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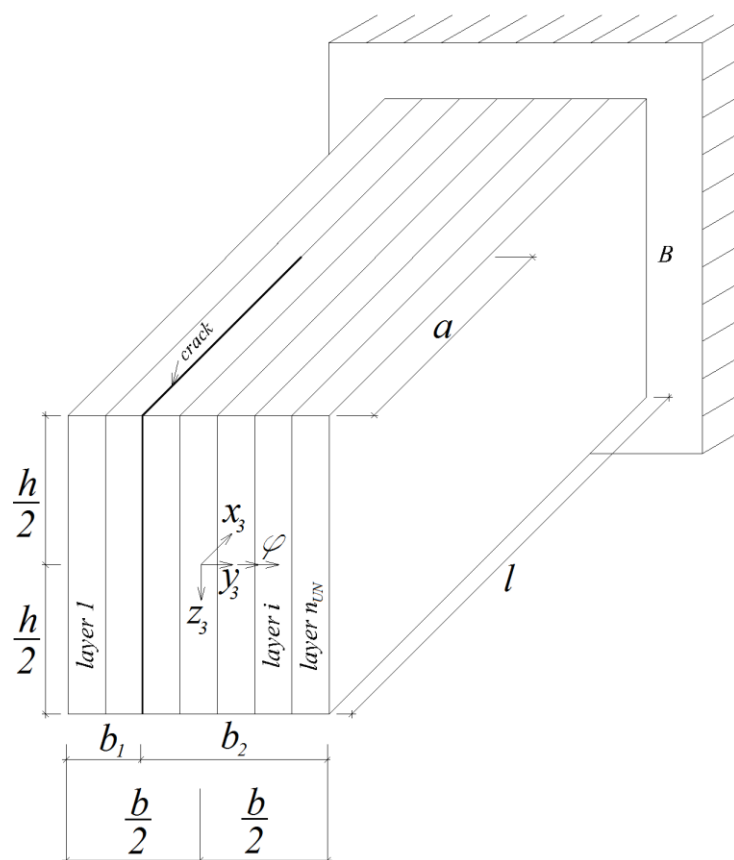
# Multilayered functionally graded beam with viscoelastic behaviour: a delamination study

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Multilayered functionally graded structural materials are made of adhesively bonded layers. The material of each layer is continuously inhomogeneous. Due to their high strength-to-weight and stiffness-to-weight ratios, the multilayered functionally graded materials are excellent candidates for light-weight load-bearing structural applications in various areas of the practical engineering.



**Figure 1.** Multilayered functionally graded viscoelastic cantilever beam configuration with a vertical delamination crack.

The basic drawback of multilayered materials and structures is the delamination fracture [1, 2, 3]. The delamination, or separation of layers, sharply deteriorates the operational performance of multilayered structural members and components. The delamination behaviour is of primary importance for the structural integrity and reliability of multilayered structures. The delamination is studied mainly in terms of the strain energy release rate. It should be mentioned that the delamination analyses of multilayered functionally graded beam configurations usually are concerned with instantaneous reaction of the structure, i.e. the time-dependent effects of the viscoelasticity on the delamination are not taken into account [4, 5]. However, the multilayered functionally graded structural members and components exhibit viscoelastic behaviour that has to be considered in the delamination studies.

Therefore, the purpose of the present paper is to develop a strain energy release rate analysis for a delamination crack in a multilayered functionally graded viscoelastic beam that is made of vertical layers (figure 1). A vertical delamination crack of length,  $a$ , is located arbitrary between layers as shown in figure 1. The viscoelastic behaviour is treated by using a linear viscoelastic model consisting of two springs and a dashpot. The moduli of elasticity of springs and the coefficient of viscosity of the dashpot are distributed continuously in the width direction of each layer (exponential laws are used for describing the distributions).

A time-dependent solution to the strain energy release rate that accounts for the viscoelastic behaviour of the functionally graded material is obtained by differentiating the time-dependent strain energy with respect to the crack area. The time-dependent strain energy cumulated in the multilayered functionally graded beam is found by integrating of the time-dependent strain energy density in the volume of the beam. A time-dependent solution to the strain energy release rate is derived also by analyzing the time-dependent compliance of the beam. The results obtained by the two solutions are identical. This fact proves the correctness of the solutions derived in the present paper.

The time-dependent solutions are applied to study the variation of the strain energy release rate with the time due to the viscoelastic behaviour of the material. The analysis indicates that the strain energy release rate increases with the time. Two three-layered cantilever beams are considered in order to evaluate the influence of the delamination crack location along the width of the beam on the strain energy release rate. Concerning the effects of material gradient along the width of layers, the calculations show that the strain energy release rate decreases with increasing of  $q_{11}$ ,  $q_{21}$  and  $q_{31}$  (the parameters,  $q_{11}$ ,  $q_{21}$  and  $q_{31}$ , control the material gradient along the width of layers). The increase of the speed of the angle of rotation of the free end of the right-hand delamination crack arm leads to increase of the strain energy release rate. The investigation reveals that the strain energy release rate decreases with increasing of the ratios of the moduli of elasticity in the layers.

The results obtained in the present paper give important information about the effects of location of the delamination crack, material gradients and the external loading on the delamination behaviour of the multilayered functionally graded viscoelastic beams.

## References

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1. Oral presentation.

2. Topic: Technics and Technologies in Mechanical, Agriculture and Transport Engineering