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States and Faults Estimation for Constrained Quasi-LPV Nonlinear System.

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***Abstract*— In this paper, we address a $\mathfrak{L}_2$ Proportional Integral (PI) observer based controller problem, for a class of quasi-LPV fuzzy system with faults and external disturbances. First, an augmented descriptor system is constructed where the augmented state vector consists of the original states, the actuator faults and sensor faults. Second, based on the quasi-LPV descriptor system, a robust $\mathfrak{L}_2$ PI observer based Estimated Dynamic State Feedback Fault Tolerant Controller *($\mathfrak{L}_2$ PI -EDSFFTC)* is designed, which is robust to external disturbances and both actuator and sensor faults. The studied system is subject to input and state constraints. An integrated robust controller strategy is adopted by a novel structure of non Parallel Distributed Compensation (non-PDC) control law. This ensures the closed-loop stability of the faulty system, and respects the given saturation constraints on the control input. The optimization problem is formulated using fuzzy Lyapunov function, and expressed on terms of linear matrix inequalities (LMIs). Finally, its application to an example is presented, to highlight the performance of the developed method.**

I. Introduction

Nowadays, the dependability and safety have become more important for practical engineering systems. However, in many real systems, the frequent occurrence of unknown faults often lead to performance degradation and even instability of the system [1-2]. In order to strengthen the system reliability and guarantee system stability, the faults estimation (FE) and fault tolerant control (FTC) have received considerable attention during the past few decades and plenty of results of these research fields have been reported in the literature [3-6]. Using FE/FTC results to design faults observer and fault estimations for nonlinear systems directly is a challenging issue.

Fortunately, quasi-LPV and LPV fuzzy model provide an effective way to express the complicated nonlinear systems via a set of local linear models interpolated by membership functions. As a result, the nonlinear control systems theory can be widely exploited to analyze and synthesize the nonlinear systems [7-8]. Therefore, there is a rapidly growing interest in FE and FTC problems for nonlinear system based on the quasi-LPV fuzzy method and many important results have been reported for the topic in the literature [9-13]. References [14] and [15] use optimization techniques to obtain the gain parameters of observers and controllers that are robust against the faults and disturbances. Meanwhile, the FE/FTC problem is considered for a class of nonlinear stochastic systems with actuators and sensors faults in [16] and [17]. Although several FE observers and FTCs for T-S fuzzy models with faults have been reported, they still have some challenging issues to be investigated, which motivated us to conduct the current work. In the LPV fault reconstruction literature, [23] proposed an observer applying the sliding mode methodology for fault estimation with application to a Boeing 747-100 LPV model with affine representation. In [25] fault estimation and LPV fault compensation are addressed, maintaining the control objectives, through the use of an affine LPV model of a two-link manipulator.[26] deals with a state observer for affine LPV systems, with a solution computed as a linear combination between the parameters and their boundaries. Furthermore, for a winding machine system, a polytopic LPV sensor fault detection filter has been developed in [18], [19], [27]. Although the works in these references are applied to quasi-LPV and LPV systems, the actuator fault estimation remains insufficiently explored. For that, the main contribution of this paper corresponds to the design of an observer in charge of the actuator and sensor fault estimation simultaneously along with the system state, with fewer disturbances than the one presents at the system dynamics. Thus, the proposed observer can be used in an FTC framework since the faults estimation and estimated states generate a saturated control signal with less disturbance corruption. This paper is dedicated to the study of the observer design based on the $\mathfrak{L}_2$ approach for quasi-LPV system with measurable premise variable, the main objective is to address the FE/FTC problem for a class of constrained quasi-LPV fuzzy systems subject to actuator and sensor faults. This reconfigurable controller can be designed in order to maintain stability, acceptable dynamic performance and steady state of the overall system, despite the presence of faults.

The second section of this paper is dedicated to brief description of some notations and problem statements, for modeling of the quasi-LPV structure of studied system, section III is devoted to the description of the proposed observer, the modified constrained FTC controller, the control problem definitions and the presentation of main results with the LMI based design conditions for $\mathfrak{L}_2$ PI observer based *EDSFFTC*. The applicability of the method is studied and illustrated through simulation example to compare and show the applicability and performance of our approach which is analyzed in section IV. Finally some conclusions and remarks are given in section V.

I. Notations and problem statements:

A. Notations: throughout this paper, the following notations are adopted to represent conveniently the different expressions, given a set of nonlinear function: $h_i(.)$, $v_{\hbar}(.)$, $i \in \{1 \dots . r\}$, $\hbar \in \{1 \dots . r_e\}$ are the nonlinear scalar functions. This work focuses on measurable premise variables grouped in the vector $z(k)$, whose measurements can be obtained from the observer design, which depends on the state vector, and can be equivalently represented by a vector of states, expressed as $h_i(z(k))$ and $v_{\hbar}(z(k))$, satisfying the convex sum property. For a vector x and z, $x(k), z(k)$defined by x_k, z_k , $x(k+1)$ defined by x_{k+}, the same for the other vector. I_r denotes the set $\{1, 2, \cdots, r\}$, I_{r_e} denotes the set $\{1, 2, \cdots, r_e\}$, $\mathfrak{R}^+$ represents the set of positive real integer. I denote the identity matrix. An asterisk $*$ symbolizes the symmetric block matrices. $\mathcal{N}_n$ denotes the set $\{1, \cdots, n\}$.

