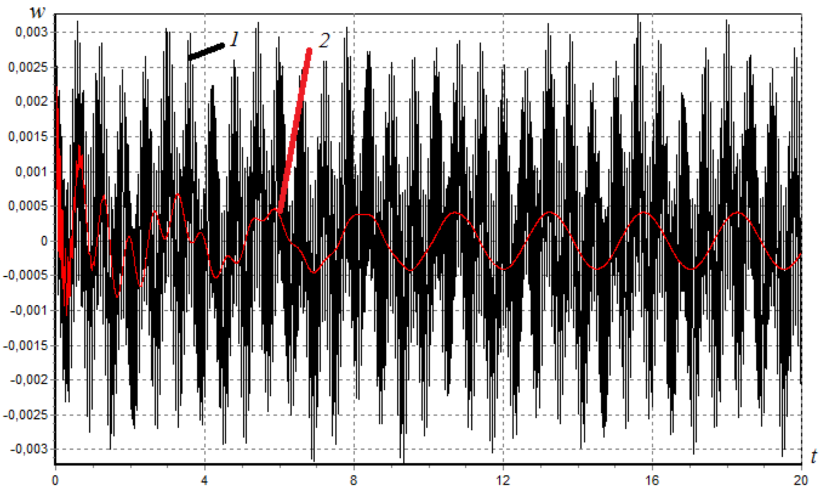

Fig.1. Modes of vibrations of the pipeline midpoint according to elastic and viscoelastic theory: A = 0 (curve 1); A = 0.1 (curve 2).

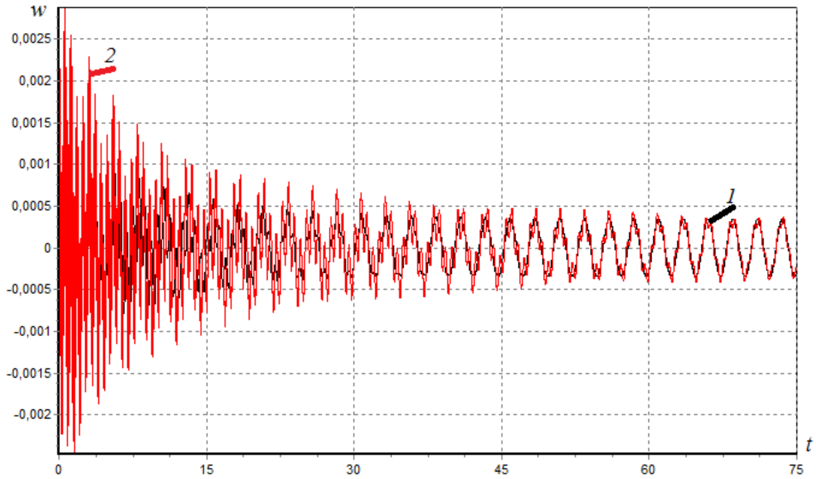

Fig.2. α=0.2 (curve 1); α=0.5 (curve 2); A=0.02; β=0.05; A_1=0; β_{fp}=0.32; ω=2.5; k_w=10; $\overline{N}_o = 4$; $\overline{P} = 5$; f=0.004; $U_f = 2$.

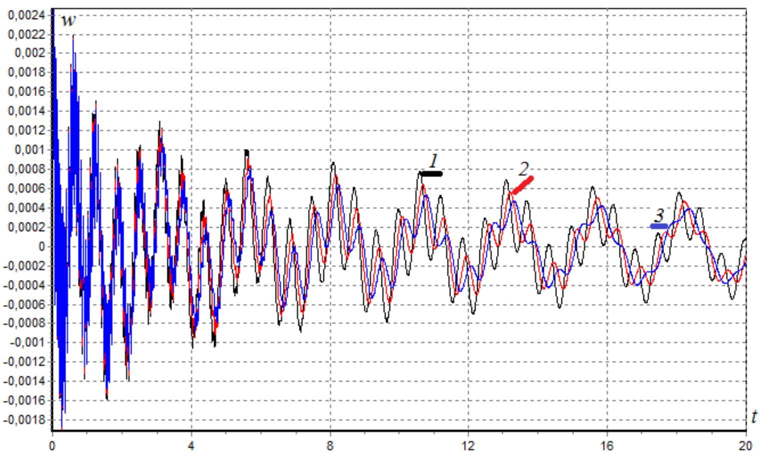

Fig.3. A_1=0 (curve 1); A_1=0.1 (curve 2); A_1=0.2 (curve 3); A=0.02; α=0.25; β=0.05; α_1=0.25; β_1=0.05; β_{fp}=0.32; ω=2.5; k_w=10; $\overline{N}_o = 4$; $\overline{P} = 5$; f=0.004; $U_f = 2$.

