A Quantitative Model of the Urban Rail Congestion Propagation under Oversaturated Conditions

Ziling Zeng and Taixun Li

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Zeng Ziling
School of traffic and Transportation
Beijing Jiaotong University
Beijing,China
17125725@bjtu.edu.cn

Li Taixun
School of traffic and Transportation
Beijing Jiaotong University
Beijing,China
13251042@bjtu.edu.cn

***Abstract*— Simulating the congestion propagation of urban rail transit system is challenging, especially under oversaturated conditions. This paper proposes a quantitative model for capturing the congestion prorogation process through formalizing the propagation by a congestion susceptibility recovery process. Apart from that, a model for calculating congestion propagation rate is constructed, which is the key parameter in the former quantitative model. A grey system model is also introduced to quantify the propagation rate under the joint effect of six influential factors: passenger flow, train headway, passenger transfer convenience, time of congestion occurring, initial congested station and station capacity. A numerical example is used to illustrate the congestion propagation process and to demonstrate the improvements after taking corresponding measures.**

Keywords—oversaturated conditions; congestion propagation model; congestion propagation rate; grey system model.

I. INTRODUCTION

Congestion, along with the expansion of urban transit networks, is becoming one of the major concerns of the transportation system agencies, especially under the oversaturated conditions. The initiatives in the transportation system consist of various management strategies and effective restrictive measures which aim to reduce congestion and avoid accidents [1-3].The analysis of congestion in road networks include macroscopic [1-2], medium [3–5] and microscopic [6-7] levels. The rearrangement of demand in the network once an element fails is a particular focus of some studies [8-11]. Generally, there is an overlook on the dynamic properties of traffic in previous papers, such as the propagation of congestion, for instance. The congestion propagation progress on highway has been studied by Sundara [12] and Zhang [13], both of whom depicted the characteristics of congestion propagation under a highway traffic incident. Based on this research, Zang[14] proposed a model to calculate the boundary and recovery rate of the congestion in the light of the traffic flow theory. Nevertheless, research in the propagation of the congestion in rail transit networks is limited in the following two categories: the propagation mechanism and the congestion simulation.

Some of the current studies emphasized on how the congestion spreads and grows in the urban rail networks. Zhou [15] proposed the concept of the propagation of passenger flow during peak hours for the first time, and Zhou analyzed the mechanism and influential factors of the congestion propagation. Tums [16] described the transmission under the oversaturated conditions in rail transit based on the combustion theory, and he proposed that the congestion propagation is similar to the ripple effect. Other researchers focused on the congestion simulation and the model construction. Duan [17] classified the urban railway stations into 3 categories: the starting, the intermediate and the terminal stations, and he extracted certain features of the oversaturated conditions. He then proposed the model of the influence of passenger densities in the station waiting areas. The two aspects: transmission dynamics of the complex network and influential factors of the congestion propagation are combined by Li [18], who proposed a coordination game model for the traffic congestion diffusion and explored the influential factors (e.g., road structure; station distribution).

The details of the influential factors of the congestion propagation are clearly analyzed, and the models for the congestion propagation are constructed with brevity. However, the quantitative research in the congestion propagation process and the influential factors like the propagation rate is limited. Wu [19] calculated the approximate rate of propagation based on time algorithm, but Wu has applied an assumed value of the congestion propagation rate in the simulation instead of calculating the value with accuracy.

In previous research, however, there is an overlook of the dynamic properties of traffic, such as the propagation of congestion. Furthermore, quantitative research in the congestion propagation process is very limited. This paper aims to provide extensive analyses and simple formulations to help understand the behavior of the congestion propagation in urban rail networks. We propose a congestion propagation model in the urban rail networks by making some simplified assumptions about the traffic behavior. This is important to reflect the decision making process of traffic controllers who need simple and applicable formulations to help react in real time.

This paper is organized as follows. The problem statement is delineated in Section 2 to illustrate the purpose of our research. Section 3 constructs the congestion propagation model, and it is modeled as a susceptible-congested-recovered process. Section 4 describes the method for calculating the congestion propagation rate. Numerical simulation on a real-world network with the field measurement data are presented in Section 5. Section 6 focuses on comparing the proposed model with the previous model presented in the literature survey part. Section 7 concludes this study.

II. Problem Statement

The passenger flow will increase rapidly when the station encounters the oversaturated condition. Due to the carrying capacity of carriages, a high volume of stranded travelers in this station and the huge passenger flow will lead to congestion. In this case, passengers waiting in the consecutive stations will not be able to board the subway in time, and it may result in serious backups at these consecutive stations and the congestion will propagate among stations.