- $\mathcal{H}(A)$ denotes the Hermitian of the matrix A, i.e.$\mathcal{H}(A) = A + A^T$.
- $\mathbb{Z} + (*)$ denotes $\mathbb{Z} + \mathbb{Z}^T$.
- $\mathcal{X}^T > 0$ means that $\mathcal{X}$ is a symmetric positive definite matrix.
- The single double or triple sums can be rewritten as:

$$\mathcal{G}_h = \sum_{i=1}^{r} h_i(z_k)\,\mathcal{G}_i \ ; \ \mathcal{G}_{hh} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z_k)\, h_j(z_k)\,\mathcal{G}_{ij}$$

B. System description: in the following section, the controller is derived using the descriptor form. Sufficient LMI constraints are derived from Lyapunov's theory. Compared to [29], in the following section a constrained controller is proposed, for this we consider the following class of quasi-LPV fuzzy model subject to input saturation, external disturbances, actuator and sensor fault:

$$\begin{cases} x_{k+} = A_h x_k + B_h sat(u_k) + F_a f_{a,k} + B_\omega \omega_k \\ y_k = C x_k + F_s f_{s,k} \end{cases} \quad (1.a)$$

Where $x_k \in \mathfrak{R}^{n_x}$,$u_k \in \mathfrak{R}^{n_u}$, $\{f_{a,k} = f_{s,k} = f_k\} \in \mathfrak{R}^{n_f}$,$y_k \in \mathfrak{R}^{n_y}$, $\omega_k \in \mathfrak{R}^{n_\omega}$ are the state, control input, actuator-sensor fault, output vector, and the exogenous disturbances respectively. The state-space matrices: A_i, B_i, C, B_ω are of the appropriate dimensions, F_a , F_s and the matrix C involved in (1.a) is assumed to be a full row rank, k is a current samples, where $i \in I_r$ represent the *i-th* linear right hand-side submodel of quasi-LPV model (1.a). System (1.a) with bounded nonlinearities can be represented by a polytopic form:

$$\begin{cases} x_{k+} = \sum_{i=1}^{r} h_i(z_k)(E_i\, x_k + B_i sat(u_k)) + F_a f_{a,k} + B_\omega \omega_k \\ y_k = C x_k + F_s f_{s,k} \end{cases} \quad (1.\text{b})$$

Where the membership function is denoted $h_i(z_k)$, and vary within the convex set Ω_1 :

$$\Omega_1 = \{h_i(z_k) \in \mathfrak{R}^r;\ h_i(z_k) = [h_1(z_k), \dots \dots, h_r(z_k)]^T; h_i(z_k) \geq 0\} \quad (2.\text{a})$$

Note that $h_i(z_k)$ depends on the variable z_k verifying the convex sum property, rewritten here by convenience:

$$\sum_{i=1}^{r} h_i(z_k) = 1 \quad (2.\text{b})$$

$r \in I_r$: is the number of sub-models, in the right-hand side.

The augmented form is adopted. The quasi-LPV system (1.a) can be equivalently rewritten in the following compact descriptor singular form:

$$\begin{cases} E^* x_{k+}^* = A_h^* x_k^* + B_h^* sat(u_k) + F_a^* f_k + B_\omega^* \omega_k \\ y_k = C^* x_k^* + F_s f_k \end{cases} \quad (3.a)$$

Where: $x_k^* = \begin{bmatrix} x_k \\ x_{k+} \end{bmatrix} \in \mathfrak{R}^{n_{x^*}}$; $E^* = diag[I \quad 0]$; $A_h^* = \begin{bmatrix} 0 & I \\ A_h & -I \end{bmatrix}$; $B_h^* = \begin{bmatrix} 0 \\ B_h \end{bmatrix}$; $F_a^* = \begin{bmatrix} 0 \\ F_a \end{bmatrix}$; $B_\omega^* = \begin{bmatrix} 0 \\ B_\omega \end{bmatrix}$. (3.b)

II. Control problem

A. Control law: In order to satisfy the desired constrained controller performance *(EDSFFTC)*, the augmented controller design, is defined for system (4.a):

$$\begin{cases} E_{x_F^*}^* x_{F,k}^* = A_{Fh}^* P_{h2}^{-1*} x_{F,k}^* + B_{Fh}^* P_{h1}^{-1*} \hat{x}_k^* \\ u_k = C_{Fh}^* P_{h2}^{-1*} x_{F,k}^* + D_{Fh}^* P_{h1}^{-1*} \hat{x}_k^* \end{cases} \quad (4.\text{a})$$

