Joint Estimation of Vessel Parameter-Motion and Sea State

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***Abstract*—We consider the problem of real-time estimation of sea state and wave-induced motions on a moving vessel using onboard inertial sensors *without knowing* vessel's dynamic parameters (*i.e.*, draught and breadth). This is crucial for vessel operational planning and performance, preventing structure failure, emissions reduction and fuel economy. This work proposes a new estimation approach by reformulating the conventional problem of sea state and vessel motion estimation (unknown input into a known dynamic system) as an *input-state-parameter* estimation problem of mass-spring-damper systems. We exploit the strong correlations between a vessel's vertical displacement and its rotation to develop a new estimation algorithm—Parameter-Sharing Extended-Augmented Kalman Filter (PS-EAKF)—for the problem to estimate the unidentified vessel parameters together with vessel motion (heave and pitch) and sea state. *Experimental data* from a scale-model vessel in regular head seas demonstrate the effectiveness and robustness of the proposed approach.**

***Index Terms*—Sea waves, Mass-spring-damper system, Input-state-parameter estimation problems, Condition monitoring.**

I. Introduction

UNderstanding the motion of a vessel affected by sea waves is a fundamental problem in maritime engineering and naval architecture [1]. This knowledge is important for the stability of vessels, improved operational performance (optimal cruising speed), preventing structure failure (due to mitigation of impulsive wave loads and periodic fatigue), emissions reduction and fuel economy (via reduced resistance) [1]–[3]. As illustrated in Fig. 1a, conventional methods [4], [5] estimate sea states —such as wave elevation and frequency— and the vessel responses–such as heave and pitch—to wave excitation by considering the vessel as a wave buoy and use the recorded responses as the input to the estimator. These methods rely on the linear relationship between the vessel response and wave excitation described by transfer functions, often derived from computational models, to estimate sea wave parameters. In [4], an adaptive Kalman filter was proposed for the real-time estimation of sea state using noisy sensor measurements (*i.e.*, displacements and accelerations) and the vessel's transfer functions computed from simulation tools to construct the measurement models. In [5], the approach was extended to the sea state estimation from a moving vessel.

Conventional methods in [4], [5] are information demanding, requiring a well-defined model of incident waves coupled with the vessel response modelled by transfer functions. The latter depends on accurate vessel geometry and its hydrodynamic characteristics (moment of inertia, buoyancy, payload) [6]. Hence, when this information is not readily available, the application of conventional methods becomes impractical due to many unknown, vessel-wave dynamic model parameters. Thus, it is highly desirable to estimate sea state *without prior knowledge* of the vessel's response described by complex transfer functions and associated parameters.

Although a formulation to estimate sea state and a vessel's response without prior knowledge of the vessel's parameters remains to be explored, such a problem would naturally lead to solving a challenging *input-state-parameter* estimation problem. In general, when system parameters are known, several optimal filtering approaches have been proposed to solve the joint input-state estimation problem [7]–[13]. However, the problem is significantly more challenging if no prior knowledge exists for all three quantities. The resulting *input-state-*

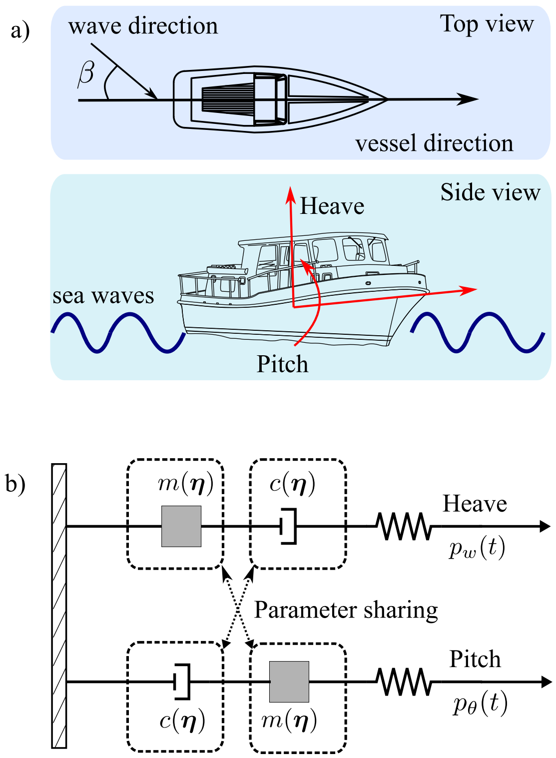

Fig. 1. a) Wave-induced vessel motions. b) An illustration of the input-state-parameter estimation problem with a pseudo-mass-spring-damper model.

parameter estimation problem has recently drawn significant interest. Naets et al. [14] included the unknown inputs and parameters in the state vector to form an augmented state vector and implemented the EKF (extended Kalman filter) to estimate the augmented state. Azam et al. [15] augmented the state vector with the unknown parameters and then utilised an unscented dual Kalman filter proposed in [16] to estimate the system's state, inputs, and parameters. Wan et al. [17] adapted Gillijins and De Moor filter to minimise covariance and avoid biased estimation. Maes et al. [18] proposed a new algorithm to adopt a time delay in the estimation and significantly reduced estimation uncertainty. Although attaining good results, current methods only consider problems *without correlation* between the unknown parameters, such as damping and mass parameters in mechanical systems.

