The Students' Use of Visual Representation for Stimulating Their Metacognitive Strategies: Teachers' Perspectives

Amal Kadan-Tabaja

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Amal Kadan-Tabaja

Haifa University, Faculty of Education, Israel; math33444@gmail.com

Visual representation is effective in enhancing mathematical learning and thinking processes. This study focuses on visual representations automatically provided by a formative assessment platform to describe students' mathematical strategies. We examined the teachers' perspectives on how the students' metacognitive strategies could be stimulated by visual representation of strategy (VRS) provided by a formative assessment platform in assignments for comparing fractions. Twenty-five teachers participated in this study. Based on different data resources, we were able to identify three categories where the teachers considered the VRS as a tool for stimulating students' use of metacognitive strategies: as part of class management, as part of task requirements, and as part of feedback information.

Keywords: Example-eliciting task, strategy, fraction, visual representation, metacognition.

Introduction and theoretical background

Mathematical strategies refer to the methods used by students to solve problems with mathematical content, whether their answers are correct or not (Hegedus & Otálora, 2022). Researchers have reported that students use different strategies to successfully compare fractions, which serve as the mathematical content of this study. These strategies were included in the mathematical curriculum and taught in the classroom. The common strategies are using the same numerator, the same denominator, the benchmark of one whole or one half, the distance of each fraction from one whole, and finding equivalent fractions by expansion or reduction algorithms to get a common denominator or numerator. Ellis et al. (2019) argued that investigating mathematical features and properties of sets of examples may shed light on the students' strategies and thinking. In this study, we used STEP (Seeing the entire picture) as a formative assessment platform. STEP enables primarily: (a) example-eliciting tasks (EETs)—a task that includes an interactive diagram and asks students to generate examples by performing dragging under given constraints (Yerushalmy & Olsher, 2020); (b) assessing students' mathematical strategies based on automated analysis of the mathematical features of the students' examples; and (c) reflecting these strategies automatically and visually, which we call "visual representation of strategy" (VRS) (Kadan-Tabaja & Yerushalmy, 2023). Research shows that the technological platforms that provide an immediate picture of students' work may support the work of teachers and allow for better representation of the mathematical content and more effective student learning (Olsher, Yerushalmy, & Chazan, 2016). Research shows that students dealing with visual representation related to examples they had constructed may be more effective for their learning and further stimulate their thinking process (Robutti, 2010). In this study, we focused on the students' metacognitive strategies as the thinking process that refers to the learners' knowledge, planning, monitoring, and evaluating their strategies for learning and thinking in the cognitive process (Pintrich, 2002).

The novelty of this study lies in integrating the formative assessment platform with the students' answers to an EET, to represent their mathematical strategies automatically and visually. We examined the teachers' perspectives on how the students used the VRS that STEP provided to stimulate their metacognitive strategies. Our research question was: From the teachers' perspective, how can the students' use of VRS within a formative assessment platform stimulate their metacognitive strategies?

Research setting

The interactive diagram of the task in this study was based on a similar representation as that mentioned in the literature (Figure 1) (Arnon et al., 2001), using the STEP platform. Arnon et al. (2001) studied and reported on students who learned with the Shemesh software, which was designed to promote conceptual learning of fractions, offering concrete representations of the fraction and the operations performed on it. Fractions are represented in the discrete Cartesian coordinate system by a point whose vertical coordinate is the numerator and its horizontal coordinate the denominator. All equivalent fractions are represented on a straight ray passing through the origin point. The origin and points on the vertical axis do not represent any fraction. All equivalent fractions are represented on a straight ray passing through the origin (e.g., $\frac{2}{5}=\frac{4}{10}=\frac{6}{15}$). Points that exist on a ray with a larger slope represent larger fractions (e.g., $\frac{6}{6}>\frac{2}{5}$). The red point (corresponding to an X in Figure 1) represents the given fraction (a fixed point), and the green point (corresponding to an empty circle in Figure 1) is the fraction that the student can drag freely to satisfy the requirement of the task.