Where: $P_{h1}^* = diag[P_{h1} \quad 0]$; $P_{h2}^* = diag[P_{h2} \quad 0]$; $A_{Fh}^* = \begin{bmatrix} 0 & I \\ A_{Fh} & -I \end{bmatrix}$; $B_{Fh}^* = \begin{bmatrix} 0 & 0 \\ B_{Fh} & 0 \end{bmatrix}$; $C_{Fh}^* = [C_{Fh} \quad 0]$; $D_{Fh}^* = [D_{Fh} \quad 0]$ and $E_{x_F^*}^* = diag[I_{x_F} \quad 0]$. (4.b)

The gains $P_{h1}^*, P_{h2}^*, A_{Fh}^*, B_{Fh}^*, C_{Fh}^*$ and D_{Fh}^* are matrices controllers to be determined, $\hat{x}_k^*$ is the estimated augmented state variable of x_k^*.
The architecture of the proposed constrained FTC controller design is based on the scheme depicted in Figure 1.

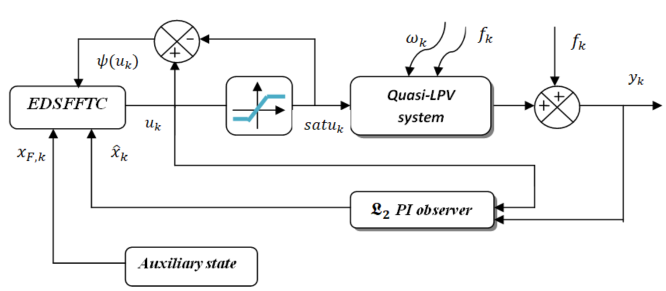

Figure 1. The proposed *EDSFFTC* design scheme.

System (1.a) achieves observability conditions, as detailed in [31]. To derive the controller laws, a *PI* observer is synthesized to estimate both faults and states for system (1.a) and has the singular form:

$$\begin{cases} E^*\hat{x}^*_{k+} = A^*_h\hat{x}^*_k + B^*_h u_k + F^*_a\hat{f}_k + L^*_{Ph}(y_k - \hat{y}_k) \\ \hat{y}_k = C^*\hat{x}^*_k + F_s\hat{f}_k \\ \hat{f}_{k+} = \hat{f}_k - L_{Ih}(y_k - \hat{y}_k) \\ u_k = C^*_{Fh}P^{-1*}_{h2}x^*_{F,k} + D^*_{Fh}P^{-1*}_{h1}\hat{x}^*_k \\ E^*_{x^*_F}x^*_{F,k+} = A^*_{Fh}P^{-1*}_{h2}x^*_{F,k} + B^*_{Fh}P^{-1*}_{h1}\hat{x}^*_k \end{cases} \tag{5.a}$$

Where: $L^*_{Ph} = \begin{bmatrix} 0 \\ L_{Ph} \end{bmatrix}$ (5.b)

L^*_{Pj}: is the augmented proportional gain for estimating the augmented variable state.

L_{Ijk}: is the integral gain for estimating the fault.

Definition 1: The equations (6) define the $\mathfrak{L}_2$ PI observer for the augmented system (3.a), for arbitrary initial conditions: $x^*(0)$, $f(0)$ and a stabilizing input u_k, the following relations are true:

$$\lim_{k\to\infty}(x^*_k - \hat{x}^*_k) = 0 \tag{6.a}$$

$$\lim_{k\to\infty}(f_k - \hat{f}_k) = 0 \tag{6.b}$$

The estimation error between the system (4.a) and the observer (5.a) is given by: $e^*_{0,k} = x^*_k - \hat{x}^*_k$ (7)

The fault error is defined by: $e_{f,k} = f_k - \hat{f}_k$ (8)

Also, we define the augmented vector: $\bar{X}_k = \begin{bmatrix} \bar{x}^*_k \\ f_k \end{bmatrix}$ and $e_k = \begin{bmatrix} e^*_{0,k} \\ e_{f,k} \end{bmatrix}$:

$$\begin{cases} \bar{\mathcal{E}}\bar{X}_{k+} = \bar{\mathcal{A}}_{hh}\bar{X}_k - \bar{\mathcal{B}}_e e_k - \bar{\mathcal{B}}_h\psi(u_k) + \bar{\mathcal{B}}_\omega\omega_k \\ y_k = \bar{\mathcal{C}}\bar{X}_k \\ u_k = \mathfrak{G}_h\mathfrak{P}^{-1}_h\bar{X}_k - \mathcal{D}_{F\hat{h}}\mathrm{P}^{-1*}_h e_k \end{cases} \tag{10.a}$$

And

$$\begin{cases} E_e e_{k+} = \mathfrak{A}_{hh}e_k - \mathcal{B}_h\psi_k + \mathcal{B}_\omega\omega_k \\ u_k = \mathfrak{G}_h\mathfrak{P}^{-1}_h\bar{X}_k - \mathcal{D}_{Fh}\mathrm{P}^{-1*}_h e_k \end{cases} \tag{10.b}$$

