# Designing and Performance Optimization of Dual Bell Nozzle

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## ABSTRACT

In order to produce rocket engines with high thrust and efficiency, dual-bell nozzle design and performance optimization are essential. Using computational fluid dynamics (CFD) simulations and analytical modelling, this paper gives a thorough investigation of the design and optimization of a dual-bell nozzle. The key considerations are the geometric optimization of the nozzle contour and the performance optimization of the expansion ratio for both the primary and secondary nozzle. Analyses are also considered to be done on how the nozzle performs under different conditions of altitude, pressure, and temperature.

Dual-bell nozzles are widely used in rocket engines, particularly for high-performance liquid rocket engines. The design of a dual-bell nozzle involves the careful selection of the contour shape and expansion ratio for both the primary and secondary nozzle. The primary nozzle is designed to operate at a high pressure and temperature, while the secondary nozzle is designed to operate at a lower pressure and temperature. The combination of the two nozzles provides a significant increase in thrust and efficiency, particularly at high altitudes where the ambient pressure is low.

The optimization of a dual-bell nozzle involves the use of analytical models and CFD simulations to determine the optimal geometry and expansion ratio for each nozzle. The geometric optimization is focused on achieving a smooth transition from the primary to the secondary nozzle, while maintaining the desired expansion ratios for both nozzles. The performance optimization involves the selection of the optimal expansion ratio for both nozzles to achieve the maximum thrust and efficiency. This paper provides valuable insights into the design and optimization of dual-bell nozzles and can aid in the development of more efficient and cost-effective rocket engines.

# INTRODUCTION

Rocket engines used in the first stage of space launchers operate from sea level to near vacuum. The nozzle's area ratio is constrained by the need to avoid uncontrolled separation and harmful side loads during lift-off. This constraint has a substantial impact when high altitudes are reached, the engine's performance suffers.

The dual-bell nozzle is an advanced rocket engine nozzle concept with an altitude-adapted capability. This nozzle concept is promising for application to existing and nearterm launch vehicles with a simple modification to the current bell-type nozzle configuration. As shown in Fig., the dual-bell nozzle has a base nozzle and an extension nozzle connected at the wall inflection where a forced, steady and symmetrical separation takes place at low altitudes. The wall inflection point, therefore, acts as an effective nozzle exit with a small nozzle area ratio at low altitudes. At high altitudes, nozzle flow is attached to the wall through the extension nozzle exit, thereby providing the use of the full area ratio. When applied to booster engines to be ignited on the ground, this nozzle is expected to provide a significant overall propulsion and flight performance gains over the conventional bell-type nozzle. This is due to its capability to adapt the nozzle exhaust flow to ambient pressure at low and high altitudes during the ascent flight. In our previous studies, we analytically evaluated the propulsion and flight performance gains of the dual-bell nozzles with taking account of weight penalties of dual bell nozzles. In that analysis, theoretical-optimum performance gains are obtained by utilizing the flow separation model in the nozzle proposed by Romine.4 In this model, the flow separation inside the nozzle is controlled by the nozzle flow adjustment to ambient pressure, not by the wall boundary layer. Using this model, we calculated the separation pressure and location through a triple-point shock altitude where the separation criterion is cleared, the separation point must jump to the extension nozzle exit within a sufficiently short period of time to avoid unfavorable side loads and/or vibration hazards. Also, the separation conditions have to be uniquely determined for the safe operation of this nozzle. To resolve the above issues on the dual-bell nozzle, we analyzed the dual-bell nozzle flow with a Navier Stokes CFD code applied to laminar axis-symmetric nozzle flows. Especially, we focused on the effect of the deflection angle at the inflection point on the dual-bell nozzle flow. According to our CFD analysis, it was found that the deflection angle should be larger than the angle evaluated using a simple Prandtl-Meyer expansion theory and the time to complete the separation point transition was estimated to be less than 0.1 [s]. Additionally, the theoretical separation criteria to be applied to the dual-bell nozzle were also assessed. In the present study, we tried to narrow down the overexpansion level at the wall inflection through our CFD analysis. Additionally, the effect of the nozzle length on the separation point transition was also investigated.

## Literature Survey

| S.N. | Tile of the Paper | Author Name | Objective |
|---|---|---|---|
| 1. | ANALYSIS OF DUAL BELL ROCKET NOZZLE USING COMPUTATIONAL FLUID DYNAMICS | 1. Balaji Krushna<br>2. Srinivasa Rao<br>3. B. Balakrishna | In the present paper one of such nozzle is selected and studied using computational fluid dynamics (CFD) and the results are synthesized for bench marking the general approach to study the Dual Bell nozzles. The result shows the variation in the Mach number, pressure, temperature distribution and turbulence intensity. |
| 2. | Design and Performances of the Dual-Bell Nozzle | 1. Hamitouche Toufik,<br>2. Sellam Mohamed<br>3. Kbab Hakim | To compare obtained results to those of studies that have been done by the National Office of Studies and Aerospace Research (ONERA-France) and the National Space Research Center (CNRS). |
| 3. | Study of expansion ratio on dual bell nozzle of LOX-RP1 engine for replacing the existing bell nozzle to dual bell nozzle | 1.Ashis Kumar Samanta<br>2.Santhosh K S<br>3.Khalid Rashid<br>4.Jayashree | To have an optimum thrust the second bell expansion ratio has been increased. Also, it is to be noted that transition behavior, side loads and cooling methods are to be studied and hot and cold flow tests are to be conducted for the DBN configuration before finalizing the design. |
| 4. | Investigation of Flow Characteristics inside a Dual Bell Nozzle with and without Film Cooling | 1.Mayank Verma<br>2.Nitish Arya<br>3.Ashoke De | In this study, we are going perform a two-dimensional axisymmetric simulation to assess the flow characteristics and understand the film cooling process in a dual bell nozzle. The secondary stream with low temperature is injected at three different axial locations on the nozzle wall, and the simulations are carried out to emphasize the impact of injection location (secondary flow) on film cooling of the dual bell nozzle. |
| 5. | Experimental Study of Dual Bell Nozzles | 1. C. Nürnberger-Génin<br>2. R. Stark | The cold tests on sub-scale nozzles have shown good stability between the two operating modes through a hysteresis effect on the transition pressure ratio from up to 20%. The transition duration was measured with two methods: pressure measurements and Schlieren optics and is in order of a few milliseconds. |
| 6. | EXPERIMENTAL STUDY OF GAS-FLOW SEPARATION IN OVEREXPANDED | 1.CHARLES R. FOSTER<br>2.FREDERICK B. COWLES | A system of equations was developed that could be solved for the area ratio of separation as a function of pressure ratio for various parametric values of the wedge angle 8 using one-dimensional supersonic nozzle theory and one-dimensional |