The task on which this study was based, students were asked to construct a fraction larger than the given fraction $\frac{2}{5}$ represented by the red point by dragging the green point (Figure 2). The task required students to construct 10 examples of fractions that fit the requirement. Examples were submitted separately and captured in STEP. In response to the students' submissions of their examples, STEP visually represented each student's examples on a single screen using blue points and provided automated analysis both to teachers and students. Automated analysis for teachers represented the students' identified strategies of choosing examples both verbally and visually (Kadan-Tabaja & Yerushalmy, 2023). The automated analysis for students included a set of statements describing strategies of comparing fractions. Students were asked to read and activate each statement, then to mark which statement they used to choose their examples. Otherwise, to indicate the method they used to choose their examples. The statements marked by the students were automatically reflected as VRSs (Figure 3).

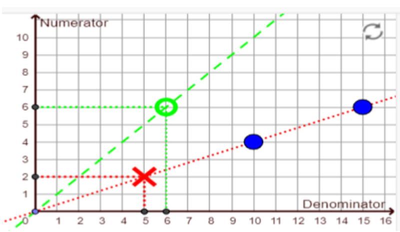


Figure 1: The interactive diagram

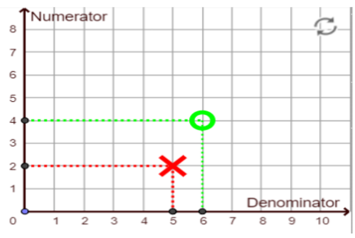


Figure 2: The interactive diagram of the task

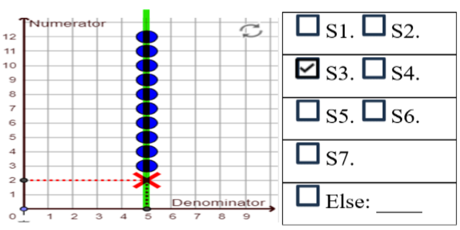
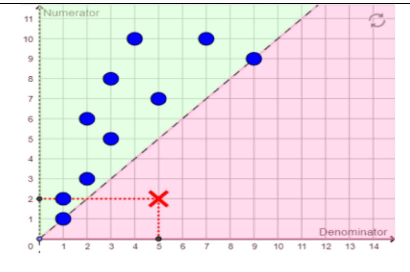
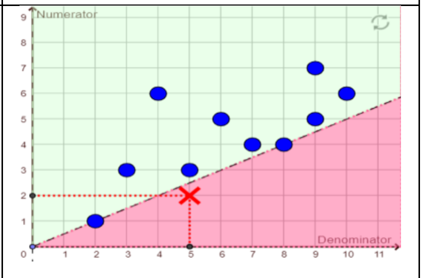
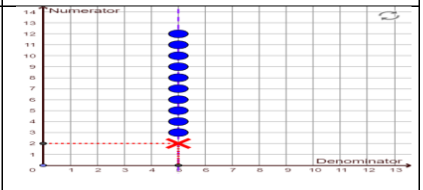
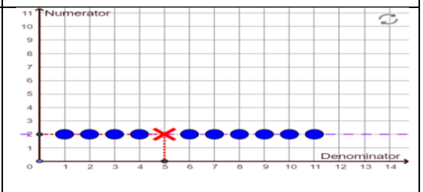
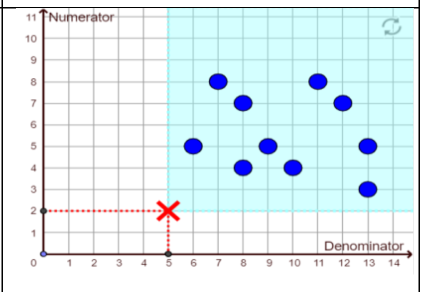
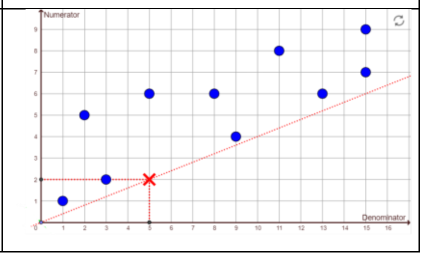
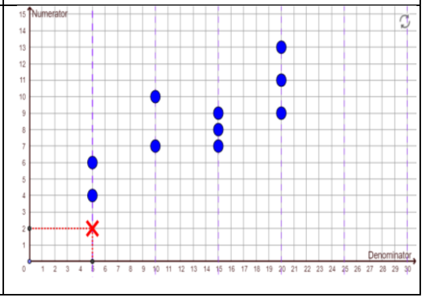