$\bar{\mathcal{E}} = diag[\bar{E} \quad I_f]$; $\bar{\mathcal{A}}_{hh} = \begin{bmatrix} \bar{A}^*_{hh} & \bar{F}^*_a \\ [0] & I_f \end{bmatrix}$; $\bar{\mathcal{B}}_h = \begin{bmatrix} \bar{B}^*_h \\ [0] \end{bmatrix}$; $\bar{\mathcal{B}}_\omega = \begin{bmatrix} \bar{B}^*_\omega \\ [0] \end{bmatrix}$ $\bar{\mathcal{B}}_e = diag[\bar{B}_e \quad [0]]$; $\bar{\mathcal{C}} = [\bar{C} \quad F_s]$; $\mathfrak{G}_h = [\bar{\mathcal{G}}_h \quad [0]]$; $\mathfrak{P}^{-1}_h = diag[\mathcal{P}^{-1*}_h \quad [0]]$; $\mathcal{D}_{Fh} = [D^*_{F\hat{h}} \quad [0]]$; $\mathrm{P}^{-1*}_h = diag[P^{-1*}_{h1} \quad [0]]$; $\mathfrak{A}_{hh} = \begin{bmatrix} (A^*_{hh} - L^*_{Ph}C^*) & (F^*_a - L^*_{Ph}F_s) \\ L_{Ih}C^* & (L_{Ih}F_s + I_{e_f}) \end{bmatrix}$. (10.c)

Now, we consider the augmented system as follows : $\eta_k = \begin{bmatrix} \bar{X}_k \\ e_k \end{bmatrix}$ (11), by combining the system (9) and the error dynamics (10), the closed-loop quasi-LPV system is obtained as follows:

$$\begin{cases} E\eta_{k+} = \mathcal{U}_{hh}\eta_k - \mathcal{M}_h\psi(u_k) + \mathcal{O}_\omega\omega_k \\ y_k = \mathcal{Q}\eta_k \\ u_k = \mathcal{J}_h\mathbb{P}^{-1}_h\eta_k \\ \psi(0) = 0 \end{cases} \tag{11.a}$$

With $: E = diag[\bar{\mathcal{E}} \quad E_e]$; $\mathcal{U}_{hh} = \begin{bmatrix} \bar{\mathcal{A}}_{hh} & -\bar{\mathcal{B}}_e \\ 0 & \mathfrak{A}_{hh} \end{bmatrix}$; $\mathcal{M}_h = \begin{bmatrix} \bar{\mathcal{B}}_h \\ \mathcal{B}_h \end{bmatrix}$; $\mathcal{O}_\omega = \begin{bmatrix} \bar{\mathcal{B}}_\omega \\ \mathcal{B}_\omega \end{bmatrix}$; $\mathcal{Q} = [\bar{\mathcal{C}} \quad [0]]$; $\mathcal{J}_h = [\mathfrak{G}_{hh} \quad \mathcal{D}_{Fh}]$; $\mathbb{P}^{-1}_h = [\mathfrak{P}^{-1}_h \quad \mathrm{P}^{-1*}_h]$; $\mathcal{O}_\omega = \begin{bmatrix} \bar{\mathcal{B}}_\omega \\ \mathcal{B}_\omega \end{bmatrix}$. (11.b)

Moreover, to handle the non-linearity $sat(u_k)$ the dead-zone function $\psi(u_k)$ will be employed:

$$\psi(u_{k(l)}) = u_{k(l)} - sat(u_{k(l)}) \tag{12}$$

We use the generalized sector condition proposed by [20] to deal with the dead zone function. Also, the set $\widetilde{\mathcal{D}}_u = \{\tilde{x}_k \in \Re^{n_{\tilde{x}}}: |u_{k(l)} - v_{k(l)}| \le u_{\max(l)}; l \in I_{n_l}\}$,(13) with the auxiliary signal $v_{k(l)} = \chi_{hv}\mathbb{P}^{-1}_h\eta_k$ used as a degree of freedom in the design conditions and the condition: $\psi(u_{k(l)})^T \tilde{S}_{h(l)}{}^{-1}[\psi(u_{k(l)}) - v_{k(l)}] \le 0$ holds. Due to the saturating actuators, only initial conditions in a subset of $\mathcal{L}_v$yield the trajectories of (3.a) to converge to the origin. Such a subset is denoted by $\mathcal{L}_v$, being called the domain of attraction. The determination of DoA is not an easy task even for small order systems since it can be (20)non-convex, open, and in some cases, unbounded [20] Therefore, an estimate of the DoA $\subseteq \mathbb{D}_\eta$ is computed, usually the largest possible. One way to construct the estimate DoA is to employ level sets taken from the Lyapunov function associated with the closed-loop system. To this end, a non quadratic Lyapunov function is considered: $V(\eta_k) = \eta^T_k E^T \Pi^{-1}_h E\eta_k \le \rho$ (14), if there exist a function given by (16) fulfilling the Lyapunov conditions for stability of (3.a), a level set associated with the Lyapunov function can be defined as in the following lemma:

