New explicit representation for the prime counting function

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Abstract

In this paper, we present a new representation for the prime counting function, it is similar of the Legendre formula for $a = 1.08633,,,$ and the gauss formula for $a = 0$.

We present this new formula :

$$\pi(x) = \frac{1}{\sqrt[x]{x.e^{-a}} - 1}$$

$\pi(x)$: denote the prime counting function

Keywords : Prime number, prime counting function.

Mathematics Subject Classification : 11N05, 11A41, 41A99

1. Introduction

Many people have devoted time trying to create formulae to generate The celebrated prime number theorem or to count primes numbers, something that at times has bordered on obsession.

This paper concerns a new representation for the Legendre formula and the Gauss function for the prime counting function $\pi(x)$.

The prime density function, the gauss conjecture, states that :

$$\pi(x) \sim \frac{x}{\ln(x)} \quad as \;\; x \to \infty \qquad (1)$$

The Legendre's conjecture regarding the $\pi(x)$ states that :

$$\pi(x) = \frac{x}{\ln(x) - a} \qquad (2)$$

$where\ the\ Legendre\ constant\ \ a = 1.08633,,,$

Theorem **:** we will present now the new representation and the proof for the prime counting function

$$\pi(x) = \frac{1}{\sqrt[x]{x.e^{-a}} - 1} \quad as \ \ x \to \infty$$

Proof theorem . Taylor series $of \ \pi(x) \ as \ \ x \to \infty$

$$\pi(x) = \frac{x}{\ln(x) - a} - \frac{1}{2} + \frac{\ln(x) - a}{12x} + O\left(\frac{1}{x^2}\right)$$

We have

$$\lim_{x\to\infty} \frac{\ln(x) - a}{12x} = 0$$

So that

$$\pi(x) = \frac{x}{\ln(x) - a} - \frac{1}{2} \quad as \ \ x \to \infty$$

or, equivalently

$$\frac{1}{\sqrt[x]{x.e^{-a}} - 1} = \frac{x}{\ln(x) - a} - \frac{1}{2} \ as \ \ x \to \infty \qquad (3)$$

The relation (3) give the new representation of the Legendre formula for $a = 1.08633,,,$

2. Main results :

Looking at the expression of (1) and (3) above, for $a = 0$ we obtain a new representation for the Gauss conjecture :

$$\pi(x) \sim \frac{1}{\sqrt[x]{x} - 1} + \frac{1}{2} \quad as \ \ x \to \infty$$

Looking at the expression of (2) and (3) above, for $a = 1.08633,,,$ we obtain a new representation for the legendre formula

$$\pi(x) = \frac{1}{\sqrt[x]{x.e^{-a}} - 1} + \frac{1}{2}$$

This completes the proof

The table shows how the functions $\pi(x)$, $\frac{1}{\sqrt[x]{x.e^{-a}} - 1} + \frac{1}{2}$ and the Legendre formula compart at powers of 10. $(for \ \ a = 1.08633,,,)$

x	$\frac{1}{\sqrt[x]{x.e^{-a}} - 1} + \frac{1}{2}$	$\frac{x}{\ln(x) - a}$	$\pi(x)$
10	8	8	4
10^2	28	28	25
10^3	171	171	168

10^4	1230	1230	1229
10^5	9590	9590	9592
10^6	78 559	78 559	78 498
10^7	665 257	665 257	664 579
10^8	5 768 892	5 768 892	5 761 455
10^9	50 924 441	50 924 441	50 847 534
10^{10}	455 798 466	455 798 466	455 052 511
10^{11}	4 125 054 147	4 125 054 147	4 118 054 813
10^{12}	37 672 316 307	37 672 316 307	37 607 912 018
10^{13}	346 653 178 885	346 653 178 885	346 065 536 839
10^{14}	3 210 287 167 276	3 210 287 167 276	3 204 941 750 802
10^{15}	29 893 179 954 460	29 893 179 954 460	29 844 570 422 669
10^{16}	279 680 917 170 575	279 680 917 170 575	279 238 341 033 925
10^{17}	2 627 594 920 124 090	2 627 594 920 124 090	2 623 557 157 654 233
10^{18}	24 776 883 130 563 108	24 776 883 130 563 108	24 739 954 287 740 860
10^{19}	234 396 314 864 306 897	234 396 314 864 306 897	234 057 667 276 344 607
10^{20}	2 223 933 570 740 069 490	2 223 933 570 740 069 490	2 220 819 602 560 918 840

Table 1

We can see above at table 1 that the formula $\frac{1}{\sqrt[x]{x.e^{-a}}-1}+\frac{1}{2}$ gives exactly the values of Legendre's formula

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