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| | EXHAUST NOZZLES FOR ROCKET MOTORS | | oblique shock theory (applicable to the local region near the wall). All of the experimental data would be included if a constant wedge angle of 183° was assumed. |
| 7. | DUAL-BELL ALTITUDE COMPENSATING NOZZLES | 1.M. Horn<br><br>2.S. Fisher | While there are some inefficiencies in the dual bell nozzle idea, there are still clear performance advantages to adopting a dual bell nozzle for specific mission applications. While other altitude compensating nozzle ideas have comparable benefits, they often suffer from mechanical complexity, cooling problems, and, ultimately, excessive weight and cost. The dual bell nozzle provides a one-of-a-kind combination of performance, simplicity, light weight, and ease of cooling, and thus merits further investigation. |
| 8. | Characteristics Analysis of Dual Bell Nozzle using Computational Fluid Dynamics | 1. Yeasir Mohammad Akib<br><br>2. Asif Kabir<br><br>3. Mahdi Hasan | This paper represents the design of a dual-bell nozzle profile and the study of the fluid parameters behaviour like Mach number, pressure, temperature, and velocity vectors under under-expanded conditions. Then we compared our result with ref only for the Mach number contours. Moreover creating a small scaled version of the dual bell nozzle can help us for testing it in real conditions. |
| 9. | Design and analysis of contour bell nozzle and comparison with dual bell nozzle | 1. Sreenath K R1<br>2. Mubarak A K | To see which bell is better that is dual bell nozzle has better overall performance than the single bell-shaped nozzle. Atmospheric pressure restricts the expansion of the exhaust gas at low altitudes so the efficiency is much higher at low altitudes. At low altitudes, a vehicle can save 25-30% more fuel by using a dual bell nozzle. |
| 10. | Experimental Study on Flow Transition in Dual Bell Nozzles | 1. Chloé Nürnberger-Génin<br>2. Ralf Stark | To understand the phenomenology of the flow by the transition from sea-level to high-altitude mode, a series of tests have been made under subscale cold flow conditions. Three nozzles with different geometries have been tested. Two of them were successively shortened and driven under the same conditions for each extension length. |
| 11. | Wall pressure and thrust of a dual bell nozzle in a cold gas facility | 1. P. Reijasse1,<br>2. D. Coponet1,<br>3. J.-M. Luyssen1,<br>4. V. Bar2,<br>5. S. Palerm2,<br>6. J. Oswald2 | To test the dual-bell nozzle in the ONERA-R2Ch wind tunnel within the CNES PERSEUS program. The wall pressure distributions and the thrust for the two flow regimes have been characterized in the nozzle pressure ratio (NPR) range from 51 up to 597. |

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| 12. | Ariane 5 Performance Optimization Using Dual-Bell Nozzle Extension | 1. Ralf Stark<br>2. Chloé Génin<br>3. Dirk Schneider<br>4. Christian From | To evaluate the impact of dual-bell nozzles on the payload mass delivered into geostationary transfer orbit by Ariane 5 Evolution Cryo-technique Type A (ECA), detailed studies were to be conducted. |
| 13. | Numerical investigation of Dual Bell Nozzle Flow Field | 1. Chlo´e G´enin<br>2. Ralf Stark<br>3. Sebastian Karl<br>4. Dirk Schneider | The simulation of the recirculation area in the separated extension during sea level mode shows the most difficulties. The calculated hysteresis effect, essential to define the flow mode stability, shows to be in good accordance with the experiment. |
| 14. | COLD FLOW TESTING OF DUAL-BELL NOZZLES IN ALTITUDE SIMULATION CHAMBERS | 1. R. Stark<br>2. Ch. Böhm<br>3. O. J. Haidn<br>4. H.Zimmermann | One focus of future dual-bell nozzle tests will be on the influence of total and back pressure fluctuations during transition regime. As big, scaled back pressure fluctuations can only be achieved with surrounding chamber systems it is mandatory to decouple the transition process and the described nozzles impulse decay. |
| 15. | Dual-Bell Nozzle Design | 1. Chloé Génin<br>2. Dirk Schneider<br>3. Ralf Stark | Over a decade of dedicated study, analytically, experimentally, and numerically, on dual-bell nozzle flow behaviour has led to a validated method for contour design and transition prediction. The influence of wake flow has shown to be critical in the flow behaviour and further work will be necessary to ensure a safe and predictable transitional behaviour. |
| 16. | Experimental and numerical study of heat flux in dual bell nozzles | 1. Chlo´e G´enin<br>2. Andre Gernoth<br>3. Ralf Stark | The characteristic contour inflection of a dual bell nozzle is the key to altitude adaption. After the transition, under high altitude conditions, the nozzle flows full, increasing the vacuum thrust. The region of the inflection is of particular interest for the adjustment of the conventional cooling methods of dual bell nozzles. The contour inflection leads to a local increase in the thermal loads. |
| 17. | Effect of Overexpansion on Thrust | John D. McKenney | If the phenomena of jet separation did not occur in a rocket motor nozzle but instead the gases expanded adiabatically all the way to the nozzle outlet, there would theoretically be a loss in thrust. Sub atmospheric pressure exists in a nozzle from the point where gas separation occurs to the nozzle exit. The reduction in thrust upon launch of a sounding rocket is not as severe as the theory predicts when jet separation is ignored because jet separation is observed to occur in severely over yexpanded |

