Modelling the Effects of Constrained Warping, Non-Linear Torsion and Anticlastic Curvature at Large Deflections of Leaf Springs

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Abstract

In precision machines, repeatable, deterministic motion behaviour at a high resolution is important. This requirement can be met by elements that allow relative motion by elastic deformation, which are applied in flexure mechanisms [1, 2, 3]. In these mechanisms, sliding or rolling contact is avoided, which leads to the reduction of friction, backlash, hysteresis and wear, which are difficult to model and to predict. Disadvantages are the limited range of motion, the larger drive stiffness and the generally lower and position-dependent support stiffness.

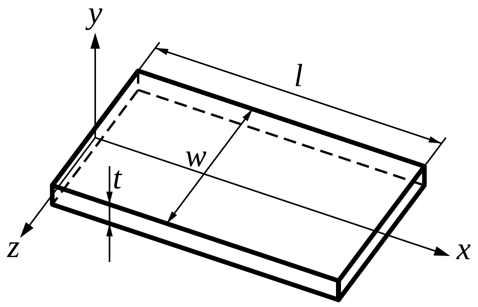

Figure 1: Example of a wide leaf spring. It is clamped at the face at $x = 0$ and loaded with constrained local displacements at the face at $x = l$, whereas the other faces are free and unloaded.

A typical elastic element is a leaf spring, as shown in Fig. 1. This is a structural element that is thin in one direction and extended in the other two directions, like a plate. It is distinguished from a plate in that it has two extended portions of its circumference that are free and two portions in which it is loaded or connected to other structural elements, like a beam. In particular, rectangular leaf springs with a constant thickness and two opposite sides that are free and two opposite sides that are connected to other elements are considered. Leaf springs have the property that they are stiff against in-plane loads and compliant for out-of-plane loads. This make them useful to constrain motion in some directions with a high stiffness, called the support stiffness, while allowing motion in other directions with a low stiffness, called the drive stiffness, so they can approximate conventional joints over a limited range of motion.

The relations between deflections and forces in the non-linear range are sought. Especially the support stiffness strongly depends on the motion in the compliant directions. Some analytical models, such as the pseudorigid body model by Howell [4], the beam constraint model by Awtar and its variations [5] and the model by Nijenhuis [6] can be used to obtain approximations for these relations. Here, a finite element description is used.

In the finite element description of flexible multibody systems [7, 8, 9], the deformation of elements was described by generalized strains defined by explicit functions of the nodal coordinates. An extension of this description was recently proposed [10], in which implicit relations between generalized strains and nodal coordinates can be used. In particular, assumed strain distributions are used and relations between the nodal points are obtained by integrating the strains over the length of the leaf spring. This extended way of describing finite elements is used to develop a finite element for modelling leaf springs with the inclusion of warping constraints, non-uniform torsion, non-linear torsional stiffness and the influence of anticlastic curvature on the flexural and torsional rigidity. Also the interactions between the different modes of deformation are included.

If only the constrained warping is included in the description, results comparable with those in [6] are obtained for a cantilever leaf spring, which was to be expected, since the same kind of interpolations and assumptions are made. Inclusion of the non-linear torsional stiffness gives more accurate results. The effects of anticlastic curvature are included by expanding the local out-of-plane displacements in Legendre polynomials up to order four in the lateral direction and Hermite polynomials in the axial direction. Results closer to those obtained by detailed finite element models are found. For moderately large relative rotations (smaller than 0.15 rad) and displacements (smaller than 0.15 times the length of the leaf spring), a model with a single element gives errors in the actuation load derivatives and error motion derivatives of the order of a few per cents. Some differences remain, even if the leaf spring is subdivided in more elements, because the description is approximate and several effects, such as the constrained lateral displacements at the ends, are not included [11].

A mass description similar to the one in [12] is used with the addition that higher-order polynomials are used to capture the higher order of the interpolation. Because of the orthogonality of the Legendre polynomials, many coupling terms are zero.

An analysis of a compliant mechanism, a parallel leaf spring mechanism, is presented, for which results are compared with those obtained by detailed finite element calculations. Forces, displacements and eigenfrequencies in a deflected stationary position are compared.

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