In this work, we formulate a new algorithm to *estimate sea state and a vessel's response without prior knowledge of the vessel's parameters*—where system parameters, the vessel's breadth and draught are *unknown*. In contrast to conventional methods, our approach investigates casting the problem as an input-state-parameter estimation problem of dual pseudo mass-spring-damper systems derived from the simplified seakeeping analysis for heave and pitch of a vessel. In particular, we exploit the correlation of motion vectors and develop a new estimation algorithm; a Parameter-Sharing-Extended-Augmented Kalman Filter (PS-EAKF) for the problem where the estimated parameters are shared across dual filters—one tasked with estimating heave and the other, pitch–at every recursion (see Fig. 1b).

The work is organised as follows. Section II describes the background of wave-induced vertical motions of a vessel. Section III formulates the problem and presents the proposed PS-EAKF algorithm. Section IV is an experimental study using a model-scale dataset. Section V draws our conclusions.

II. Background

A. Notations

We denote scalar values using normal letters (*e.g.*, x, A), vectors and matrices using bold letters (*e.g.*, $\mathbf{x}, \mathbf{D}$). Additionally, if the displacement is denoted by x, then the corresponding velocity and acceleration are $\dot{x}$ and $\ddot{x}$, respectively. Here, we denote $\widehat{(\cdot)}$ as the estimated value of $(\cdot)$, $(\cdot)^H$ as the transpose of the vector or matrix $(\cdot)$ and $\mathrm{diag}(\cdot)$ as the diagonal matrix created from the elements of $(\cdot)$.

B. Seakeeping Analysis–Wave-Induced Vessel Motion

In the scope of our study, we are interested in: *i)* estimating wave characteristics and the vertical motions of a vessel, *i.e.*, the vertical displacement at the centre of gravity (heave) and rotation (pitch), induced by sea waves, as illustrated in Fig. 1a; and *ii)* investigating the development of a new algorithm for the resulting input-state-parameter estimation problem; and *iii)* validating using experimental data. We employ the simplified seakeeping analysis in [19] describing vessel motions for heave w and pitch θ under the influence of a unidirectional *regular wave*. Notably, as we demonstrate in Section IV, the model remains valid for the scaled-model experiments we conducted. The equations of motion can be written in the form described in [19] and given below:

$$\frac{2T}{g}\ddot{w} + \frac{A^2}{kB\alpha^3\omega}\dot{w} + w = aP_w\sin(\varpi t + \phi_w) = p_w(t), \quad (1)$$
$$\frac{2T}{g}\ddot{\theta} + \frac{A^2}{kB\alpha^3\omega}\dot{\theta} + \theta = aP_\theta\sin(\varpi t + \phi_\theta) = p_\theta(t). \quad (2)$$

Here T and B are the draught and the breadth of the vessel, $g = 9.8$ m/s^2 is gravitational acceleration, ω is the wave frequency, $k = \omega^2/g$ is the wave-number, A is the dimensionless sectional hydrodynamic damping ratio, α is dimensionless parameter depends on the Froude number (speed–length ratio) $Fn = V/\sqrt{gL}$ and vessel length L, a is wave amplitude, $p_w(\cdot)$ is the input heave force with amplitude aP_w and phase offset ϕ_w, $p_\theta(\cdot)$ is the input pitch moment with amplitude aP_θ and phase offset ϕ_θ, ϖ is the encountered frequency. The relationship between the encountered frequency (that experienced by the vessel) and the sea wave frequency is given by the Doppler shift:

$$\varpi = \omega - kV\cos(\beta) = \omega - \omega^2 V\frac{\cos(\beta)}{g}, \quad (3)$$

where V is the vessel forward speed, and β is the encountered angle between the vessel heading and the wave propagation direction. The other vessel parameters are given by [19, pp. 63]:

$$\alpha = 1 - V\sqrt{\frac{k}{g}}\cos(\beta),$$
$$A = 2\sin(\frac{1}{2}kB\alpha^2)\exp(-kT\alpha^2),$$
$$k_e = |k\cos(\beta)|,$$
$$\kappa = \exp(-k_e T),$$
$$f = \sqrt{(1-kT)^2 + \big(\frac{A^2}{kB\alpha^3}\big)^2},$$
$$P_w = 2\kappa f \sin(\frac{1}{2}k_e L)/(k_e L),$$
$$P_\theta = 24\kappa f \frac{\sin(\frac{1}{2}k_e L) - \frac{1}{2}k_e L\cos((\frac{1}{2}k_e L)}{k_e^2 L^3}$$

III. Problem Formulation

A. Problem Statement

In this work, we are interested in estimating the characteristics of a *regular wave* (the *input*), including its amplitude a and frequency ω impacting on a *moving* vessel via onboard *displacement sensors measuring heave and pitch* at the vessel centre of gravity. We also want to estimate the *encountered* wave characteristics impacting on the vessel, including the input heave force $p_w(\cdot)$ and the input pitch moment $p_\theta(\cdot)$ and the vessel's vertical displacements of heave w and pitch θ over time (the *state*). Importantly, we aim to estimate the *waterline* breadth (B) and draught (T), which are: i) bounded and time-varying, and ii) depend on the instantaneous wetted surface, vessel buoyancy, payload, and hydrodynamic characteristics (the *parameters*). For simplicity, we consider a vessel encounter angle $\beta \geq \pi/2$ rad, there is a 1-to-1 mapping between the encountered frequency ϖ and the input frequency ω.