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| | | | nozzles. First efforts to cause separation at a Several attempts to achieve the requisite area ratio have been successful, and forcing the jet to separate at two points may increase a sounding rocket's performance. |
| 18. | Performance of dual Bell Nozzle | 1. Masafumi Miyazawa<br>2. Hirotaka Otsu | The nozzle contour design criterion that will offer dependable nozzle operation without potential side loads and/or vibration threats was found by the CFD calculations. The association between the separation criterion and transition altitude, which affects the performance of the dual-bell nozzle, was also established by the CFD analysis. When we construct the nozzle contour in accordance with the design criterion established here, the dual-bell nozzle has demonstrated to deliver significant launch vehicle flight performance benefits. We have also demonstrated that a radiation-cooling system can be fitted into the extension-nozzle wall using lightweight materials, resulting in the efficient light-weight dual-bell nozzle. |
| 19. | Transitional Behaviour of Dual Bell Nozzles: Contour Optimization | 1.Chloe genin,<br>2.Ralf Stark<br>3.Dirk Schneider | The dual bell nozzle has two modes of operation that allow for altitude adaptation. The transitional behaviour between one operating mode and another is defined by the shape of the nozzle extension. For a quick transition, the wall pressure distribution along the extension contour is especially important. Theoretically, an ideal transition is guaranteed by a contour extension with a constant wall pressure (CP) or a positive pressure gradient (PP). |
| 20. | Experimental and Numerical study of dual bell nozzle flow | 1.M. Ivanov<br>2. I. Lipatov, D.<br>3.Knight, R.<br>4.Stark, M. Frey<br>5.C. Génin, K.<br>6.Quering, O.<br>7.Haidn, P. Reijas | For security reasons, the main stage engine of current launch vehicles using parallel configuration, such as the European launch vehicle Ariane 5, must be ignited on the ground.The nozzle model is in sea level mode when the nozzle pressure ratio (NPR) is between 20 and 24.2.The air is joined at the base nozzle and separated at the contour incision.The separation point starts moving |

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| | | | inside the extension nozzle as the NPR value (24.7) rises.A planar dual bell nozzle model has been put to the test in both hot and cold environments.The calculated separation position is positioned further upstream than was determined by experiment for higher NPR levels. Despite these slight variations from the experiments, a good simulation is produced by the numerical method |
| 21. | Numerical study of heat flux in dual bell nozzle | 1.Ralf Stark<br>2.Andreas Gernoth<br>3.Chloe Gén | The parallel staged European heavy lifter Ariane 5 has two solid boosters that produce the majority of the lift-off thrust, supporting a cryogenic main stage.The flow is attached in the base nozzle and separates at the contour inflection for low values of the NPR.A planar dual bell nozzle has been the subject of an experimental research to determine the thermal flux in the wall.Along the nozzle contour, the local heat flux derived from temperature readings drops.The value of the heat flux obviously increases in both operation modes in the area upstream of the contour inflection. |

## Bell Nozzle Design

A nozzle is constructed using three curves as shown in fig below: an initial, large circle coming from the combustion chamber to the throat, a smaller circle exiting the throat, and a parabola to extend the approximated bell contour to the exit plane.

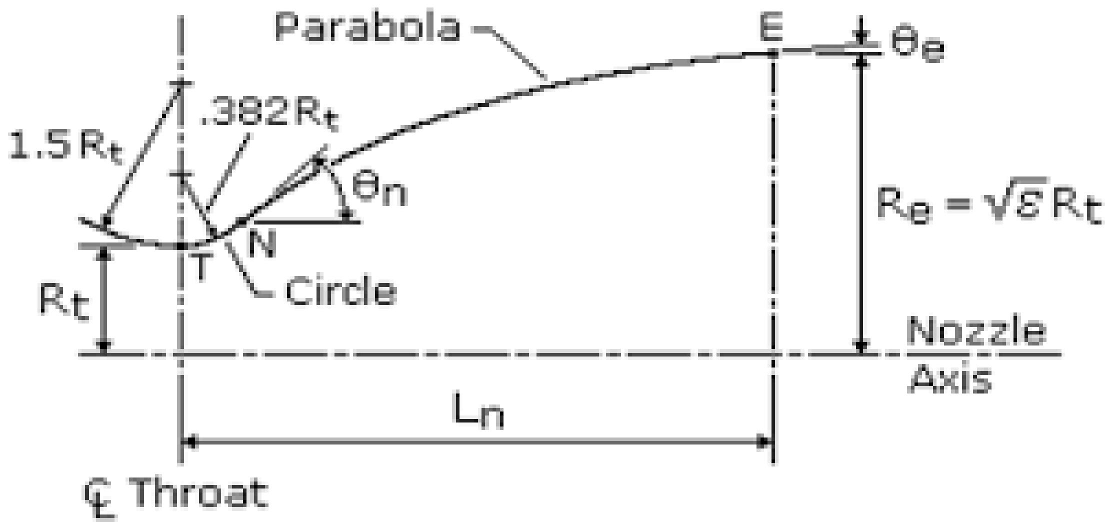


**Fig: Rao method of Nozzle Design**

A nozzle, using Rao coefficients to define the circular curves entering and exiting the throat, equal to 1.5Rt and 0.382Rt, was used as a baseline nozzle for this project. The curves were modeled using MATLAB. The Rao parabolic nozzle is defined by three curves, the length of the nozzle, and the throat radius. The length of the nozzle is determined by

$$L_n = \frac{K(\sqrt{\varepsilon}-1)R_t}{\tan(\theta_e)}$$

where K is a value chosen based on the percent of the length of a conical nozzle with a 15° half angle, the flow deflection angle at the exit, sqrt e, and the throat radius, Rt. In order to define the nozzle further, a coordinate system is defined with the axial (x) axis passing through the line of symmetry and the radial (y) axis going through the center of the throat. The first and second curves define the entrance and exit of the throat of the nozzle, and are based on circular curves. The first curve into the nozzle is determined by the equation:

$$x^2 + (y - (R_t + 1.5R_t))^2 = (1.5R_t)^2$$

which can then be solved for y. Note the curve is defining the bottom half of the circle, and therefore is negative.

