A metrics for ‘Point-Ellipse’ distance evaluation in 2D

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Abstract

This document proposes a method to define the distance between a set of points and an ellipse, which can be used as cost function of a Genetic Algorithm - or any other kind of solvers - to solve the ellipse fitting problem. This method involves a metrics to retrieve the correct polygon edge to be compared with each point of the set, featuring linear time complexity.

Keywords: `Ellipse Fitting, Geometry Codes`

1. Introduction

Finding the best geometrical fit to a set of scattered data is a common problem that finds applications in several fields. Let's use the example of data describing a human trajectory, which can be sampled in order to obtain a set of scattered points: an interesting application would be to retrieve a specific geometrical shape that might resemble what the user has drawn. This particular scenario finds important applications in the rehabilitation field, where describing a patient's trajectory with geometrical information might provide an indication about the performance of a therapy task. Furthermore, other fields may be interested in such a solution, like image processing, pattern recognition, computational metrology, or robot guidance.

This document will describe an efficient method to evaluate the distance between a set of points and an ellipse in the bidimensional space. Such a procedure can be used as a metrics to retrieve the ellipse that best fits with the data. The present paper can be assumed as an appendix and extension of the author's previous work "*Convex polygon fitting in robot-based neurorehabilitation*" [1], which describes in detail the methods to fit convex polygons to a set of scattered points in 2D. Ellipses were not investigated in the previous work since they are the most common shapes evaluated in the literature [2, 3, 4], with methods that involve *quadratic form* [5], *least square fitting* [6], *Bayesian estimator* [7], and *Hough transformation* [8]. Yet, the author hereby proposes a simple and fast method that might be used to obtain a good fitting, which has already been used in other works in the robotic-assisted rehabilitation field [9, 10, 11].

The next sections will address the geometric problem of describing a suitable metrics in the ellipse fitting context, named *Radial metrics*. Detailed descriptions, images and algorithms will be presented, correlated with examples and computational complexity estimation. Note that all the algorithms will show the optimized code; the operations concerning translation and rotation of points will be featured only for the x or y coordinate that will actually be used.

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2. ‘Point-Ellipse’ Distance (Radial Metrics)

With reference to the “Polar metrics” described in [1], which is used to evaluate the ‘point-polygon’ distance, the evaluation on ellipses is less demanding due to the following motivations:

- ellipses are not composed by segments, so no space partitioning is required;
- no special cases are included, such as distance from vertices rather than lines; and
- there is no difference between a point inside the ellipse and a point outside.

The algorithm is then simpler than the Polar metrics, and has been defined as *Radial metrics* since it involves angles and rotational calculations. Note that knowing the details of the Polar metrics implementation is not required for understanding the following sections.