B. Formulation of Input-State-Parameter Estimation

We can reformulate the problem of estimating a regular wave and the vertical motions of a vessel as a joint *input-state-parameter* estimation problem for a simplified hydrodynamical system [14], [18], [20], [21]. Since we can assume that the coupling between motion components is negligible [19], the heave and pitch in (1) and (2) can be considered as two single-degree-of-freedom (SDOF) spring-mass-damper systems, typically used to model the structural dynamics [11]. In particular, the system can be modelled as two *pseudo* mass-spring-damper systems (see Fig. 1b). Thus, we can formulate a model for the vessel experiencing heave and pitch motion, wherein the state x and the input $p(t)$ can be either w and $p_w(t)$ or θ and $p_\theta(t)$, respectively, as:

$$m(\boldsymbol{\eta})\ddot{x}(t) + c(\boldsymbol{\eta})\dot{x}(t) + x(t) = p(t) \tag{4}$$

where $\boldsymbol{\eta} \triangleq [B, T, \omega]^H \triangleq [\eta_1, \eta_2, \eta_3]^H \in \mathbb{R}_+^{n_\eta}$ is the unknown parameter with $n_\eta = 3$, and

$$m(\boldsymbol{\eta}) = \frac{2\eta_2}{g},\ c(\boldsymbol{\eta}) = \frac{g[A(\boldsymbol{\eta})]^2}{[\eta_1[\eta_3]^3[\alpha(\boldsymbol{\eta})]^3]} \tag{5}$$

are *pseudo-mass* and *pseudo-damping* constant of the structural system with *pseudo-stiffness* coefficient equal to unity,

$$\alpha(\boldsymbol{\eta}) = 1 - V\eta_3\frac{\cos(\beta)}{g}, \tag{6}$$
$$k(\boldsymbol{\eta}) = \frac{[\eta_3]^2}{g}, \tag{7}$$
$$A(\boldsymbol{\eta}) = 2\sin(0.5\eta_1 k(\boldsymbol{\eta})\alpha^2(\boldsymbol{\eta}))\exp(-\eta_2 k(\boldsymbol{\eta})[\alpha(\boldsymbol{\eta})]^2). \tag{8}$$

Importantly, our formulation is *different* from the common normalisation technique for estimating parameters in [18], [20], [21] where $\eta_2 = \frac{m(\eta_2)}{m_{\text{truth}}}$ and $\eta_1 = \frac{c(\eta_1)}{c_{\text{truth}}}$ which assumed that there is no *correlation* between the unknown mass $m(\cdot)$ and the unknown damping $c(\cdot)$, in addition to the *linear* relationship between the unknown damping $c(\boldsymbol{\eta})$ and $\boldsymbol{\eta}$. In contrast, as shown in (5) and (8), our formulation accommodates the *correlation* between pseudo-mass $m(\cdot)$ and pseudo-damping $c(\cdot)$ which depends on η_2, and the relationship between $c(\boldsymbol{\eta})$ and $\boldsymbol{\eta}$ is *non-linear*.

Selecting the state vector $\mathbf{x} = [x, \dot{x}]^H \in \mathbb{R}^{n_x}$ with $n_x = 2$, (4) can be discretised to obtain:

$$\mathbf{x}_{k+1} = \mathbf{D}(\boldsymbol{\eta})\mathbf{x}_k + \mathbf{E}(\boldsymbol{\eta})p_k + \mathbf{Q}_k \tag{9}$$

where

$$\mathbf{D}(\boldsymbol{\eta}) = \begin{bmatrix} 1 & \triangle \\ -\triangle/m(\boldsymbol{\eta}) & 1 - c\triangle/m(\boldsymbol{\eta}) \end{bmatrix},$$
$$\mathbf{E} = \begin{bmatrix} 0 \\ \triangle/m(\boldsymbol{\eta}) \end{bmatrix},$$

Here, $\triangle$ is the measurement time step and $\mathbf{Q}_k \sim \mathcal{N}(0, \boldsymbol{\Sigma}_\mathbf{Q})$ is the process noise. $\mathcal{N}(\mu, \Sigma)$ denotes a Gaussian distribution with a mean of μ and a covariance of Σ.