$$y = -\sqrt{(1.5R_t)^2 - x^2} + 2.5R_t$$

The second curve begins at the throat where the derivative of both curves is equal to zero. The second curve is also a circle defined by the equation:

$$x^2 + (y - (R_t + 0.382R_t))^2 = (0.382R_t)^2$$

which leads to the equation for the second circle:

$$y = -\sqrt{(0.382R_t)^2 - x^2} + 1.382R_t$$

In order to ensure a smooth transition from the combustion chamber to the throat, there needs to be continuity between the curve defining the combustion chamber and the entrance to the throat. That is, the derivative for both points needs to be equal:

$$\frac{dy}{dx} = \tan(\theta_1) = \frac{x_1}{\sqrt{(1.5R_t)^2 - x_1^2}}$$

where Λ1 is the angle at the start of the curve ( x = -0.0184 m ), and x1 is a function of the throat radius and Λ1. The curve leading from the combustion chamber to the throat curvature begins at x1, which equals:

## Dual Bell Nozzle History

The dual-bell concept was first introduced in literature in 1949 by F. Cowles and C.Foster Horn and Fisher tested four contour combinations to find the extension contour that provided the most favorable flow transition characteristics and high altitude performance when compared to the performance of two baseline contours. In their testing, a 16:1 expansion ratio Rao optimized contour was used as the base contour for each test nozzle. The extension contours that were tested were selected based on the pressure gradients that were produced, since this gradient affects overall performance as well as flow transition characteristics. They tested conical and Rao contours, which both produce a negative pressure gradient, a “constant pressure” contour that produced no pressure gradient, and an overturned contour, which produced a positive pressure gradient. They concluded that a constant pressure contour extension provided the most beneficial combination of flow characteristics over the course of a SSTO flight. However, they also demonstrated that real dual-bell nozzles fall short of the theoretical optimum due to losses sustained from aspiration drag, earlier-than-ideal flow separation, and a non-optimal contour for high altitude flight. Even with these additional losses, Horn and Fisher found that a dual-bell nozzle could provide enough thrust to carry 12.1% more payload than a conventional nozzle of the same area ratio. The Rao’s method of characteristics uses the method of characteristics to design nozzles. A kernel flow, flow in the initial expansion region of the nozzle, is generated with the method of characteristics for a wide variety of flow angles.

Next, the curvature of the throat is defined and a nozzle curve is generated using other given parameters such as the area ratio and the length of the nozzle. The contour is created by picking points on the flow field that result in a smooth, theoretically shock less flow back to the throat. This process is rather complex, and the resulting thrust optimized contour can only be defined by a coordinate list. Rao decided to approximate this contour from the inflection point to the nozzle exit with a parabola.

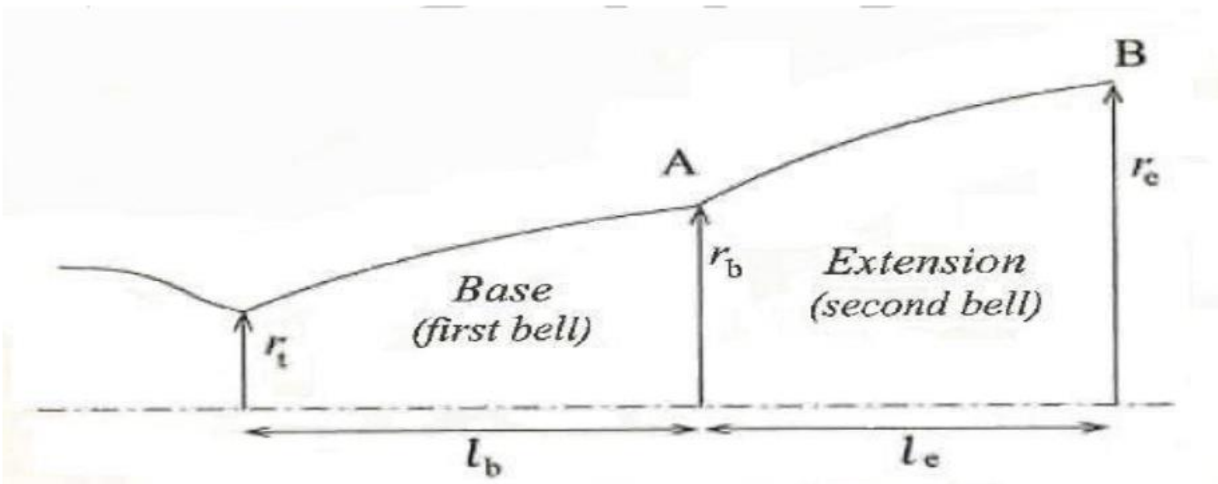


**Fig: Dual Bell Schematic**

### Dual-Bell Nozzle Design

The dual-bell contour design adds a fourth curve to the conventional Rao design by adding a second parabola to connect two Rao that share the same throat area, but are optimized for different altitudes. The second parabola defines the second bell section and connects the two contours thereby achieving a greater expansion ratio. The dual-bell nozzle was defined similarly to the contour of the Rao nozzle, with the same throat entrance and exit parameters as bell nozzle. The parabola coefficients for the first parabola were found using the same method as the Rao contour.