In this section, we use an experimental example to demonstrate the purpose of our research and approaches for analyzing and anticipating the propagation of congestion. In Fig. 1, station 3 represents the initial congested station and is also a transfer station. It is crucial to analyze the impact on the whole network caused by the initial congested station once the train moves to the next station.

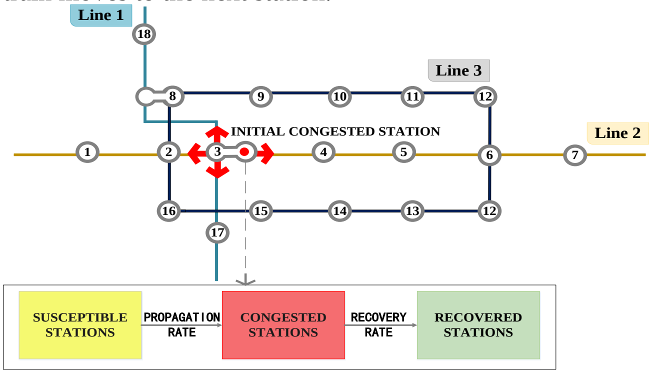


Fig. 1. A toy example of congestion propagation

The stations in the urban transit network can be divided into four classes: the oversaturated stations (e.g., station 3), also known as congested station; the susceptible stations, which are usually adjacent to the initial congested stations and are easily affected and delayed by congestion (e.g., station 2,4,8,17); the recovered stations, which are recovered from the congestion status and last, the uncrowded stations. In the following model, we focus on the first three statuses to simulate the congestion propagation.

This paper aims to fill the gap of previous research by providing a quantitative research on congestion propagation. We propose a congestion propagation model and use the increment of the number of congested stations to evaluate the congestion incidence of the network. As a result, the model generates the proper measurements for different oversaturated conditions. Furthermore, in order to simulate the congestion propagation process, we propose a separate method to calculate the value of congestion propagation rate in which the outcome produces a quantitative analysis of the efficiency of measurements taken to improve the oversaturated conditions

III. Congestion Propagation Model

Table 1 lists the indices, sets and parameters used in the mathematical formulations.

TABLE I. NOTATION OF CONGESTION PROPAGATION MODEL

Symbol	Definition
t	A time step
Δt	The headway between two subways.
$\boldsymbol{S}$	Set of stations in the network
k	Congestion propagation rate
r	The ratio of stations recovered from congested conditions
$\boldsymbol{N}_i$	The number of stations connected to station $i \in \boldsymbol{S}$
$I(t)$	The number of congested station at t
$S(t)$	The number of susceptible stations at t
$R(t)$	The number of stations recovered from the congestion at t

These variables $S(t)$, $I(t)$ and $R(t)$ represent three different types of stations at a particular time respectively. $S(t)$ represents the number of susceptible stations, $I(t)$ represents the number of congested stations, and $R(t)$ represents the number of recovered stations. Assume that the congestion propagation rate is k, recovery rate is r. During the time period [t, t+Δt], the congested station increment is $kI(t)S(t)\Delta t$, and the recovered station increment is $rI(t)\Delta t$. The state transition equations can be formulated as follows, and figure 2 shows the congestion propagation.

$$S(t+\Delta t)-S(t)=-kI(t)S(t)\Delta t \tag{1}$$

$$I(t+\Delta t)-I(t)=kI(t)S(t)\Delta t-rI(t)\Delta t \tag{2}$$

$$R(t+\Delta t)-R(t)=rI(t)\Delta t \tag{3}$$

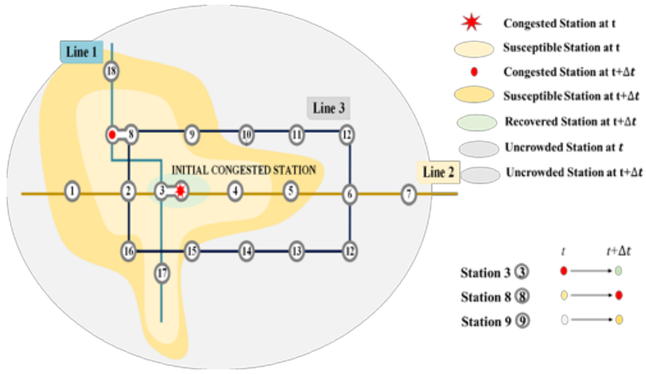


Fig. 2. Congestion propagation between t to t+Δt

The formulas above can be represented in continuous time as follows:

$$S'(t)=-kI(t)S(t) \tag{4}$$

$$I'(t)=kI(t)S(t)-rI(t) \tag{5}$$

$$R'(t)=rI(t) \tag{6}$$

Where $S(t_0)=S_0>0, I(t_0)=I_0>0, R(t_0)=R_0=0$.