Lemma 1: suppose that $V(\eta_k)$ given in (14) is a Lyapunov function for system (14.a). Then, a possible level set is given by:

$$\mathcal{L}_v = \cap_{z\in\Omega_1}\mathcal{E}(E^T\Pi^{-1}_h E, \rho) = \cap^r_{i=1}\mathcal{E}(E^T\Pi^{-1}_h E, \rho) \tag{15}$$

For > 0 , and :

$$\mathcal{E}(E^T\Pi^{-1}_h E, \rho) = \{\eta_k \in \Re^{n_\eta}: \eta^T_k E^T\Pi^{-1}_h E\eta_k \le \rho\} \tag{16}$$

For the proof, see of this lemma can be founded in [21].

Assumption 1: The state trajectories of quasi-LPV descriptor system (14.a) are contained within the following polyhedral set (validity domain), $\mathbb{D}_\eta \in \Re^{n_\eta \times n_\eta}$ defined as follows:

$$\mathbb{D}_\eta = \{\eta_k \in \Re^{n_\eta}: \mathcal{N}^T_q\eta_q \le 1, q \in I_{n_q}\} \tag{17}$$

Where the given matrix $\mathcal{N}_q \in \Re^{n_\eta}$, represents the state constraints of system (7.a), with: $\mathcal{N}_q = [\mathcal{N}_m^* \quad 0_{n_u\times n_{x_F}} \quad 0_{n_u\times n_f} \quad 0_{n_u\times n_{e_0}} \quad 0_{n_u\times n_{e_f}}]$; $\mathcal{N}_m^* = [\mathcal{N}_m \quad 0]$ (18)

B.LMI-Based design conditions of constrained descriptor system: this work is concerned with proposing a systematic method to design a controller such that the closed-loop system satisfies the following properties, presents a new LMI-based method to design an estimated constrained FTC, besides a sufficient condition to solve the following control problem:

Property 1 ***[local stability]:*** Given a scalar α', the initial condition $\eta(0)$ belong to a specific set in the state-space, which $\mathcal{E}(E^T\Pi_h^{-1}E,\rho)$ is a region of asymptotic stability (RAS) for the saturated system (3.a). In the presence of disturbances, the controller guarantees that the trajectories of (3.a) are bounded, there exist a matrix $\tilde{P}_i > 0$ and a positive scalar $\rho > 0$ such that, for any η (0) $\in \mathcal{E}(E^T\Pi_h^{-1}E,\rho)$ and $\omega_k \neq 0$, the trajectories of the saturated system remains inside the polyhedral set $\mathbb{D}_\eta$, and do not leave the ellipsoid $\mathcal{E}(E^T\Pi_h^{-1}E,\rho)$, and converges exponentially to the equilibrium point with a decay rate less than α', satisfying the following property:

Property 2: ***[$\mathfrak{L}_2$ gain-performance]*** Given vector $\mathcal{N}_q$ defined in assumption 1, and a positive scalar δ depending in the type of disturbances involved in the dynamics of system (3.a), see [20] we distinguish two following control problems:

Control problem 1: when $\omega_k \neq 0$.There exist positive scalar ρ and γ such that $\forall\, \eta_k \in \mathcal{L}_v \backslash \{0\}$, the corresponding closed-loop trajectory (14.a) remains inside the validity domain $\mathbb{D}_\eta$ defined in (19). Moreover the $\mathfrak{L}_2$-gain of the state vector $\tilde{x}_k$ is bounded as follows:

$$\|\eta_k\|_2^2 < \gamma^2\|\omega_k\|_2^2 + \rho, \forall k > 0 \qquad (19)$$

where the objective is to attenuate the effects of exogenous input ω_k on the augmented state space by minimizing γ and ρ.

Control problem 2: Consider the quasi-LPV descriptor model design with a polytopic controller (14.a) such that: $\mathcal{L}_v \subseteq \mathbb{D}_\eta \cap \mathbb{D}_u$ as large as possible, and is a contractively invariant set with respect to the closed-loop system. The problem is reformulated to design the observer gains to guarantee asymptotic convergence to zero despite the mismatches.