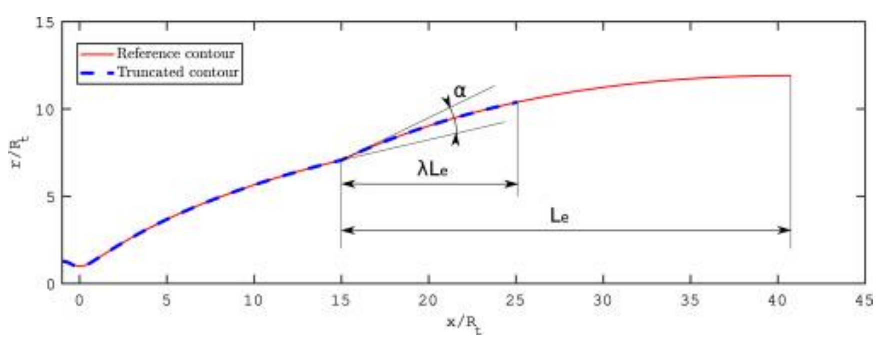


**Fig: Method of Dual Bell Nozzle Design**

## Design of Dual-Bell Nozzle Contour

- The base nozzle length and the total length of dual bell nozzle are assumed to be 80% of a conical nozzle with a half-angle of 15 [deg.].
- The parabola for the base nozzle is determined using the coordinates of nozzle throat and inflection point along with the deflection angle $\theta B$ at the base nozzle exit.
- The parabola for the extension nozzle is determined using the coordinates of inflection point and extension nozzle exit along with the deflection angle $\theta ip$ at the inflection point.
- The deflection angle at the inflection point, $\theta ip$, is determined using the Prandtl-Meyer function.

$$\boldsymbol{\theta ip = \theta B + \alpha\,(\nu E - \nu B)}\ ,$$

where Prandtl-Meyer angles, **$\nu E$** and **$\nu B$**, are calculated using one-dimensional isentropic flow relation as shown below.

$$\boldsymbol{\nu = \sqrt{[(\gamma + 1)/(\gamma - 1)]}\ \tan^{-1} \sqrt{[(\gamma - 1)/(\gamma + 1)\,(M^2 - 1)]} - \tan^{-1} \sqrt{(M^2 - 1)}}$$

**Geometry Parameter**

| | | |
|---|---|---|
| Throat radius | $R_{th}$ | 8 mm |
| Base length | $L_b/R_{th}$ | 16.25 |
| Extension Length | $L_e/R_{th}$ | 15 |
| Area Ratio | $E_b$ | 3.95 |
| | $E_e$ | 7.97 |
| Inflection angle | $\alpha$ | $15^0$ |

**Table: Geometrical Parameter of nozzle**

**From traditional method of design**

A Matlab code was developed to plot the coordinates of the plot required to design the nozzle by inputting the required parameters like length, inlet pressure, exit angle, area ratio, etc.

**Fig: CAD-Model of Dual Bell Nozzle**

## CFD Setup

The governing equations for the present CFD analysis are the full Navier-Stokes equations for laminar symmetric flow. The exhaust gas inside the nozzle is assumed to be perfect gas. The viscosity coefficient and thermal conductivity are calculated using Sutherland's formula.

Although this formula is valid for pure air, the effect of viscosity on the shock-induced flow separation inside the nozzle is expected to be negligible. In fact, we investigated the effect of viscosity on the flow separation with varied viscosity coefficients from 1% to 10000% of the original value. The results indicated no significant difference.

The conditions in the combustion chamber and the exhaust gas are taken from the table. For our CFD code, we selected a technique to obtain a higher order accuracy the governing equations are discretized using the finite volume method and integrated in time by the Euler explicit method. The computational domain is separated into two areas: one is inside the nozzle and the other is outside the nozzle as shown in Figure. Inside the nozzle, the componational grid has 300 points along the nozzle wall and 100 points along the line normal to the nozzle centerline. Outside the nozzle, the computational grid has 150 along the line of symmetry and 130 points along the line normal to the line of symmetry. The computational grid is shown in Fig.4 and 5.

As for boundary conditions, along the nozzle surface, the adiabatic wall is assumed and the inflow condition is imposed on the nozzle throat. The physical properties at the nozzle throat are calculated from the combustion champbeer conditions using the isentropic relations with the Mach number of 1.0. On the axis of symmetry, the symmetric condition is applied. At the end of the computational domain, 0th order extrapolation is used for all physical propretires. In order to determine the pressure where the seeporation point transition takes place, the ambient pressure ranges from 100 [kappa] to 1 [kappa] and the pressure was deermined to an accuracy of 0.01 [kappa].

## Results and Discussion

### Ansys Setup

| | | |
|---|---|---|
| Solver: | Type: | Density Based |
| | Space: | Axisymmetric |
| | Velocity Formulation: | Absolute |
| | Time: | Steady |
| | Energy Equation: | On |
| | Viscous Model: | SST k-omega |
| Fluid Material: | Name: | Air |
| | Density: | Ideal Gas |
| | Viscosity: | Sutherland |
| Operating Conditions: | Pressure: | 0 Pa |
| | Gravity: | Not Checked |
| Pressure Inlet: | Gauge Total Pressure: | 446925 Pa |
| | Supersonic/Initial Pressure: | 446925 Pa |
| | Temperature: | 300 K |
| | Direction Specification Method: | Nomal to Boundary |
| | Prevent Reverse Flow: | Not Checked |
| Pressure Outlet: | Gauge Pressure: | 446998 Pa /NPR |
| | Backflow Total Temperature: | 300K |
| | Backflow Direction Specification Method: | Nomal to Boundary |
| | Prevent Reverse Flow: | Not Checked |
| Solution Methods: | Formulation: | Implicit |
| | Flux Type: | Roe-FDS |
| | Gradient: | Least Squares Cell Based |
| | Flow: | Second Order Upwind |
| | Turbulent Kinetic Energy: | Second Order Upwind |
| | Specific Dissipation Rate: | Second Order Upwind |
| Solution Controls: | Courant Number: | 1 through 5 |
| | URF: | Default |
| Solution Initialization: | Method: | Hybrid |