The increment of the number of congested station is applied to evaluate the congestion propagation; it can be defined as follows:

$$\Delta I(t+\Delta t) = N_p k - rI(t) \tag{7}$$

Where *p* represents the departure station of the metro line.

These formulas describe the increment or decrement of the number of stations among three different categories over time, which can be used to simulate the congestion propagation process.

IV. Propagation Rate Calculating Method

Propagation rate represents the congestion probability between two adjacent stations when one of them encounters oversaturated condition. The influential factors of the propagation rate can be classified into six classes: passenger flow characteristic, train departure interval, passenger transfer convenience, the time of congestion occurring, the initial congested station and station capacity [20]. We divided these parameters into two classes: parameter *A* associated with passenger transfers and parameter *B* associated with time.

A. Formula Construction

We use the total passenger flow in one station and the transfer passenger flow to represent the five influential factors: passenger flow characteristic, average passenger transfer time for transfer convenience, congestion occurring time t, the station capacity F_j at station *j* and the average train headway. The passenger arrival is set as homogeneous for the station so that the application of the average train headway to represent the passenger waiting time is feasible (e.g., 2mins)

Table 2 lists the general indices, sets and parameters used in the mathematical formulation.

TABLE II. NOTATION OF CONGESTION PROPAGATION MODEL

Symbol	Definition
U	Set of transfer routes between two stations
LL	Set of lines(such as line4)
r	Indexes of routes in ***U***
i	Indexes of stations in ***S***
j	Indexes of stations in ***S***
k	Indexes of stations in ***S***
$Flow_j$	Total passenger flow in station $j \in \boldsymbol{S}$
F_j	The maximum passenger flow in station $j \in \boldsymbol{S}$
$Flow_{i,j}$	Transfer passenger flow from station $i \in \boldsymbol{S}$ to $j \in \boldsymbol{S}$
$Transfer_{i,j}$	Average Transfer walking time from station $i \in \boldsymbol{S}$ to $j \in \boldsymbol{S}$
W_j	Average Waiting time for train arrival in station $j \in \boldsymbol{S}$
$Travel_{j,k}$	Average Travel time from station $j \in \boldsymbol{S}$ to $k \in \boldsymbol{S}$
T_{ll}	Average total running time for Line $ll \in \boldsymbol{LL}$
A	Passenger- transfer-associated parameter
B	Time-associated parameter
α	Calibration value of A
β	Calibration value of B

The quantitative model was constructed as follows.

$$k = \alpha \frac{\sum_{i=1}^{N_j} Flow_{i,j} x_1 + (Flow_j - \sum_{i=1}^{N_j} Flow_{i,j}) x_2}{Flow_j} + \beta \frac{Transfer_{i,j} x_1 + Travel_{j,k} + W_j}{T_{ll}} \tag{8}$$

s.t.

$$x_1 = \begin{cases} 1 & \text{passengers coming from other metro lines} \\ 0 & \text{passenger coming directly to station without transfer} \end{cases} \tag{9}$$

$$x_2 = \begin{cases} 1 & \text{passengers coming directly to station without transfer} \\ 0 & \text{passenger coming from other metro lines} \end{cases} \tag{10}$$

$$x_1 + x_2 = 1 \tag{11}$$

$$\text{Flow}_j \le F_j \tag{12}$$

Where, $Travel_{j,k} + W_j$ represents the time spent between two adjacent stations in the same metro line. If passengers come from another metro line, we add $Transfer_{i,j} x_1$, namely, walking time in the transfer channel to the formula (8).

We define parameter A and parameter B as follows:

$$\text{A}= \frac{\sum_{i=1}^{N_j} Flow_{i,j} x_1 + (Flow_j - \sum_{i=1}^{N_j} Flow_{i,j}) x_2}{Flow_j} \tag{13}$$

$$\text{B}= \frac{Transfer_{i,j} x_1 + Travel_{j,k} + W_j}{T_{ll}} \tag{14}$$

B. Parameter Calibration

There are three methods for the weight analysis of the parameter calibration: analytic hierarchy process, expert scoring method and variation coefficient method. In previous literatures, however, the definition of the interrelation among parameters is limited.