Figure 3: The automated analysis for students

The VRSs are based on automated analysis executed by mathematical algorithms that the researchers set up in STEP when they designed the task. In Table 1, based on Kadan-Tabaja and Yerushalmy (2023), we describe the strategies for comparing fractions (marked in underlined lowercase letters) and the automated VRSs that STEP provides. The teachers were asked to refer to these VRSs when they related to the students' use of the visual representations.

Table 1: Strategies for comparing fractions and the VRSs

The strategies for comparing fractions	VRS
<u>S1. Comparing fractions by using a benchmark of one whole.</u> The VRS takes the shape of a ray that separates between two regions. The green/red regions represent all the fractions that are larger/smaller than one, respectively. The ray between them represents fractions equivalent to one whole. The red fraction is less than one, and the submitted examples are equal to or larger than one. Thus, each blue fraction is larger than the red one.	
<u>S2. Comparing fractions by using a benchmark of one-half.</u> The VRS takes the shape of a ray that separates between two regions. The green/red regions represent all fractions that are larger/smaller than one-half, respectively. The ray between them represents fractions equivalent to one-half. The red fraction is less than one-half, and the submitted examples are equal to or larger than one-half. Thus, each blue fraction is larger than the red one.	
<u>S3. Comparing fractions by using the same denominator.</u> The VRS takes the shape of a ray that is parallel with the vertical axes in X= (the denominator of the red fraction). All submitted examples (blue) have the same denominator as the given red fraction.	
<u>S4. Comparing fractions by using the same numerator.</u> The VRS takes the shape of a ray parallel with the horizontal axes in Y= (the numerator of the red fraction). All submitted examples (blue) have the same numerator as the given red fraction.	
<u>S5. Comparing fractions based on "the numerator and the denominator of the larger fraction are larger than the numerator and the denominator of the smaller fraction, respectively."</u> The VRS takes the shape of a region that represents all fractions that have a larger numerator and denominator than the given red fraction. This is a misconception, and each blue fraction can be larger or smaller than the red one.	
<u>S6. The blue fraction is larger than the red one.</u> This strategy is true when the student's example fulfills the requirements of the tasks. In the visual representation, each blue fraction is above the red ray which represents all the	

fractions that are equivalent to the red fraction, which means that each blue fraction is larger than the red one.	
<u>S7. Comparing fractions based on "the denominator of the larger fraction is a multiple of an integer of the denominator of the smaller fraction."</u> The VRS takes the shape of rays that represent all fractions that are parallel with the vertical axes, X= a × (the denominator of the red fraction), where a is an integer that is not equal to zero. Thus, each blue fraction can be larger than the red one.	

Methodology

Twenty-five elementary and secondary teachers volunteered to participate in the study. The teachers took part in 30 hours of teacher development workshops (10 sessions, 3 hours each) conducted by the first researcher, after which they conducted 4 activities on fractions that contained about 12 tasks. For this study, we focused on one task. Some of the sessions were held online, while others were conducted offline (Table 2).