**Simulation Result of Traditional method design nozzle**

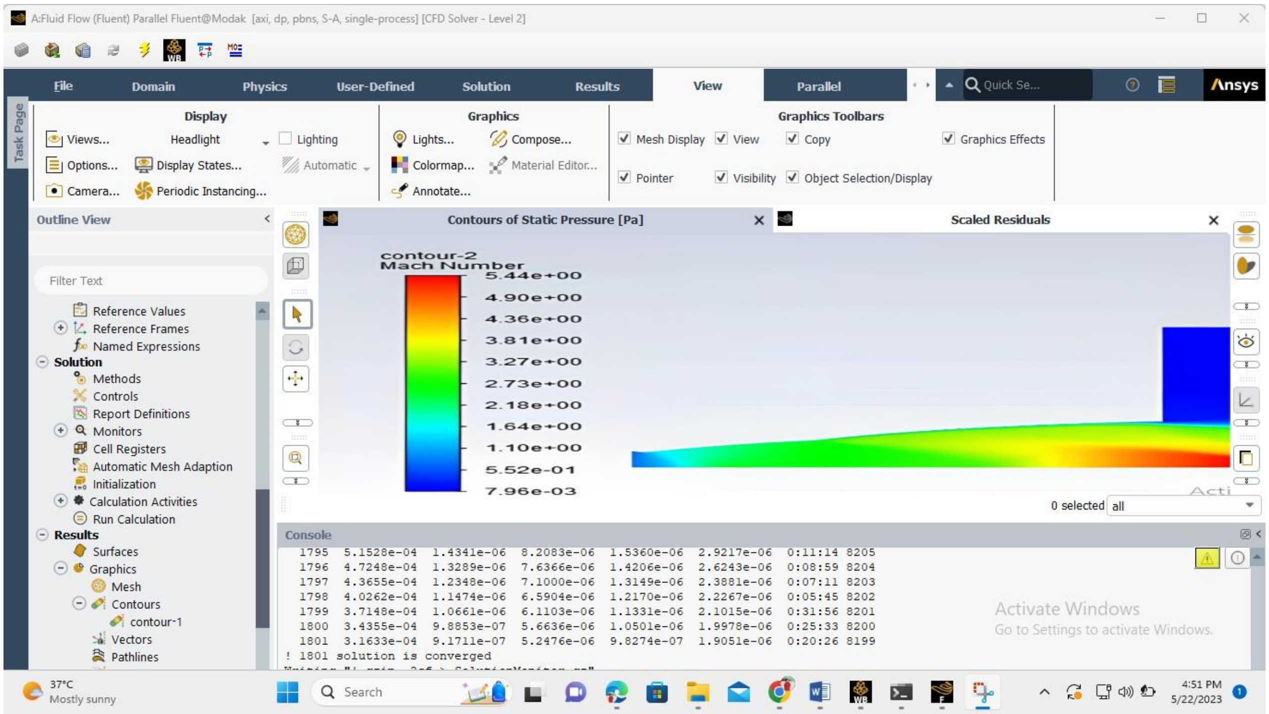


**Fig: Simulation of traditional method of Dual bel nozzle**

Following the traditional method of nozzle design and inputting all the calculated parameters, most of which are already mentioned in the ansys setup table and a few in the geometrical table above, we found the Mach contour to be 5.44, i.e., 5.44 times the speed of sound. As we already know, a dual bell nozzle is an altitude-adaptive nozzle that helps streamline flow by providing a wall for better performance. We use a dual bell nozzle in vacuum, and it is mostly used by launch vehicles in their upper stages.

**Simulation result of Rao Method of Nozzle design**

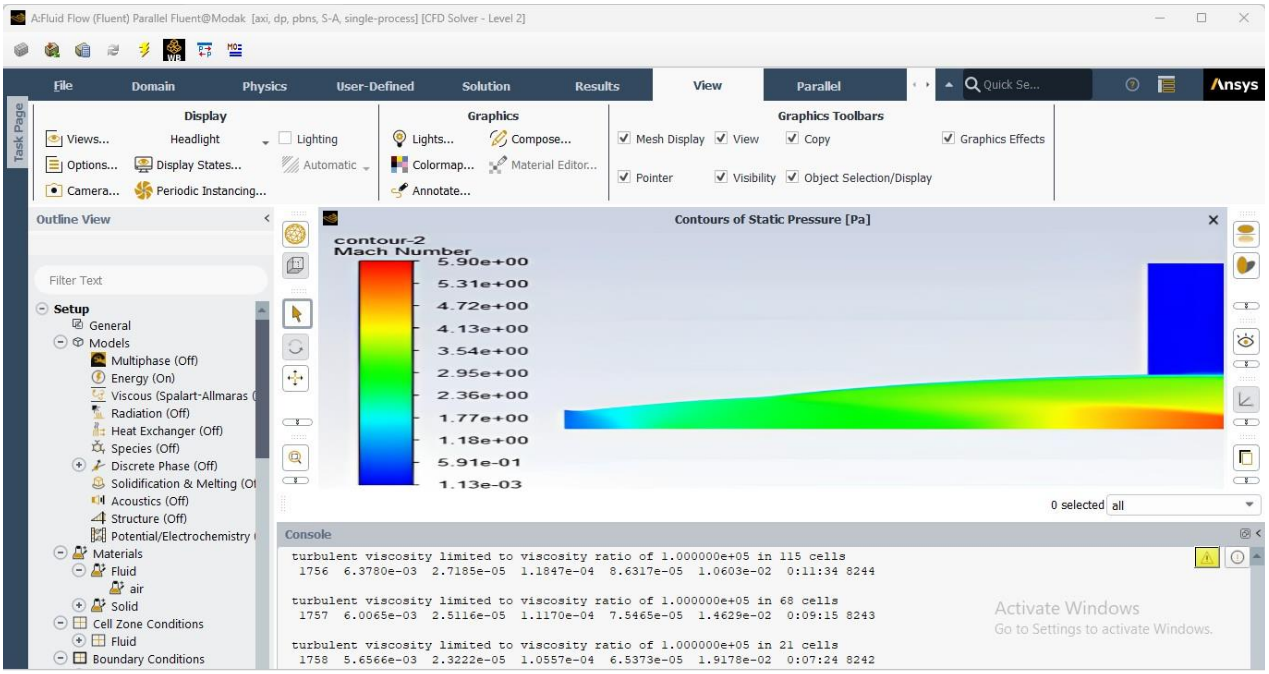


**Fig: Simulation result of Rao Method of Nozzle design**

Rao Method of nozzle design is most efficient method of nozzle design and eliminate the error of traditional method of nozzle design. Following the Rao method of nozzle design and inputting all the calculated parameters, most of which are already mentioned in the ansys setup table and a few in the geometrical table above, we found the Mach contour to be 5.90, i.e., 5.90 times the speed of sound. As we already know, a dual bell nozzle is an altitude-adaptive nozzle that helps streamline flow by providing a wall for better performance. We use a dual bell nozzle in vacuum, and it is mostly used by launch vehicles in their upper stages.