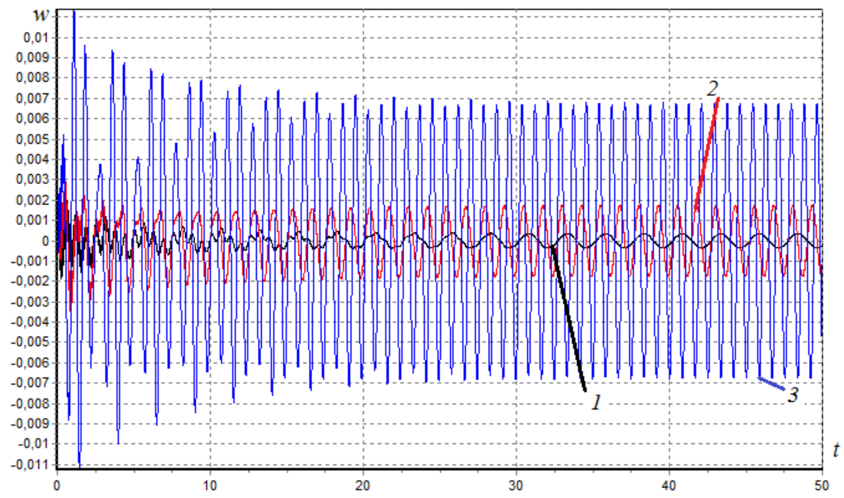

Fig.4. ω=2.5 (curve 1); ω=5 (curve 2); ω=7.5 (3); A=0.02; α=0.25; β=0.05; A_1=0.1; α_1=0.25; β_1=0.05; β_{fp}=0.32; k_w=10; $\overline{N}_o = 4$; $\overline{P} = 5$; f=0.004; $U_f = 2$.

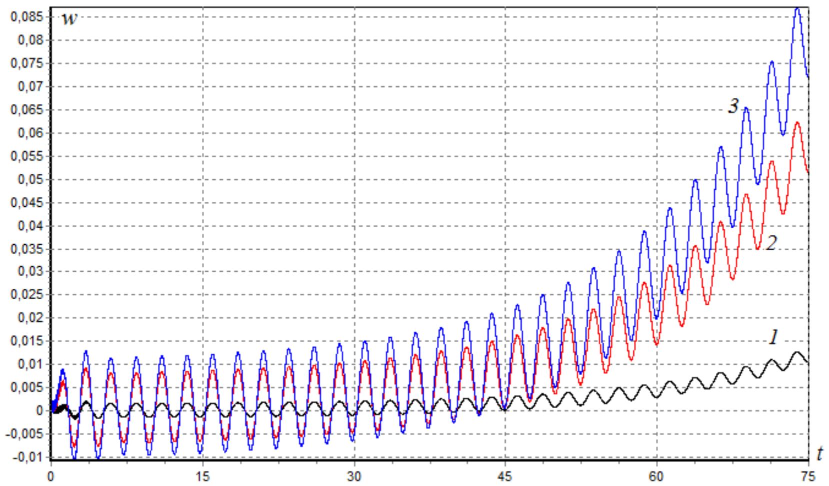

Fig.5. f=0.001 (curve 1); f=0.005 (curve 2); f=0.007 (curve 3); A=0.02; α=0.25; β=0.05; A_1=0.1; α_1=0.2; β_1=0.05; β_{fp}=0.32; ω=2.5; k_w=10; $\overline{N}_o = 4$; $\overline{P} = 5$; $U_f = 9.5$.

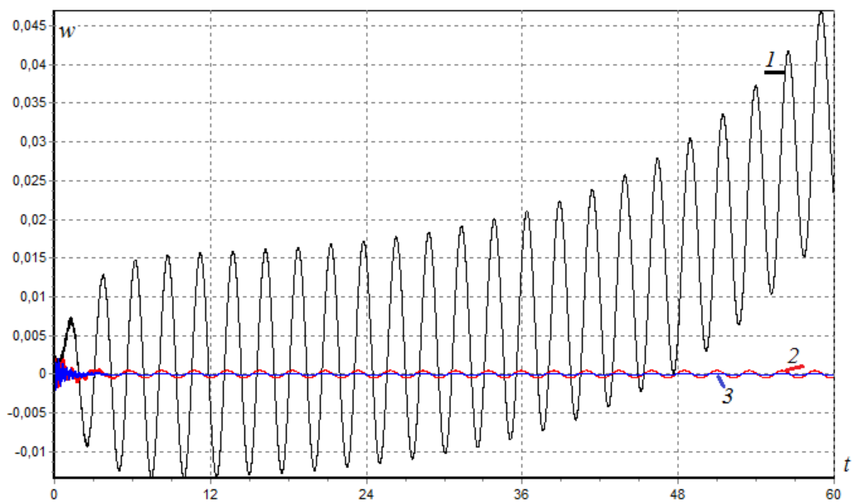

Fig.6. k_w=0 (curve 1); k_w=100 (curve 2); 500(3); A=0.02; α=0.25; β=0.05; A_1=0.1; α_1=0.2; β_1=0.05; β_{fp}=0.32; ω=2.5; f=0.004; $\overline{N}_o = 4$; $\overline{P} = 5$; $U_f = 9.5$.

Fig. 6 shows the effect of the parameters of the Winkler bases k_w on a viscoelastic pipe vibrations. The *y*-axis represents the deflection value *w*, and the abscissa axis represents *t* (time). Analysis of simulation results for a pipeline without considering the base parameter $(k_w = 0)$, shows that vibrations with a sharply increasing amplitude are observed; such vibrations are undesirable, since they lead to damage generation and accumulation in a structure.

5. Conclusion

An integro-differential equation of transverse vibrations of a rectilinear viscoelastic pipeline with a medium moving inside it is derived. A computational algorithm has been developed to solve the problems of the dynamics of a viscoelastic pipe taking into account unsteady external forces. Based on the developed computational algorithm, applied computer programs have been created that allow numerical studies of dynamic strain of the pipeline conveying fluid flow, based on the Winkler base, with account for viscosity properties of the structure and pipe base material, axial forces, internal pressure and external forces.

When modeling linear problems, a number of new dynamic effects were investigated: it was found that an account for viscoelastic properties of the material and the pipeline base leads to a decrease in the amplitude and frequency of pipe vibrations; it was revealed that with an increase in frequency of external loads, the amplitude and frequency of pipe vibrations increase; it was shown that an increase in external load parameter leads to an increase in vibration amplitude, and to a decrease in vibration frequency.

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