Sparse Decomposition Method for Image Denoising

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Abstract —The sparse representation of data has been made an evolution on the part of the signal processing community. In this work, we will present a different notions related to sparseness, and explain the integration and contribution of the image denoising application.

The general goal of sparse representations of data is to find the best approximation of a target signal using a linear combination of a few elementary signals from a fixed collection. For this fact, several methods of the decomposition of the signal have been used like PCA (Principal Component Analysis), ICA (Independent components Analysis), Duet method, MP (Matching pursuit), Least Angle Regression (LARS),.... and some noise reduction schemes are offered: DCT_OMP, DCT_BOMP, Log Gabor_BOMP, DCT_OCMP and Wavelet_OMP

Mots-clés— Sparse representations, Denoising, Matching pursuit (MP), OMP, LARS.

I. Introduction

The approach for image denoising used in this work is based on sparse redundant representations, and it is compared to trained dictionaries. Several algorithms are used to build this type of dictionaries. Among them, the K-SVD algorithm is used to obtain a dictionary that can effectively describe the image. In addition, some greedy algorithms are used to perform sparse-coding of the signal.

Since the K-SVD is limited in handling small image fixes, we are expanding its deployment to arbitrary image sizes by defining a global front image that forces sparse fixes at each location in the image. We show how these methods lead to a simple and efficient denoising algorithm. This leads to a denoising performance equivalent to and sometimes superior to the most recent alternative denoising methods.

The first problems are divided according to the type of imagery, indeed, biomedical images are generally volumetric images (3D) and sometimes have an additional temporal dimension (4D) and / or several channels (4-5D) (for example, multiple sequence MRI images). The variation of biomedical images is very different from that of a natural image (for example a photograph), because clinical protocols aim to stratify the way in which an image is acquired (for example, a patient is lying on his back, head is not tilted, etc.) In their analysis, we aim to detect subtle differences (i.e. a small region indicating an abnormal result).

II. Formulation

The general goal of sparse representation is to find an approximate representation of a target signal using a linear combination of a few elementary signals from a fixed collection. There are several sparse decomposition algorithms used to solve this type of problem in practice.

The problem is to find the exact decomposition which minimizes the number of non-zero coefficients:

$$\min_x \|x\|_0 \quad under\ the\ contraint\ y = Dx \quad (1.1)$$

$x \in \mathbb{R}$, and K is the sparse representation of y. And $\|x\|_0$ the norm l_0 of x and corresponds to the number of non-zero values of x. The **dictionary** D is made up of K columns dk, $k = 1, \ldots, K$, called atoms, each **atom is** supposed to be normalized.

In theory, there are an infinity of solutions to the problem, and the goal is to find the sparsest solution possible, that is to say the one with the lowest number of non-zero values in x.

In practice, we seek an approximation of the signal and the problem becomes (1.1):

$$\min_x ||\, y - Dx \,||^2 \quad (1.2)$$

, under the constraint:

$$||\, x \,||_2 \leq L$$

with L > 0 the constraint of sparseness, that is to say an integer representing the maximum number of non-zero values in x .

II. SPARSE DECOMPOSITION ALGORITHMS

Many approximation techniques have been proposed for this task. We have used the following algorithms:

2.1 Matching pursuit (MP),
2.2 The Orthogonal Matching Pursuit (OMP) algorithm,
2.3 The LASSO algorithm,
2.4 The LARS (least angle regression) algorithm,

In order to find the approximate solutions:

2.1 Matching Pursuit (MP) :

$$\min_{\alpha \in \mathbb{R}^m} \frac{1}{2} || x - D\alpha ||_2^2 \quad s.t. \quad ||\alpha||_0 \leq L$$

1. Initialization: $\alpha = 0$; residual $r = x$
2. while $||\alpha||_0 \leq L$
3. Select the element with maximum correlation with the residual

$$\hat{\imath} = \arg\max_{i=1,\dots,m} |d_i^T r|$$

4. Update the coefficients and residual

$$\alpha_{\hat{\imath}} = \alpha_i + d_i^T r$$
$$r = r - (d_i^T r)\, d_i$$

5. End while.

2.2 Orthogonal Matching Pursuit:

$$\min_{\alpha \in \mathbb{R}^m} \frac{1}{2} || x - D\alpha ||_2^2 \quad s.t. \quad ||\alpha||_0 \leq L$$