### Performance Optimized Nozzle

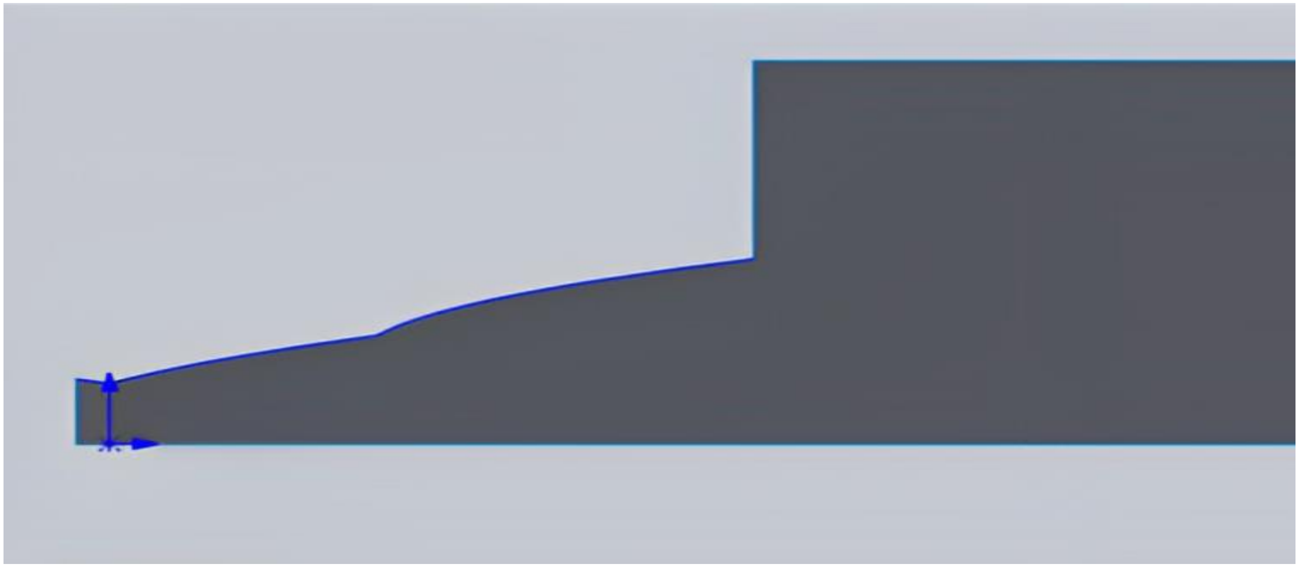

**Fig: CAD-Model of Performance Optimized Nozzle**

Performance optimised nozzles are designed by validating certain expressions and equating them with a standard method. The method of charecterisitic was taken into consideration before substituting any standard parameter. From the standard method of nozzle design we have a length calculation formula which is dependent of of standard parameter and the formula is shown below:

$$L_n = \frac{K\left(\sqrt{\varepsilon}-1\right)R_t}{\tan(\theta_e)}$$

And as we have stated in our abstract that the performance optimisation of dual bell nozzles is based on geometrical optimisation, we deduced a certain scalar function from the length formula that is dependent on the geometry, and deriving such a scalar function will result in changes to the entire geometry. We are getting some phenomenal results, which are further satisfied by our analytical method.

$$L_n = \frac{K(\sqrt{\varepsilon}-1)R_t}{\tan(\theta_e)} \quad - \quad \textbf{Constant}$$

The above mentioned formula was firstly based on our assumption, and then verified using both analytical and numerical method.