To elaborate the relationship among different parameters, we deduct the grey system model to calculate the values of α and β .

The grey system model aims to calculate the degree of association between the behavior factor (i.e., congestion propagation), and the relevant factors (i.e., passenger flow and passenger behavior). If the developing trend between the behavior and the relevant factors is consistent, the degree of grey incidence would be large. If the trend is less well defined, the degree of grey incidence would be small [21]. The grey system model is considered to be an analysis of the geometric proximity among different factor sequences and the behavior sequence. The proximity is described by the degree of grey incidence, which is regarded as a measure of the similarities of data that can be arranged in sequential order. In this model, data are collected for behavior sequence X_0 and relevant factor sequence *Xi* over the same time period.

The notation for parameter calibration is shown as follows.

TABLE III. NOTATION FOR PARAMETER CALIBRATION

Symbol	Definition
X_0	Behavior sequence
X_i	Relevant factor sequence of parameter $i \in \boldsymbol{W}$
$x_i(j)$	The elements of X_i
$\gamma(X_p, X_q)$	The grey correlation degree of X_p and X_q
$\Delta_{0i}(j)$	The absolute deviation of $x_0(j)$ and $x_i(j)$
$\Delta_{\min}$	The bipolar minimum deviation of $x_0(j)$ and $x_i(j)$
$\Delta_{\max}$	The bipolar maximum deviation of $x_0(j)$ and $x_i(j)$
λ	

$\boldsymbol{W}$	An arbitrary given number $\lambda \in (0,1)$
$\boldsymbol{V}$	Set of uncalibrated parameters
i	Set of input data
j	Index of uncalibrated parameters $\boldsymbol{W}$
p	Index of input data $\boldsymbol{V}$
q	Index of uncalibrated parameters $\boldsymbol{W}$
	Index of uncalibrated parameters $\boldsymbol{W}$

Definition 1: According to the notation we can define α and β as follows.

$$\alpha=\gamma(X_0, X_A) \tag{15}$$

$$\beta=\gamma(X_0, X_B) \tag{16}$$

Definition 2: Assume that the behavior data sequence is defined as follows:

$$X_0 = (x_0(1), x_0(2), \cdots, x_0(n))^T \tag{17}$$

Besides, X_0 equals to k when α and β equal to 1

And the correlation factor data sequence is assumed to be:

$$X_i = (x_i(1), x_i(2), \cdots, x_i(n))^T . \tag{18}$$

$$\vdots \qquad \vdots$$

$$X_m = (x_m(1), x_m(2), \cdots, x_m(n))^T .$$

Where $x_i(j) > 0,\ i = 0,1,2\cdots m,\ j = 1,2\cdots n.$

Suppose that $\gamma(X_0, X_i) = \frac{1}{n}\sum_{j=1}^{n}\gamma(x_0(j), x_i(j))$ (19)

Satisfies the following four properties:

(i)Normality $0 < \gamma(X_0, X_i) \le 1, when\ X_0 = X_i \Leftrightarrow \gamma(X_0, X_i) = 1$

(ii)Integrity: $\gamma(X_p, X_q) \ne \gamma(X_q, X_p) when\ p \ne q,$

where $X_p, X_q \in X = \{x_k \mid k = 0,1,2\cdots m, m \ge 2\}.$

(iii)symmetry: $\gamma(X_p, X_q) = \gamma(X_q, X_p) \Leftrightarrow x = \{X_p, X_q\}$

(iv)Proximity: the smaller $|x_0(j) - x_i(j)|$, the bigger $\gamma(x_0(j), x_i(j))$.

Then, $\gamma(X_0, X_i)$ is called the grey correlation degree of X_0 and X_i , $\gamma(x_0(j), x_i(j))$ represents the correlation coefficient of X_0 and X_i at the *j*-th point, the four properties (i),(ii),(iii)and (iv) is called four axioms of the grey correlation.

Definition 3: Assume that $X_i = (x_i(1), x_i(2), \cdots, x_i(n))$ which is defined by the above Definition 1, where *i*=1,,2…m. we define:

$$\Delta_{0i}(j) = | x_0(j) - x_i(j) | \tag{20}$$

$$\Delta_{\min} = \min_i \min_j \Delta_{0i}(j) \tag{21}$$

$$\Delta_{\max} = \max_i \max_j \Delta_{0i}(j) \tag{22}$$

Then $\Delta_{0i}(j)$ is called absolute deviation of $x_0(j)$ and $x_i(j)$, $\Delta_{\min}$ and $\Delta_{\max}$ are called bipolar minimum deviation and bipolar maximum deviation respectively.