Suppose that we can measure the displacement of x, then its velocity and acceleration can be derived from displacement by definitions, *i.e.*,:

$$\dot{x}_k = \frac{x_k - x_{k-1}}{\triangle}, \quad \ddot{x}_k = \frac{\dot{x}_k - \dot{x}_{k-1}}{\triangle}.$$

Since all the velocity and acceleration measurements can be computed from the displacement measurements, the measurement vector $\mathbf{o} = [o, \dot{o}, \ddot{o}]^H \in \mathbb{R}^{n_o}$ with $n_o = 3$ follows:

$$\mathbf{o}_k = \mathbf{G}(\boldsymbol{\eta})\mathbf{x}_k + \mathbf{J}(\boldsymbol{\eta})p_k + \mathbf{R}_k \tag{10}$$

where

$$\mathbf{G}(\boldsymbol{\eta}) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1/m(\boldsymbol{\eta}) & -c(\boldsymbol{\eta})/m(\boldsymbol{\eta}) \end{bmatrix},$$
$$\mathbf{J} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m(\boldsymbol{\eta})} \end{bmatrix},$$

and $\mathbf{R}_k \sim \mathcal{N}(0, \boldsymbol{\Sigma}_\mathbf{R})$ is the measurement noise.

Algorithm 1: Extended-Augmented KF (EAKF)

Input: $\mathbf{x}^a_{k-1},\ \mathbf{P}_{k-1},\ \mathbf{o}_k,\ f(\cdot),\ h(\cdot)$
Output: $\mathbf{x}^a_k,\ \mathbf{P}_k$
/* 1. Time update */
Compute $\bar{\mathbf{D}}^a_{k-1}$ via (13)
$\widehat{\mathbf{x}}^a_k = f(\mathbf{x}^a_{k-1})$; $\widehat{\mathbf{P}}_k = \bar{\mathbf{D}}^a_{k-1} P_{k-1} (\bar{\mathbf{D}}^a_{k-1})^H + \mathbf{\Sigma}^a_Q$
/* 2. Measurement update */
Compute $\bar{\mathbf{G}}^a_k$ via (13)
$\mathbf{L}_k = \widehat{\mathbf{P}}_k (\bar{\mathbf{G}}^a_k)^H \big(\bar{\mathbf{G}}^a_k \widehat{\mathbf{P}}_k (\bar{\mathbf{G}}^a_k)^H + \mathbf{\Sigma}_R\big)^{-1}$;
$\mathbf{x}^a_k = \widehat{\mathbf{x}}^a_k + \mathbf{L}_k\big(\mathbf{o}_k - h(\widehat{\mathbf{x}}^a)\big)$; $\mathbf{P}_k = \widehat{\mathbf{P}}_k - L_k \bar{\mathbf{G}}^a_k \widehat{\mathbf{P}}_k$.

We propose using the Extended Augmented Kalman Filter (EAKF) by augmenting the unknown input force p with $n_p = 1$ and the unknown parameter $\boldsymbol{\eta}$ into the state, *i.e.*,

$$\mathbf{x}^a_k = [\mathbf{x}^H_k \quad p_k \quad \boldsymbol{\eta}^H_k]^H.$$

From (9), (10), the augmented state equation is obtained as:

$$\mathbf{x}^a_{k+1} = \mathbf{D}^a(\boldsymbol{\eta})\mathbf{x}^a_k + Q^a_k, \tag{11}$$

$$\mathbf{o}_k = \mathbf{G}^a(\boldsymbol{\eta})\mathbf{x}^a_k + R_k, \tag{12}$$

where

$$\mathbf{D}^a(\boldsymbol{\eta}) = \begin{bmatrix} \mathbf{D}(\boldsymbol{\eta}) & \mathbf{E}(\boldsymbol{\eta}) & \mathbf{0}_{n_x \times n_\eta} \\ \mathbf{0}_{n_p \times n_x} & \mathbf{I}_{n_p} & \mathbf{0}_{n_p \times n_\eta} \\ \mathbf{0}_{n_\eta \times n_x} & \mathbf{0}_{n_\eta \times n_p} & \mathbf{I}_{n_\eta} \end{bmatrix},$$

$$\mathbf{G}^a(\boldsymbol{\eta}) = \Big[\mathbf{G}(\boldsymbol{\eta}) \quad \mathbf{J}(\boldsymbol{\eta}) \quad \mathbf{0}_{n_o \times n_\eta}\Big],$$

$Q^a_k \sim \mathcal{N}(0, \mathbf{\Sigma^a_Q})$ is the Gaussian process noise with zero mean and a covariance matrix

$$\mathbf{\Sigma^a_Q} = \begin{bmatrix} \mathbf{\Sigma_Q} & \mathbf{0}_{n_x \times n_p} & \mathbf{0}_{n_x \times n_\eta} \\ \mathbf{0}_{n_p \times n_x} & \Sigma_p & \mathbf{0}_{n_p \times n_\eta} \\ \mathbf{0}_{n_\eta \times n_x} & \mathbf{0}_{n_\eta \times n_p} & \mathbf{\Sigma}_{\boldsymbol{\eta}} \end{bmatrix};$$

with $\Sigma_p \in \mathbb{R}^{n_p}_+$ is the initial estimation covariance noise of the input force p; $\mathbf{\Sigma}_{\boldsymbol{\eta}} = \mathrm{diag}([\Sigma_{\eta_1}, \Sigma_{\eta_2}, \Sigma_{\eta_3}]) \in \mathbb{R}^{n_\eta \times n_\eta}_+$ is the covariance of unknown parameters $\boldsymbol{\eta} = [\eta_1, \eta_2, \eta_3]^H$.