The objective now is to compute the gains of observers based controller (5.a), to ensure the stability of the closed loop system (14.a) guarantying the trajectories tracking performance for all $\omega_k \neq 0$, sufficient conditions to achieve this objective are given through the following theorem:

Theorem 1 : For a given the discrete-time quasi-LPV system (3.a) with a nonlinearities parameter uncertainties $z_k \in \Omega_1$ and Ω_2 under input saturation with the proposed observer based controller (5.a), whose validity domain is defined by $\mathbb{D}_\eta$, is locally exponentially stable if there exist a matrices: $P_{i1} = P_{i1}^T > 0$, $P_{i2} = P_{i2}^T > 0$, $\{P_{i1}, P_{i2}\} \in \Re^{n_x\times n_x}$, $\chi_{j1} \in \Re^{n_u\times n_x}$, $\chi_{j2} \in \Re^{n_u\times n_x}$, $A_{Fj} \in \Re^{n_x\times n_x}$, $B_{Fj} \in \Re^{n_x\times n_x}$, $C_{Fj} \in \Re^{n_u\times n_x}$, $D_{Fj} \in \Re^{n_u\times n_x}$
for any diagonal gain matrix $S_j \in \Re^{n_u\times n_u}$ a positive scalars: $\bar{\gamma} = \sqrt{\gamma^2}, \rho$, where $(i,j) \in (I_r \times I_r)$, such that the following inequalities hold:

$$\begin{cases} \min \bar{\gamma}, \rho \\ E^T\mathbb{P}_i^{-1}E > 0 \end{cases} \qquad (20)$$

$$\rho + \gamma^2\delta < 1 \qquad (21)$$

$$\begin{bmatrix} -E^T\mathbb{P}_i^{-1}E & * \\ \mathcal{J}_{j(l)} - \chi_{j(l)} & -u^2_{\max(l)}/\rho \end{bmatrix} < 0 \qquad (22)$$

$$\begin{bmatrix} -E^T\mathbb{P}_i^{-1}E & * \\ \mathcal{N}_{q(l)} & -1/\rho \end{bmatrix} < 0 \qquad (23)$$

$$\begin{cases} \mathfrak{T}_{ii} < 0 \\ \dfrac{2}{r-1}\mathfrak{T}_{ii} + \mathfrak{T}_{ij} + \mathfrak{T}_{ji} < 0 \quad (i,j) \in (I_r \times I_r), \qquad i \neq j \end{cases} \qquad (24)$$

Where the quantity $\mathfrak{T}_{ij}$ is defined in (26):

$$\mathfrak{T}_{ij} = \begin{bmatrix} \Psi & * \\ \Theta_{12}^T & -\Theta_{22} \end{bmatrix} < 0 \qquad (25.a)$$

where:

$$\Psi = \begin{bmatrix} \Psi^{11} & * & * & * & * \\ \mathbb{P}_i & -I_{n_\eta\times n_\eta} & * & * & * \\ \chi_j & 0_{n_u\times n_\eta} & -2S_j & * & * \\ 0 & 0_{n_u\times n_\eta} & 0_{n_u\times n_u} & -\gamma^2 I & * \\ \mathcal{U}_{ij}\mathbb{P}_i & 0_{n_\eta\times n_\eta} & -\mathcal{M}_i S_j & \mathcal{O}_\omega & -\mathbb{P}_i \end{bmatrix} < 0 \qquad (25.b)$$

$$\Theta_{12}^T = \begin{bmatrix} [L_j^* C^* & 0_{n_\eta\times n_\eta} & 0_{n_u\times n_u} & 0_{n_u\times n_u} & 0_{n_\eta\times n_\eta}] \\ [0_{n_\eta\times n_\eta} & 0_{n_\eta\times n_\eta} & 0_{n_u\times n_u} & 0_{n_u\times n_u} & I_{n_\eta\times n_\eta}] \end{bmatrix} \qquad (25.c)$$

$$\Theta_{22} = \begin{bmatrix} \varepsilon(2I - \mathbb{P}_i) & * \\ 0_{n_{x^*}\times n_{x^*}} & \varepsilon^{-1}(2I - \mathbb{P}_i) \end{bmatrix} \qquad (25.d)$$

and: $\mathcal{J}_j = [\mathfrak{G}_j \quad \mathfrak{D}_{Fj}] = [D_{Fj}^* \quad C_{Fj}^* \quad 0 \quad D_{Fj}^* \quad 0]$ (25.e)

$\chi_{j(l)} = [D_{Fj}^* \quad C_{Fj}^* \quad 0 \quad D_{Fj}^* \quad 0]$ (25.f)

With : $\mathbb{P}_i = \Pi_i$

Remark : The LMI set (25) can brings conservatism into the observer design due to the LMI dimension, the number of models, and the requirement of a Lyapunov matrices P_{h1}, P_{h2}. To reduce the conservatism, relaxed conditions can be obtained. Then, without loss of good compromise between complexity and conservatism, the relaxation LMIs set given by [31] can be considered.

Proof: the demonstration is omitted for the sake of brevity.