Table 2: Teacher development workshops

Online/offline sessions	Duration	The process of teachers' works in the session
Online	12 hours	The teachers worked on the tasks in small groups (including the task in our study). They discussed the task requirements, the conceptions or misconceptions, the learning process, the different kinds of feedback, the use of the automated information that the platform enables. Each group documented the discussion in Google Slides and represented it in front of all development workshop participants.
Online	6 hours	The teachers were acquainted with the terms "students' self-regulation," "thinking process," and "metacognitive actions and reflection process."
Offline	12 hours	In the offline sessions of the workshops, each teacher was asked to observe one student engaging with the tasks through the automated VRS that STEP enabled, specifically with the task of this study. Then, the teachers were asked to answer a semi-structured questionnaire containing 10 questions.

Data resources and analysis

To answer the research question, we made use of three resources: (a) video recordings of discussions in small groups (of five teachers each), with the teachers working on the task using the automated VRS in the online session. The transcripts of video recordings were analyzed to extract the statements showing the teachers' perceptions of how their students handle the task and the automated VRSs and how these representations may stimulate the students' metacognitive strategies. (b) While observing the group discussions, the first researcher took field notes. (c) Each teacher responded to a semi-structured questionnaire after observing one student working on the task using the automated VRS. The questionnaire contained ten questions regarding the examples and strategy the student used;

challenges; difficulties faced by the student and the way the student handled them; and the insights that the teachers gained from the engagement with the task and the VRS to stimulate the students' thinking and learning. The analysis of the responses to the questionnaire and repeated reading of the data from the transcripts allowed classifying the teachers' perceptions into categories The final categorization of the data was checked by other researchers for consistency (the questions are listed in Table 3).

Table 3: List of questions in the questionnaire the teachers answered after observing one student working on the task using the automated VRSs

1.	Provide background information about the previous knowledge of the student.
2.	Is there a special reason for choosing this student?
3.	What were the characteristics of the fractions that the student chose?
4.	Which strategy did the student choose while constructing the examples?
5.	Were you able to identify difficulties, common mistakes, or misconceptions while the student was working on the task?
6.	Did the student change his/her choices while constructing his/her examples, and if yes, why?
7.	When the student's responses when he/she was exposed to the VRS?
8.	Do you have any suggestions for changes to the current task or the follow-up tasks? Explain.
9.	What are the insights that you gained from the task and the automated assessment? Explain.
10.	In your opinion, how can the use of the VRSs enhance the student's learning and thinking process?

We used a qualitative approach to analyze the data. In an open coding process, we examined excerpts from the teacher's responses and transcripts to describe the categories that would allow us to learn about how, in the teachers' opinion, the use of the VRS stimulated the students' metacognitive strategies. Based on Schoenfeld's (2013) metacognitive framework, we identified the following phrases and sentences that may reflect the metacognitive strategies (Table 4).

Table 4: Metacognitive action and statement

Metacognitive strategy	**When did it occur?**	**The following statement or action may be an example of a metacognitive strategies**
Planning	Before beginning a task	To understand what makes a correct answer… To set specific strategies before beginning a task… To make it easy… To reread the problem…
Monitoring	During the learning and feedback processes	To check the answers and the strategy while working on the task... To ask questions…
Evaluating	After a learning episode	To summarize the learning or thinking after finishing… To evaluate the conclusion that was reached…

Results

The findings show that the teachers' statements according to which the students' use of the VRS may stimulate their metacognitive strategies can be classified into three categories. Below we describe these categories, giving examples from the teachers' statements, and point out the metacognitive strategies that may be stimulated.

a. The VRS is part of classroom management. This category is related to the use of the VRS when students work on the task individually, in pairs, in small groups, or whole-class discussion. For example, one teacher said in the course of an online session: "I think that working on different VRSs in small groups or pairs may help students rethink their answer and compare it with those of others." Another teacher answered in the questionnaire: "It will be very interesting to have a discussion about the different visual representations and to connect them with the strategies for comparing fractions in the classroom." These two examples show that monitoring is a metacognitive strategy that may be stimulated by using the VRSs.