Let $f(\mathbf{x}^a) = \mathbf{D}^a(\boldsymbol{\eta})\mathbf{x}^a$ and $h(\mathbf{x}^a) = \mathbf{G}^a(\boldsymbol{\eta})\mathbf{x}^a$, by linearising the component $\boldsymbol{\eta}$ in (11) and (12), we have:

$$\bar{\mathbf{D}}^a_{k-1} = \frac{\partial f}{\partial \mathbf{x}^a}\bigg|_{\mathbf{x}^a_{k-1}}, \quad \bar{\mathbf{G}}^a_k = \frac{\partial h}{\partial \mathbf{x}^a}\bigg|_{\widehat{\mathbf{x}}^a_k}. \tag{13}$$

For completeness, the Extended-Augmented Kalman Filter (EAKF) is provided in Algorithm 1.

C. Formulation of Parameter Sharing Estimation

As discussed in Section III-B, the heave and pitch motions can be modelled using two decoupled SDOF systems. Consequently, we can use Algorithm 1 to estimate the input-state-parameters using either heave or pitch measurements. Interestingly, we observe in (1), (2), (5) that both heave and pitch systems share the same *unknown* value of $\boldsymbol{\eta}$ for pseudo-mass $m(\boldsymbol{\eta})$ and pseudo-damping constant $c(\boldsymbol{\eta})$. Therefore, to improve estimation accuracy, we propose *sharing* the estimated $\boldsymbol{\eta}$ value across two SDOF systems (heave and pitch).

Algorithm 2: Parameter-Sharing Extended-Augmented Kalman Filter (PS-EAKF)

Input: $\mathbf{w}^a_{k-1},\ \mathbf{P}_{w,k-1},\ \mathbf{o}_{w,k};\ \boldsymbol{\theta}^a_{k-1}, \mathbf{P}_{\theta,k-1},\ \mathbf{o}_{\theta,k};\ f(\cdot),\ h(\cdot)$
Output: $\mathbf{w}^a_k,\ \mathbf{P}_{w,k}, \boldsymbol{\theta}^a_k,\ \mathbf{P}_{\theta,k}$
/* 1. Compute heave */
1 $\bar{\boldsymbol{\eta}}_{w,k-1} := \boldsymbol{\eta}_{\theta,k-1}$ // share $\boldsymbol{\eta}_{\theta,k-1}$ from pitch
2 $\bar{\mathbf{w}}^a_{k-1} := [\mathbf{w}^H_{k-1} \quad p_{w,k-1} \quad \bar{\boldsymbol{\eta}}_{w,k-1}]^H$ // update heave
3 $\mathbf{w}^a_k,\ \mathbf{P}_{w,k} :=$ EAKF $\Big(\bar{\mathbf{w}}^a_{k-1}, \mathbf{P}_{w,k-1}, \mathbf{o}_{w,k}, f(\cdot), h(\cdot)\Big)$
/* 2. Compute pitch */
4 $\bar{\boldsymbol{\eta}}_{\theta,k-1} := \boldsymbol{\eta}_{w,k}$ // share $\boldsymbol{\eta}_{w,k}$ from heave
5 $\bar{\boldsymbol{\theta}}^a_{k-1} := [\boldsymbol{\theta}^H_{k-1} \quad p_{\theta,k-1} \quad \bar{\boldsymbol{\eta}}_{\theta,k-1}]^H$ // update pitch
6 $\boldsymbol{\theta}^a_k,\ \mathbf{P}_{\theta,k} :=$ EAKF $\Big(\bar{\boldsymbol{\theta}}^a_{k-1}, \mathbf{P}_{\theta,k-1}, \mathbf{o}_{\theta,k}, f(\cdot), h(\cdot)\Big)$
/* 3. Compute $\eta_3 \triangleq \omega$ via FFT and (3) */
7 **if** $k \geq N^{(1)}_\omega$ & $\mathrm{mod}(k, N^{(2)}_\omega) = 0$ **then**
8 $\quad \widehat{\varpi} :=$ FFT$(p_{\theta,k})$;
9 $\quad \eta_{3,\theta,k} :=$ Solution of (3) given $\widehat{\varpi}$.