III. Illustrative example:

This section gives design examples for the nonlinear model following control. Recall the simple nonlinear system defined in [32], with matrices system as follows :

$$A_1 = \begin{bmatrix} 1 & -\beta \\ -1 & -0.5 \end{bmatrix}\ ; A_2 = \begin{bmatrix} 1 & \beta \\ -1 & -0.5 \end{bmatrix}\ ;\ B_1 = \begin{bmatrix} 5+\beta \\ 2\beta \end{bmatrix}\ ;\ B_2 = \begin{bmatrix} 5-\beta \\ 2\beta \end{bmatrix}\ ;\ B_\omega = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}\ ;\ F_a = \begin{bmatrix} 1 \\ 0 \end{bmatrix}\ ;\ C = [1 \quad 0.2];\ F_s = 2 \text{ and } \beta = 2.65. \quad (26)$$

The membership function's :

$$h_1 = \frac{x_{1,k}+\beta}{2\beta}\ ;\ h_2 = (1-h_1). \quad (27)$$

The initial conditions are: $x_0 = [0.3 \quad 0.1]$, $\hat{x}_0 = [0 \quad -0.1]$.The maximal saturation level $u_{max} = 0.01$. The state constraints are : $\mathbb{D}_\eta = \{|x_{1,k}| \le 1, |x_{2,k}| \le \beta\}$, an the actuator and sensor fault are assumed to be an additive signal (abrupt fault) such as:

$$f_k = \begin{cases} 0.1 & ;\ if\ 8s \le t \le 20s \\ 0 & ;\ otherwise \end{cases} \quad (28)$$

Solving the LMIs of theorem 1, the unknown gains of the PI observer are obtained. The constant α', δ and ε are selected: 0.05 ; 0.2 and 1, since the main objective is to estimate the state variables and the faults, the value of matrix L_{Pj}, $L_{Ij}, A_{Fj}, B_{Fj}, C_{Fj}$ and D_{Fj} are selected as:

$L_{P1} = \begin{bmatrix} 3.0026 \\ 2.0030 \end{bmatrix}$; $L_{P2} = \begin{bmatrix} 1.0161 \\ -2.0308 \end{bmatrix}$; $L_{I1} = -0.3961$;$L_{I2} = -0.3979$.

$A_{F1} = 10^{-5} \begin{bmatrix} -0.1403 & 0.2828 \\ 0.2822 & -0.5659 \end{bmatrix}$; $A_{F2} = 10^{-4} \begin{bmatrix} -0.0075 & -0.0017 \\ 0.0633 & -0.1264 \end{bmatrix}$;

$B_{F1} = \begin{bmatrix} -0.0075 & -0.0017 \\ -0.0012 & -0.0128 \end{bmatrix}$; $B_{F2} = \begin{bmatrix} -0.0075 & -0.0017 \\ 0.0633 & -0.1264 \end{bmatrix}$

$C_{F1} = 10^{-5}[0.4251 \quad 0.2861]$;

$C_{F2} = 10^{-5}[0.3509 \quad -0.7099]$.

$D_{F1} = [-0.0040 \quad -0.0247]$; $D_{F2} = [0.0040 \quad 0.0243]$.

$P_{1\_1} = \begin{bmatrix} -0.8003 & -0.1244 \\ -0.1244 & -0.2034 \end{bmatrix}$; $P_{1\_2} = \begin{bmatrix} -0.4992 & 0.1401 \\ 0.1401 & -0.2947 \end{bmatrix}$;

$P_{2\_1} = \begin{bmatrix} 0.4337 & -0.0020 \\ -0.0020 & 0.3514 \end{bmatrix}$; $P_{2\_2} = \begin{bmatrix} 0.4333 & -0.0056 \\ -0.0056 & 0.3557 \end{bmatrix}$.

Which are used to construct the PI observer (5.a), implemented in simulation. The simulation results a carried out with level attenuation $\gamma = 3.3379$, the obtain constant: $\rho = 0.2930$.

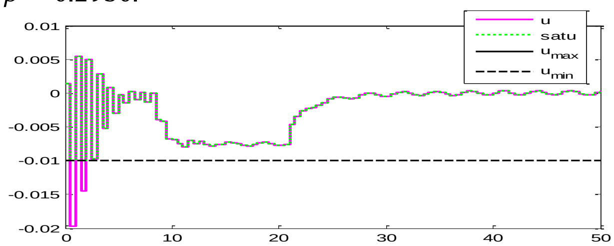

Figure 2: the control inputs.

The input disturbance signal , is a persistent sinusoidal signal defined by: $\omega_k = 0.1\sin(k)$. In this case, the measurement fault is shown in Fig.6, Fig.3 shows states estimation, which we observe that the actuator fault is well attenuated, and Fig.2 shows the stabilizing control input. Fig.4 the output signal and its estimate, the estimation and fault error in Fig.5 and Fig.7 respectively converge towards zero. These simulation results demonstrate the applicability of the method for estimating actuator and sensor faults, states of quasi-LPV system, which is stabilized despite the presence of fault and with an important input saturation appearing in the beginning as well as the disturbance and actuator fault are well attenuated.