**Simulation result of Geometrical Optimized Nozzle**

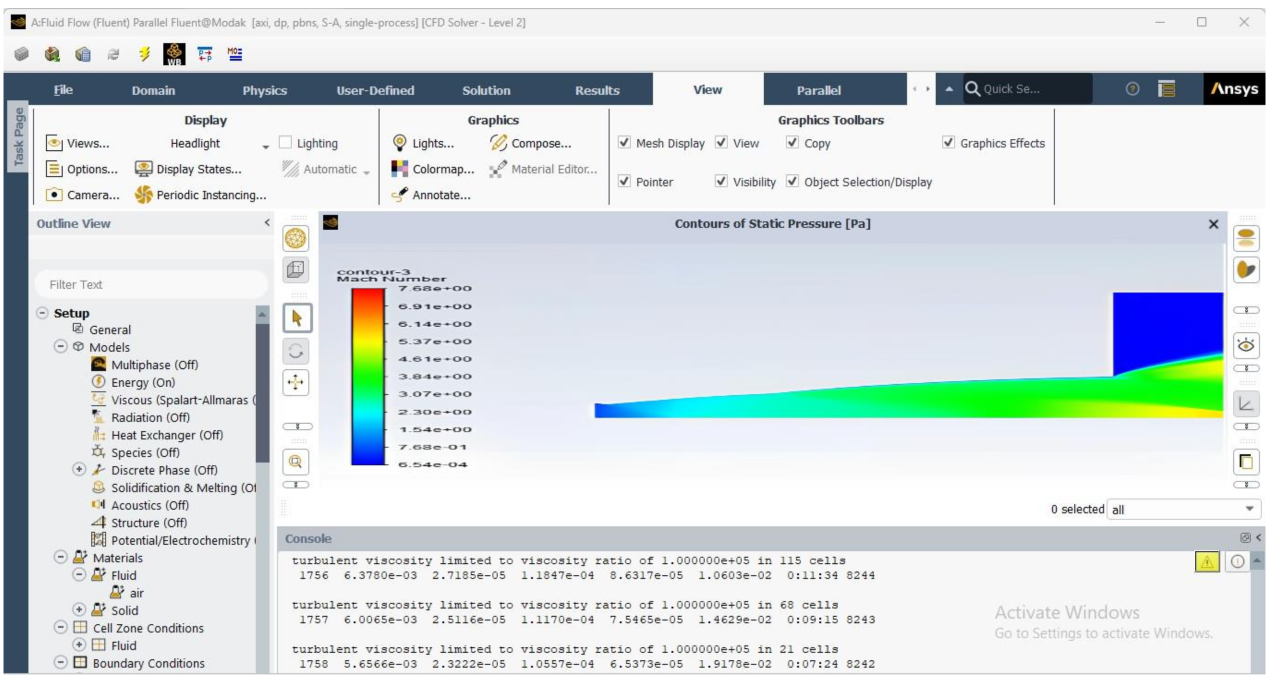


**Fig: Simulation result of Geometrical Optimized Nozzle**

The geometrically optimised nozzle is based on an assumption that is further verified by using analytical and numerical methods. And then inputting all the calculated parameters, most of which are already mentioned in the ansys setup table and a few in the geometrical table above. We found the Mach contour to be 7.68, i.e., 7.68 times the speed of sound.

As we already know, a dual bell nozzle is an altitude-adaptive nozzle that helps streamline flow by providing a wall for better performance. We use a dual bell nozzle in vacuum, and it is mostly used by launch vehicles in their upper stages.

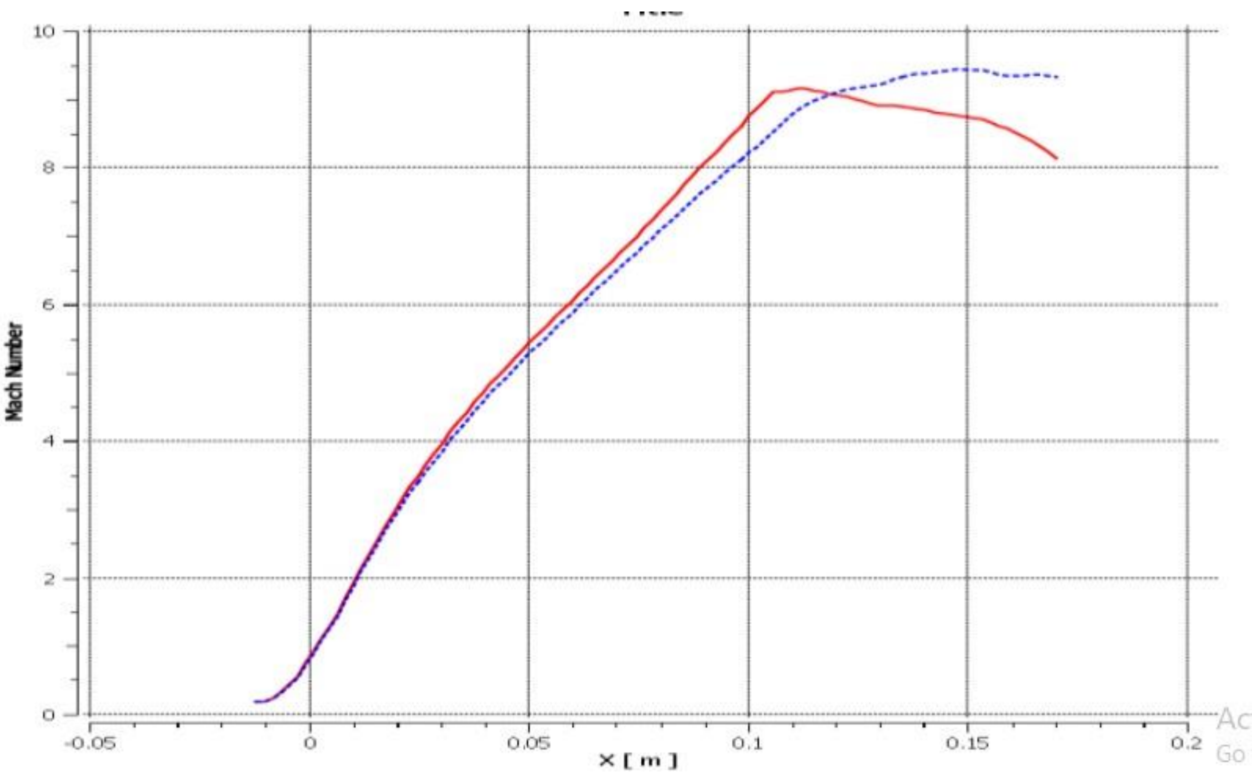


**Fig: Graph plot of Rao method vs Geometrical Optimized**

## Conclusion

We have designed the dual bell nozzle based on standard parameters. The methods that existed to design dual bell nozzles were the traditional method of nozzle design and the Rao method of nozzle design. Rao's method of nozzle design is an efficient way of nozzle design. We followed both methods and simulated their designs. The Mach contour for the traditional method of nozzle design is found to be 5.55, i.e., 5.44 times the speed of sound. Whereas for the same operating condition and even the same environmental parameter from the Rao method of nozzle design, we got the Mach contour of 5.90, i.e., 5.90 times the speed of sound. Later on, as per the statement mentioned in our abstract, the key considerations are the geometric optimisation of the nozzle contour and the performance optimisation of the expansion ratio for both the primary and secondary nozzles. We deduced the scalar function from the standard length calculation formula of the nozzle. The scalar function is dependent on both the area ratio and the exit angle. After designing the geometrically optimised nozzle, we simulated the Mach contour of the nozzle to be 7.68, i.e., 7.68 times the speed of sound, which is 1.78 times higher than the Mach contour of the most efficient nozzle design method, i.e., the Rao method of nozzle design.

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