1. Initialization: $\alpha = 0$ residual $r = x$ active set $\Omega = \emptyset$
2. While $||\alpha||_0 \leq L$
3. Select the element with maximum correlation with the residual

$$\hat{\imath} = \arg\max_{i=1,\dots,m} |d_i^T r|$$

4. Update the active set, coefficients and residual

$$\Omega = \Omega \cup \hat{\imath}$$
$$\alpha_\Omega = (d_\Omega^T d_\Omega)^{-1} d_\Omega^T r$$
$$r = x - d_\Omega\, \alpha_\Omega$$

5. End while

2.3 The LASSO algorithm

Another fundamental approach to sparse approximation replaces the combinatorial function $l_0$in the mathematical programs of subsection 1 by the standard l_1, which gives convex optimization problems admitting exploitable algorithms. the norm l_1 is the convex function closest to the function l_0, this relaxation is therefore very natural.

2.4 LARS (least angle regression) algorithm

A fast algorithm called near-angle regression (LARS) can make a small modification to solve the LASSO problem and its computational complexity is very close to that of other methods. However, LARS also only allows you to choose one atom in the atom selection process, which strongly encourages us to select more atoms in each iteration to speed up the algorithm convergence.

III. DICTIONARY LEARNING

The quality of a sparse representation of a signal depending on the space in which it is represented, learning the dictionary is a key point to make atoms as efficient as possible for a particular type of data. It has been shown that a learned dictionary has the power to provide better reconstruction quality than a predefined dictionary. This section addresses the problem of dictionary learning. Several algorithms are presented: learning dictionaries without constraint, dictionaries themselves sparseness, or dictionaries with a constraint of non-negativity. Finally, the case of structured dictionaries is dealt with.

For dictionary learning we chose the K-SVD algorithm.

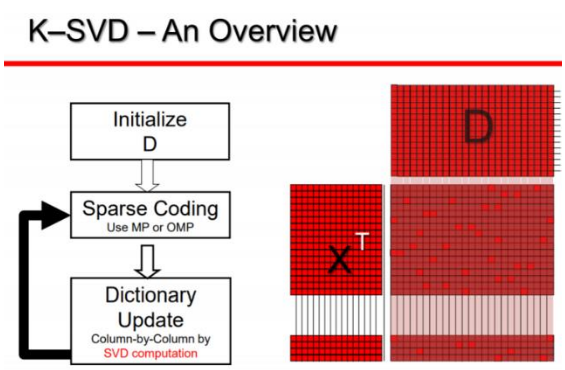

Figure 1 - Principle of the K-SVD algorithm

IV. SIMULATION

Image denoising is a classic problem and has been studied for a long time. However, this remains a difficult and open task. Mathematically, the nature of image denoising is an inverse problem, meaning that its solution is not unique, and additional hypotheses must be formulated to obtain a practical solution. As the ideal hypothesis is difficult to find for all images, a lot of research is being carried out and various techniques are developed to promote denoising performance.

In this paper, we will give the mathematical formulation of the image denoising problem. We aim to look for approximations which gives the most approximate representation possible.

The aim of our work is to find and compare the method which gives the best approximation. We choose several images for the tests, for the first test, we used the OMP algorithm for an image by fixing the number of atoms, and changing the pixel number values, and for the second test, we used the OMP algorithm for the same image by setting the pixel number and changing the atom number values.

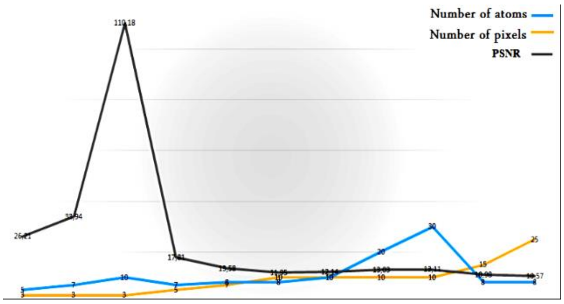

Figure 2 – Comparison at PSNR level

We also notice that, if the number of atoms increases, the calculation time also increases, and the same thing happens for the second method.