Let $\mathbf{w} = [w, \dot{w}]^H$ be the heave state, and $\mathbf{w}^a = [\mathbf{w}^H_k \quad p_{w,k} \quad \boldsymbol{\eta}^H_{w,k}]^H$ be the augmented heave state with the unknown heave input p_w and heave parameter $\boldsymbol{\eta}_w$, and $\mathbf{o}_w$ be the heave measurement vector. Likewise for pitch, let $\boldsymbol{\theta} = [\theta, \dot{\theta}]^H$ be the pitch state vector, and $\boldsymbol{\theta}^a = [\boldsymbol{\theta}^H_k \quad p_{\theta,k} \quad \boldsymbol{\eta}^H_{\theta,k}]^H$ be the augmented pitch state vector. Then, the proposed Parameter-Sharing Extended-Augmented Kalman Filter (PS-EAKF) is provided in Algorithm 2. In particular, the two SDOF-system estimations are executed sequentially from heave to pitch. Further, we share the estimated $\boldsymbol{\eta}_{\theta,k-1}$ at time $k-1$ from the pitch estimation results to improve the heave estimation results at time k (see lines $1-3$), and share the newly estimated $\boldsymbol{\eta}_{w,k}$ from the heave estimation results at time k to improve the pitch estimation results at time k (see lines $4-6$).

Estimating Wave Frequency (ω). The unknown input forces, p_w and p_θ, are estimated directly as part of the state in the PS-EAKF filter. Subsequently, we apply the fast Fourier transform (FFT) to extract its amplitude ($\widehat{p}_{w,\max}, \widehat{p}_{\theta,\max}$), encountered frequency ($\widehat{\varpi}$), and phase ($\widehat{\phi}_w, \widehat{\phi}_\theta$). For a vessel encounter angle $\beta \geq \pi/2$ rad, the input wave $\widehat{\omega} \triangleq \widehat{\eta}_3$ can be estimated directly from $\widehat{\varpi}$ using (3). Therefore, in this work, we use the FFT algorithm to directly estimate $\eta_{3,k}$ from the estimated input force $\widehat{p}_{w,k}$ and $\widehat{p}_{\theta,k}$ when $k \geq N^{(1)}_\omega$ (see lines 7). Notably, since the input frequency ω is an unknown constant and performing the FFT is computationally expensive, we propose estimating ω every $N^{(2)}_\omega$ steps instead of every time step to improve computational efficiency.

Estimating Wave Amplitude (a). Given the estimated $\widehat{\boldsymbol{\eta}}$, we can compute $\widehat{P}_w(\boldsymbol{\eta})$ and $\widehat{P}_\theta(\boldsymbol{\eta})$ (see Section II-B). Hence, we can compute the estimated wave amplitude $\widehat{a}$ as:

$$\widehat{a} = \frac{1}{2}\bigg(\frac{\widehat{p}_{w,\max}}{\widehat{P}_w(\widehat{\boldsymbol{\eta}})} + \frac{\widehat{p}_{\theta,\max}}{\widehat{P}_\theta(\widehat{\boldsymbol{\eta}})}\bigg). \tag{14}$$

Subsequently, we can re-update the input force amplitudes from the newly computed wave amplitude $\widehat{a}$ in (14), *i.e.*:

$$\widehat{p}_{w,\max} = \widehat{a}\widehat{P}_w(\widehat{\boldsymbol{\eta}}); \quad (15)$$

$$\widehat{p}_{\theta,\max} = \widehat{a}\widehat{P}_\theta(\widehat{\boldsymbol{\eta}}) \quad (16)$$

IV. Seakeeping Experimental Validation

A. Scale-Model Experimental Settings

Physical measurement experiments were performed in the Australian Maritime College's Towing Tank facility using a 1:5 scale model of a rigid hull inflatable boat (referred to hereafter as the *AMC dataset*). These experiments included regular head seas seakeeping tests at various encounter frequencies and wave heights. A single flap-type wave maker, fitted at one end of the tank, was used to generate the desired sea conditions. The model was secured to the Towing Tank carriage and towed along the length of the tank at a constant speed, and allowed to freely heave and pitch in response to encountered waves (all other DOFs were constrained). The model and experiments were prepared and conducted according to the International Towing Tank Conference procedures [22], [23].

TABLE I
Vessel Particulars

Length (m)	L	7.00
Breath (m)	B_0	2.77
Draught (m)	T	0.35
Longitudinal Centre of Gravity (m)	CoG_x	2.11
Vertical Centre of Gravity (m)	CoG_z	0.79

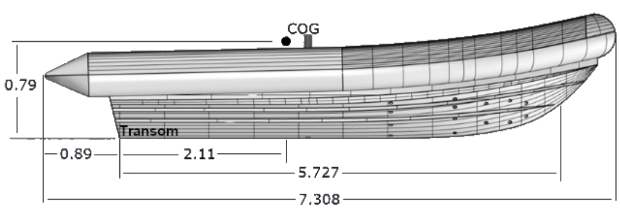

Fig. 2. Vessel's dimensions in meters. COG is the vessel's centre of gravity.

In order to demonstrate the application of the proposed PS-EAKF, we consider the case of a regular wave with amplitude $a = 0.15$ m and wave frequency of $\omega = 2.109$ rad/s[1]. The full-scale particulars of the vessel provided in Fig. 2 and Table I, including the vessels with length $L = 7$ m, maximum waterline breadth $B_0 = 2.77$ m, longitudinal centre of gravity $CoG_x = 2.11$ m, vertical centre of gravity $CoG_z = 0.79$ m. Notably, the *averaged* breadth $B = 1.47$ m and *averaged* draught $T = 0.35$ m are *unknown* to the filter in our problem setting and need to be estimated. The vessel is moving with a

[1]Notably, due to wave reflection at the end of the tank, the generated wave may contain other frequencies and the wave amplitude may not be at 0.15 m.