Theorem 1: For an arbitrary given number $\lambda \in (0,1)$, we define:

$$\gamma(X_0(j), X_i(j)) = \frac{\Delta_{\min} + \lambda\Delta_{\max}}{\Delta_{0i}(j) + \lambda\Delta_{\max}} \tag{23}$$

$$\gamma = \gamma(X_0, X_i) = \frac{1}{n}\sum_{j=1}^{n}\gamma(x_0(j), x_i(j)) \tag{24}$$

Then the value $\gamma = \gamma(X_0, X_i)$ can be calculated by using formula (24).

V. Case Study

In this section we focus on how to use the congestion propagation model and the propagation rate to conduct the quantitative analysis of the above proposed model.

The case study is based on the oversaturated condition occurred on Sep 8th 2014 at Beijing Xizhimen Station, where Line 2 was travelling pass station 1 and 3 without stopping between 20:50 to 21:15. The rail transit network is shown in Fig. 3.

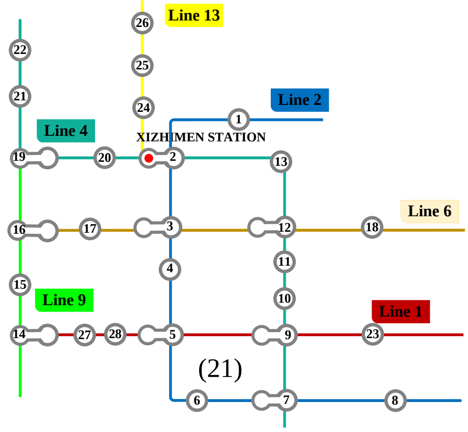


Fig. 3. A part of Beijing Metro network

A. Data preparation

In this section, we use Matlab solver on a PC with an Intel Quad Core i5 processor and 4 GB of RAM. to calculate the values of α and β.

According to definition 2, when the value of parameter α and β equal to 1, the behavior data sequence X_0 equals to the value of k. The data collected on Sep 8th 2014 at Xizhimen Station are applied to calculate the values of factor sequences and results are shown as follows:

TABLE IV. VALUE OF INPUT SEQUENCES

Metro lines	feature data sequence X_0	correlation factor data sequence X_A	correlation factor data sequence X_B
4 -2	3.73	0.13	0.19
13-2	6.63	0.36	0.28
2 -4	3.83	0.27	0.10
13-4	5.27	0.31	0.12
2-13	6.78	0.4	0.25
4-13	8.12	0.06	0.27

We assume λ =0.5, by using formula (20-24), the calibration degree can be calculated as follows.

$$\alpha = \gamma(X_0, X_A) = 0.54$$

$\beta = \gamma(X_0, X_B) = 0.46$.

Based on these values, the propagation rate k can be calculated by formula (8) and the value is 0.287.

B. *Propagation Simulation*

In the literature, the methods of calculating recovery rate r are limited. In order to simulate the propagation rate, we assume that the congestion recovery rate is 0.10 [17]. From the network graph, some parameters are given, $N_{Xizhimen}$=5, I_0=1,r=0.10. In order to calculate the propagation rate, we refer to formula (8) and the correlation degrees α *and* β are given by section 5.2 (α= 0.54, β=0.46).

Inputting this value above into formulas (1-4), the congestion propagation process can be simulated as follows.

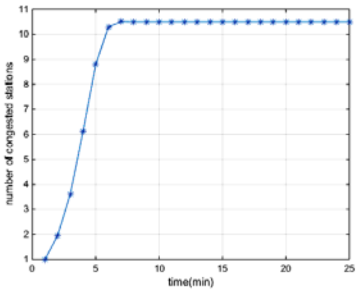
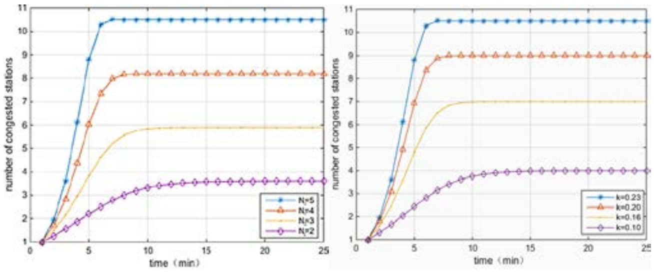