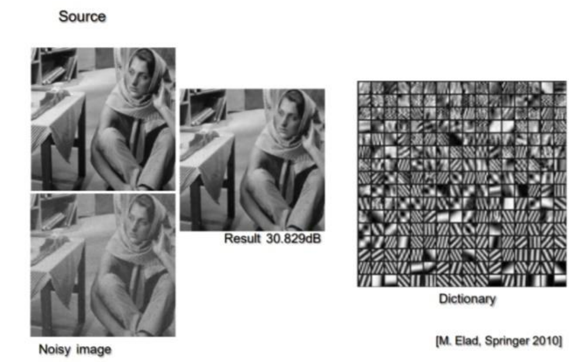

Figure 3 - The principle of dictionary learning

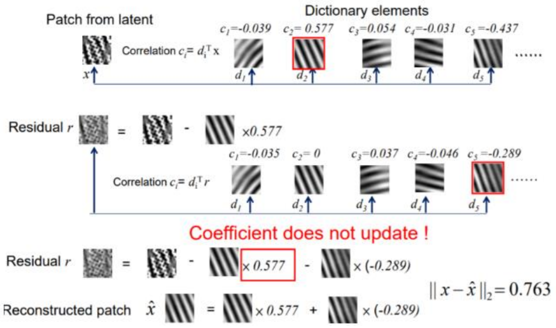

Figure 4 – Dictionary elements

For our application, we have chosen different tests for learning the dictionary for the same image: Number of pixels 20, Number of atoms 2, We find:

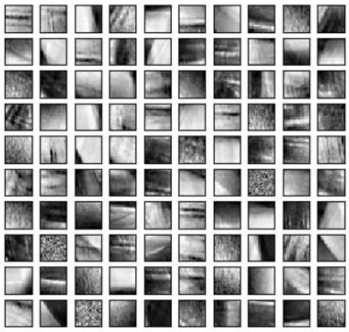

Figure 5- Example of dictionary

Comparison between OMP and LARS

From the tests it is clearly noted that LARS is very effective than OMP in terms of the efficiency of the results and in terms of PSNR but the drawback is that the computation time is very slow compared to the first method, these diagrams clearly show the difference between two methods at PSNR level and the calculation time:

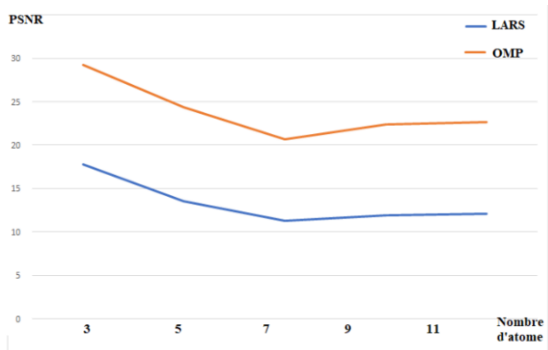

Figure 6 - Number of atoms

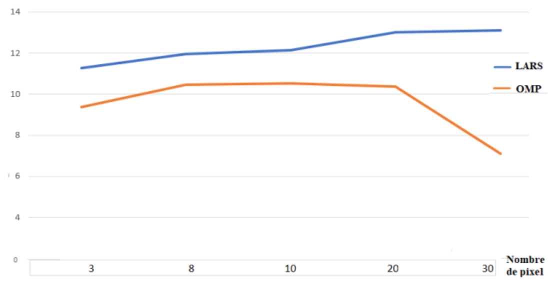

Figure 7 -Pixel number level

V. CONCLUSION

The KSVD is a quick approximation tool for updating the dictionary, which deepens the dictionary learning algorithm. The experimental results demonstrate the superior performance of the proposed method in terms of training in a better dictionary and reducing computer complexity. This learning algorithm is therefore perfectly suited to certain signal processing applications.

Finally, there are several methods for image noise reduction by sparse decomposition, but OMP and LAR remain the most efficient and KSVD is the best approximation for dictionaries.

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