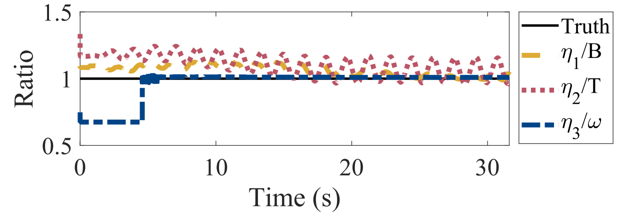

Fig. 3. The ratio of estimated unknown parameter $\boldsymbol{\eta} \triangleq [B, T, \omega]^H$ and its ground truth over 20 MC trials using PS-EAKF.

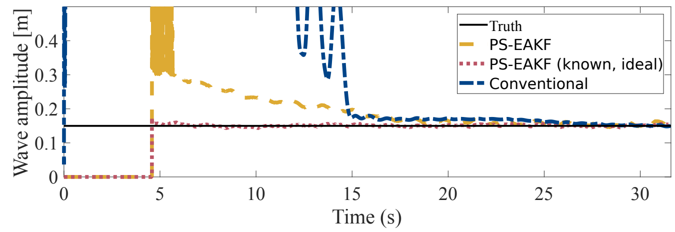

Fig. 4. Comparison results for estimating the input wave amplitude using different filtering methods over 20 MC trials (note: the y-axis is truncated).

forward speed $V = 4$ m/s with an encounter angle $\beta = \pi$ rad. The vertical displacement (heave) and rotation (pitch) of the vessel about the centre of gravity (CoG) is obtained using linear variable displacement transducers. The measurement sampling rate is $F_s = 447.2$ Hz, and the total time is 31.6 s. The unknown $B \triangleq \eta_1$, $T \triangleq \eta_2$, $\omega \triangleq \eta_3$ are initialised by sampling from the uniform distribution $\mathcal{U}\big([\frac{B_0}{2}, \frac{2B_0}{3}]\big)$ m, $\mathcal{U}\big([\frac{CoG_z}{8}, CoG_z]\big)$ m and $\mathcal{U}\big([0,3]\big)$ rad/s, respectively.

We compare the proposed PS-EAKF (*unknown* B and T) with an ideal counterpart using *known* B and T values, namely PS-EAKF (known, ideal). Additionally, we compare our results with the conventional method where B and T are *known* and the discretised unknown input frequency ω for estimation ranges from $[0.1, 0.2, \ldots, 2.5]$ rad/s, where sea wave state is estimated using the Adaptive Kalman Filter [4], [24] (named Conventional). Additionally, to demonstrate the stability of our PS-EAKF, we report mean values from 20 Monte-Carlo (MC) trials.

B. Results

Fig. 3 shows the ratio of the estimated unknown parameter $\boldsymbol{\eta}$ versus its ground truth over 20 MC trials for the AMC dataset using the proposed PS-EAKF filter. It demonstrates that the estimated $\widehat{\boldsymbol{\eta}} = [\widehat{\eta}_1, \widehat{\eta}_2, \widehat{\eta}_3]^H$ gradually converges to its ground-truth value, *i.e.*, its ratio converges to unity. Fig. 4 depicts the estimated values of wave amplitude ($\widehat{a}$) over time using different methods; the results further confirm the effectiveness of our proposed algorithm. Notably, PS-EAKF converges faster than the conventional method with known vessel parameter values. The detailed comparison results are provided in Table II; here, the estimated vessel parameters are averaged over the last 2.98 s (equal to one wave period) such

TABLE II
ESTIMATED RESULTS VERSUS TRUTH USING DIFFERENT FILTER METHODS 20 MC TRIALS.

	Input-State -Parameters	Truth	Estimates		
			PS-EAKF	PS-EAKF (known, *ideal*)	Conventional
Vessel Parameters	B (m)	1.470	1.492	-	-
	T (m)	0.350	0.373	-	-
Sea Wave	ω (rad/s)	2.109	2.129	2.129	2.092
	a (m)	0.150	0.156	0.151	0.151
Encountered Wave	ϖ (rad/s)	3.921	3.977	3.977	3.877
	$p_{w,\max}$ (m)	0.071	0.070	0.070	0.073
	$p_{\theta,\max}$ (rad)	0.039	0.040	0.040	0.040
	ϕ_w (rad)	4.263	4.675	4.489	4.225
	ϕ_θ (rad)	5.834	5.950	5.753	5.795