b. The VRS is part of task requirements (or task design), as when using the automated VRS as part of self-reflection before submitting the examples or when the task requirement is specifically related to the visual representation. One teacher stated: "When the student is asked to reflect on the VRS as part of the task requirements, this may be a clue for the student in the choice of examples." Another teacher said in the online session discussion: "We can use the visual representations in tasks to help students rethink and ask questions about their strategies for comparing fractions." According to the first example, planning is the metacognitive strategy that may be stimulated by using the VRS. The second example suggests that monitoring is the metacognitive strategy that may be stimulated by using the VRSs.

c. The VRS is part of feedback information. This category is related to the use of the automated VRS when it is related to the correctness, characteristics, and strategy chosen, and to misconceptions or common mistakes in their examples. For example, one teacher stated that "when the student was exposed to the VRS, he wondered which VRS he might have received had he used another strategy. So, the feedback visual representation made the student think again about the task and about his examples." Another teacher claimed: "The visual representations helped the student check whether his examples were correct or not; he was also able to check the strategy he used for comparing fractions visually." The first example demonstrates that monitoring is the metacognitive strategy that may be stimulated by using the VRSs. The second example demonstrates that evaluating is the metacognitive strategy that may be stimulated by using the VRSs.

Table 5 shows categorization of the metacognitive strategies described by the teachers.

Table 5. Categorization of the metacognitive strategies described by the teachers

The metacognitive strategy	**The visual representation of the strategy is part of class management**	**The visual representation of the strategy is part of task requirements**	**The visual representation of the strategy is part of feedback information**
Planning		The students' use of the VRS may help them understand what makes a correct answer for the task; it may encourage them to look at a broader range of rich strategies and to compare the information across different strategies; and it may help	

		them visualize the sequence of steps of their answer.	
Monitoring	Clustering the students in pairs or small groups for work on tasks using the VRS may help them present, compare, and discuss their strategies with other students. This may enhance the student's mathematical discourse. The students' use of VRSs in the classroom discussion may help them rethink their answers and see the problem from different perspectives.	The students' use of the VRS as part of the task requirement may help them compare, rethink, and adjust their strategy. It may help expose students to strategies and initiate an inquiry process to comply with the task requirements; it may encourage students to generate new strategies based on reasoning and exploration. It may support students in assuming ownership of their learning process and responsibility for it. It may help students enrich their strategic example space. It may encourage students to think in various modes and understand the concept from multiple perspectives.	The students' use of the VRS as part of the feedback information may lead them to assume responsibility for their learning and become independent thinkers. It may help students rethink their examples and ask questions.
Evaluating	The students' use of the VRS in the classroom discussion may expose them to new strategies or interesting answers, which may be used for clarification of their questions or comments.	The students' use of the VRS may help them assemble the lists of evaluation criteria of different strategies by which to assess their examples. It may encourage students to reconsider their answers before submitting them.	The students' use of the VRS as part of feedback information may help them evaluate the correctness and characteristics of their answers, the strategy they used to choose their examples, and the misconceptions or common mistakes in them.

Discussion and conclusion

In this study, we used Schoenfeld’s (2013) metacognitive framework to describe, from the teachers’ perspectives, how the visual representation of strategy in an automated assessment platform may stimulate the students’ metacognitive strategies. Based on the teachers’ responses, we were able to identify three categories that related to the students’ use of VRSs included in the formative assessment platform and the way each category may stimulate the students' metacognitive strategies—the use of the visual representation when it was part of classroom management, part of task requirements, and part of the feedback information. In each category, teachers stated that monitoring and evaluating actions may stimulate metacognitive strategies. Planning was identified in the students' use of the visual representation when it is part of the task requirements. The findings are consistent with the literature, which reported that visual representation may stimulate the students' mathematical thinking (Boonen, 2016) and play a central role in the formative assessment process, as perceived by the teachers (Kadan-Tabaja, & Yerushalmy, 2023).

From the teachers' perspective, integrating EETs that automatically and visually reflect students' mathematical thinking into the formative assessment platform appears to stimulate students' metacognitive strategies. The results of this study open new possibilities for using such VRSs in the automated assessment processes in the mathematics classroom.

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