Fig. 4. The process of congestion propagation

The process of congestion propagation in Fig. 4 illustrates that the increment of congested stations increases rapidly within the first 5 minutes and that there is a decline at time step 8 and a slight decline between time step 8 to 25. We adjust the parameter N_i in order to compare the result with the actual circumstances and the simulation is shown in figure 5 (a). It illustrates the alternatives of measures taken by operators which include travelling pass station 1 and 3 without stopping, the adjacent stations of the initial congested station (N_1) reduce from time step 5 to 3, as a result, the number of congested station declines to 6 within 25 minutes and the increment begins to stabilize between time step 10 to 25. The adjustment of its value (e.g., 5, 4, 3, 2) provides a better illustration of the impact of parameter N_i and the simulation is shown in figure 5 (a).

Fig.5 (b) shows that the evolutions of propagation rate equals to 0.23, 0.20, 0.16 and 0.10 respectively. From this figure we can conclude that, if we reduce the propagation rate, the total number of congested station will decline. However, comparing (b) with (a), it is clear that the reduction of adjacent stations will significantly reduce the increase of the total number of congested stations and the improvement is more efficient.



(a) (b)

Fig. 5. simulation under different value of parameters

It is shown that when combined with the above analysis of the propagation rate, the propagation of congested station will expand when the propagation rate increases. Thus, it is more efficient to reduce the number of adjacent stations (reduce N_i) by travelling pass some of the stations without stopping. Furthermore, there are other measures that can improve the oversaturated condition by reducing the propagation rate k, which includes: restricting passenger flow at the station entrance, adjusting the train interval, improving transfer convenience and facilitating access to stations.

VI. COMPARISON

Liu [21] introduced the time algorithm to calculate the congestion propagation rate.

$$k = \frac{t_1 - t_2}{T} \tag{25}$$

In this equation, T represents the train operating time in the network, t_1 represents the time point when an oversaturated condition begins at a station (eg. Xizhimen Station), t_2 represents the time point when an oversaturated condition begins at the connected station (eg. Chegongzhuang Station).

It is challenging to apply this formulation to practical use as the value of parameter t_1 and t_2 are difficult to measure. Hence, the result may lack accuracy. Moreover, the data of parameters cannot be collected before the occurrence of oversaturated conditions. Consequently, this method is not available and lacks the ability to prevent oversaturated conditions.

By comparison, the method proposed in section 4 can be used in a variety of applications and the data of parameters can be collected more easily. For example, the method can be used in generating effective measures for the oversaturated condition. Firstly, the parameter data and possible measures are the input. Secondly, the operators can calculate the propagation rate and the cumulative number of congested station. Finally, the operators can select the most effective measure, by referring to the value of congestion propagation rate and the simulated amount of congestion stations.

VII. CONCLUSION

The propagation theories of the oversaturated conditions in rail transit enlightened the quantitative study. We propose a congestion propagation model and a method of calculating the propagation rate. A numerical example is used to illustrate the congestion propagation process and to demonstrate the improvements after taking corresponding measures. The application of the congestion propagation model aims to simulate the propagation process of the whole network under oversaturated conditions. In this paper, a separate method that calculates the congestion propagation rate is studied and presented for the first time. The congestion propagation rate model is used to generate measures to solve the oversaturated condition, improve congestion and finally, enable us to analyze the incidents for future reference. The two models establish a solid foundation for the study of large-scale network congestion propagation. The influential factors of the propagation rate can be classified into six classes: passenger flow characteristic, train departure interval, passenger transfer convenience, the time of congestion occurring, the initial congested station and station capacity. Nevertheless, there are some other influential factors for these models. Hence, further research on other influential factors is suggested to extend the models. The congestion propagation model provides an extensive analysis of the propagation process related to congestion. Moreover, the model depicts the forecasting and trends of congestion so that traffic controllers obtain a quantitative observation of the process. The congestion propagation rate model distinguishes the efficiency of the alternatives related to congestion improvement measures. As a result, traffic controllers evaluate the outcome and subsequently, select the optimal measure to resolve the oversaturated conditions. The expansion of urban transit networks magnifies the propagation of congestion and oversaturated conditions. Thus, the models proposed in this paper optimize the recovery measures and more importantly, simulate the current circumstances and forecast the propagation trends in urban transit systems.

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