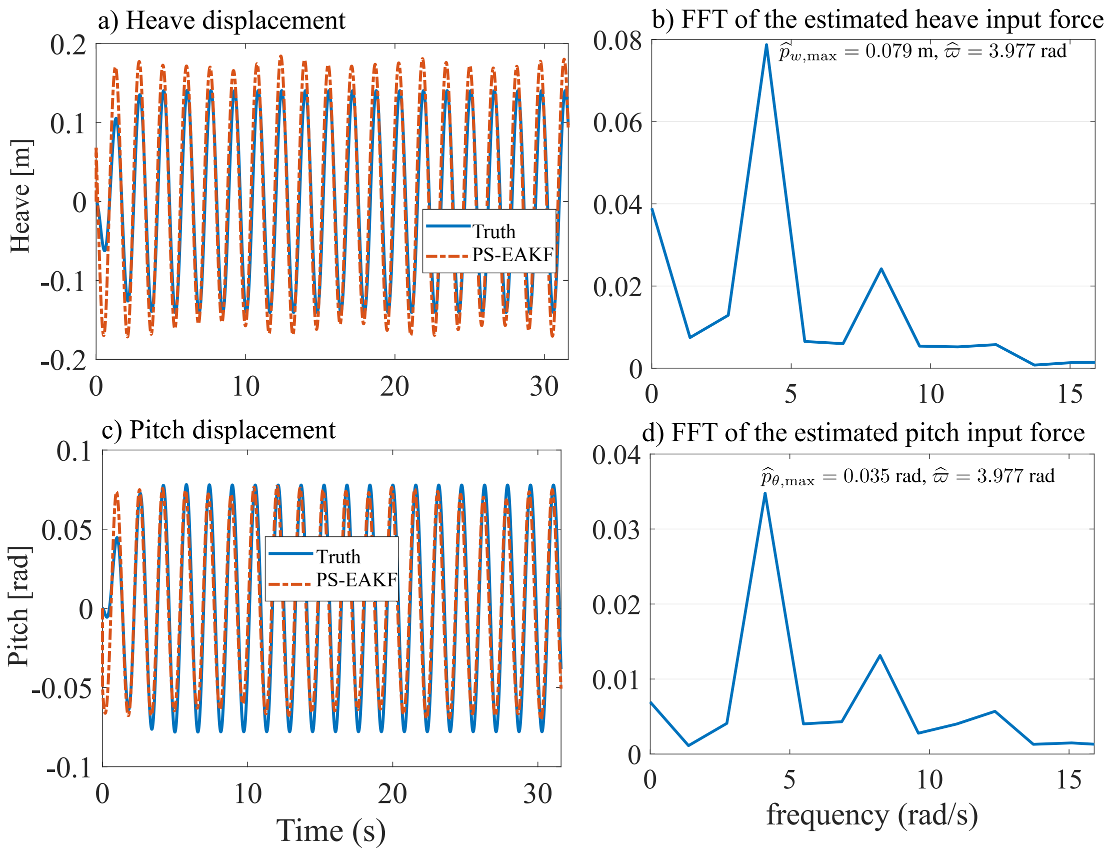

Fig. 5. Estimated results using the PS-EAKF filter for estimating: (top) **heave**, and (bottom) **pitch** using the **AMC dataset**.

that all estimation methods have converged and subsequently averaged over 20 MC trials.

The results demonstrate that PS-EAKF can estimate the *input-state-parameters* of this system reasonably accurately. In particular, we can correctly estimate the vessel parameters, B and T, with a less than 6% estimation error. Further, the proposed PS-EAKF can reliably estimate the sea wave and the encountered wave, with results comparable with the PS-EAKF (known, ideal) and the Conventional filtering methods, even though we do not need to know the vessel parameters.

Furthermore, Fig. 5 provides the detailed estimation results of displacement and input force using the PS-EAKF filter for heave and pitch. The results show that using the PS-EAKF filter; we can estimate the input-state of this system reasonably accurately. Notably, the proposed method can accurately estimate the input wave frequency ω with an estimation error less than 1%. We also observe that the PS-EAKF filter overestimates the amplitude of the heave input force, while underestimate the amplitude of the pitch input force. This can be attributed to our measurement noise assumption not matching the real-world noise from the AMC dataset. We also observe multiple peaks from the FFT plots in Fig. 5b and Fig. 5d. One hypothesis is that the measurement data from AMC dataset are noisy and non-Gaussian leading to errors in the frequency estimation and resulting multiple peaks in the FFT plots.

V. Conclusions

We reformulated the problem of estimating sea-state with onboard sensor measurements from a moving vessel when parameters of the wave-vessel model (*i.e.*, draught and breadth) are *a priori* unknown and need to be dynamically estimated as an input-state-parameter estimation of pseudo-mass-spring-damper systems problem. We developed a Parameter-Sharing Extended-Augmented Kalman Filter for solving the problem and validated its favourable performance. Our experimental results from a model-scale test program demonstrated the proposed algorithm's capability to achieve a similar estimation performance compared to the conventional methods for which accurate information of the dynamic characteristics of the vessel is available. Our future work aims to investigate more challenging sea conditions (*e.g.*, irregular seas) and examine PS-EAKF performance relying solely on indirect measurements (*e.g.*, using accelerometers) and augment the process noise as a part of state [8].

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