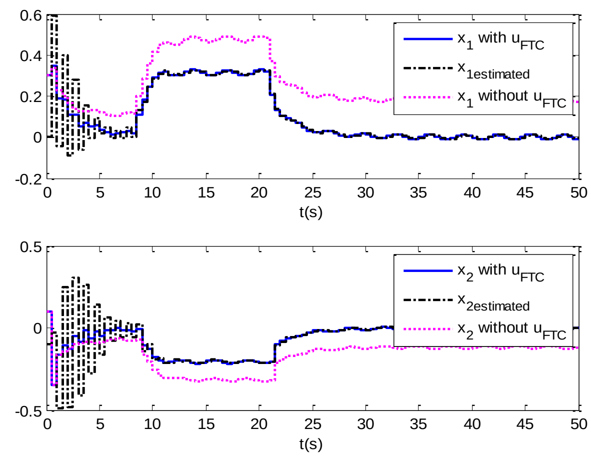

Figure 3: States variables and theirs estimate.

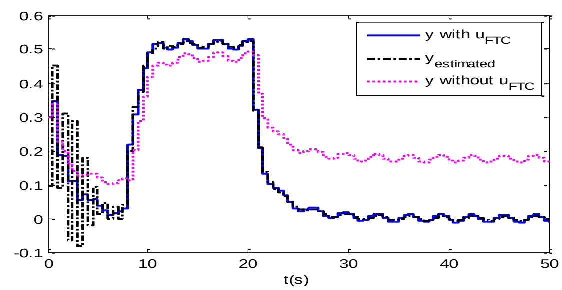

Figure 4: The output signal and its estimate.

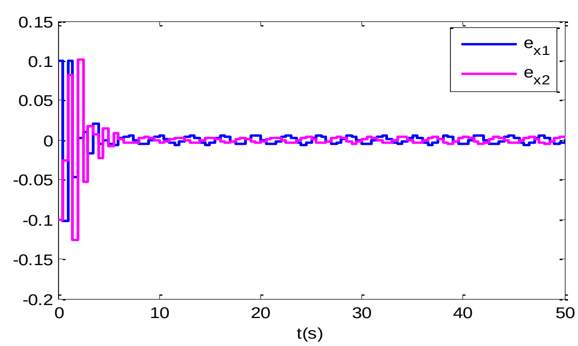

Figure 5: The estimation errors.

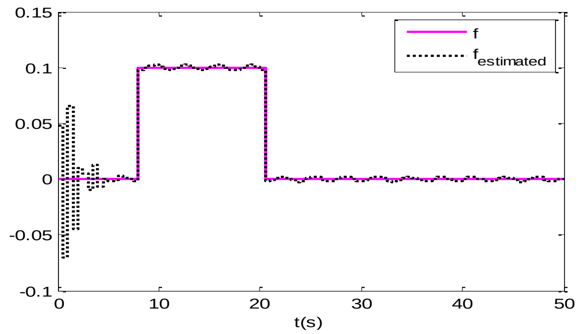

Figure 6: The fault and its estimate.

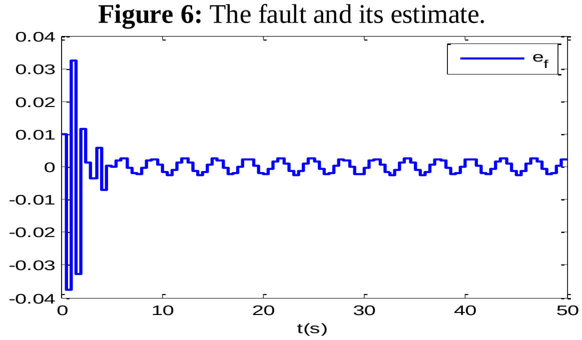

Figure 7: The fault error.

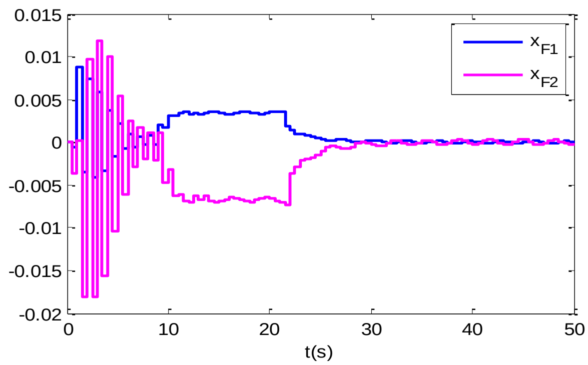

Figure 8: Auxiliary state's variable.

IV. Conclusion

In this work, a $\mathfrak{L}_2$PI observer based controller for state, actuator and sensor faults estimation was proposed. It was considered that the quasi-LPV system was affected by external disturbance in the input of the system. The used strategy was based on the $\mathfrak{L}_2$ performance criteria to be robust against disturbance and faults. Furthermore, it was demonstrated that the proposed approach is suitable to estimate system states and actuator and sensors faults by a quasi-LPV Proportional-Integral observer based controller and stabilize the states and controller into equilibrium point. Finally, a numerical example was presented to show the effectiveness and applicability of the proposed approach.

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