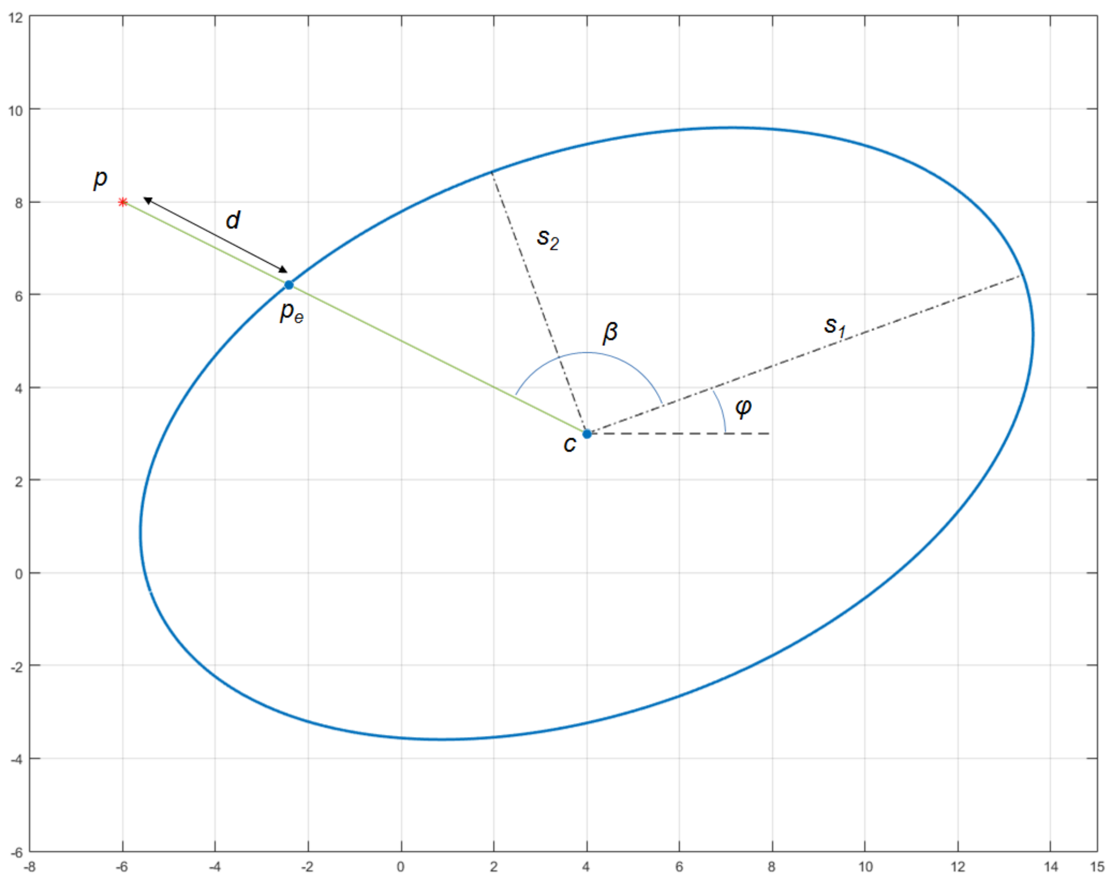


Figure 1: An ellipse defined by its parameters.

2.1. Distance Definition

Considering an ellipse in the current frame, the algorithm takes as parameters:

- the length of the two semi-axes s_1 and s_2;
- the orientation of the shape expressed as the angle φ;
- the point c which defines the center of the ellipse in the space; and
- the point p, whose distance must be calculated;

and provides as result:

- the distance d between p and the ellipse defined by $\langle s_1, s_2, \varphi, c \rangle$.

With reference to Fig. 1, the distance d is defined between p and the point p_e, which is the point where the straight line joining p and c intersects the ellipse outline. This line features an angle β from the ellipse horizontal semi axis; however, retrieving this value might be problematic when φ and the center c are not zero, since the result may vary according to the Cartesian quadrant where p belongs. On the other hand, considering an ellipse whose angle φ is zero and with no displacement from the origin of the current frame - axes coinciding with the current frame - the angle β is simply the angle between the ellipse horizontal axis and the line joining p and c.

Algorithm 1: Distance with Radial

input : The point p to be compared with the ellipse, having x and y coordinates
input : The length of the first semi-axis s_1
input : The length of the second semi-axis s_2
input : The ellipse orientation φ
input : The center of the ellipse c
output: The distance d

1 **begin**
2 $p'.x \leftarrow p.x \cdot \cos(-\varphi) - p.y \cdot \sin(-\varphi)$;
3 $p'.y \leftarrow p.x \cdot \sin(-\varphi) + p.y \cdot \cos(-\varphi)$;
4 $p'' \leftarrow p' - c$;
5 $\beta \leftarrow \mathrm{atan2}(p''.y \cdot s_1, p''.x \cdot s_2)$;
6 $p''_e.x \leftarrow s_1 \cdot \cos(\beta) + c.x$;
7 $p''_e.y \leftarrow s_2 \cdot \sin(\beta) + c.y$;
8 $d \leftarrow \sqrt{(p''_e.x - p''.x)^2 + (p''_e.y - p''.y)^2}$;

Algorithm 1 presents the procedure to retrieve the distance d exploiting rotational and translational computations. By first rotating p of an angle $-\varphi$, and then translating the result in order to reach c, the algorithm retrieves a point p'' expressed in the ellipse reference frame, having an orientation equals to zero. The angle β is than obtained considering the orientation of the straight line joining p'' and c. Note that the function atan2 at line 5 is the arctangent function with two arguments, used to to gather information on the signs of the inputs and return the appropriate quadrant of the computed angle, which is not possible for the single-argument arctangent function. The angle β is used to obtain the point p''_e, which is the point p_e expressed in the new reference system. Finally, d is calculated as the euclidean distance between p'' and p''_e.

2.2. Overall Procedure

Algorithm 2 indicates how to execute the overall function for a set of scattered points.

Algorithm 2: Distance Points-Ellipse

input : The array of points P to be compared with the ellipse, having n points
input : The length of the first semi-axis s_1
input : The length of the second semi-axis s_2
input : The ellipse orientation φ
input : The center of the ellipse c
output: The overall distance value fit, calculated as the mean value of the distances between P and the ellipse

begin
  fit ← 0;
  for i ← 1 **to** n **do**
    fit ← fit + distanceWithRadial(P[i], s_1, s_2, φ, c);
  fit ← fit/n;

Fig. 2 shows a graphical example of distance calculated by the Radial metrics over a set of points. In this image, the red points represent a set of points $p \in P$, the green points show a set of points $p_e \in P_e$, while the red lines, which are drawn between p and the corresponding p_e, show the distance between them.

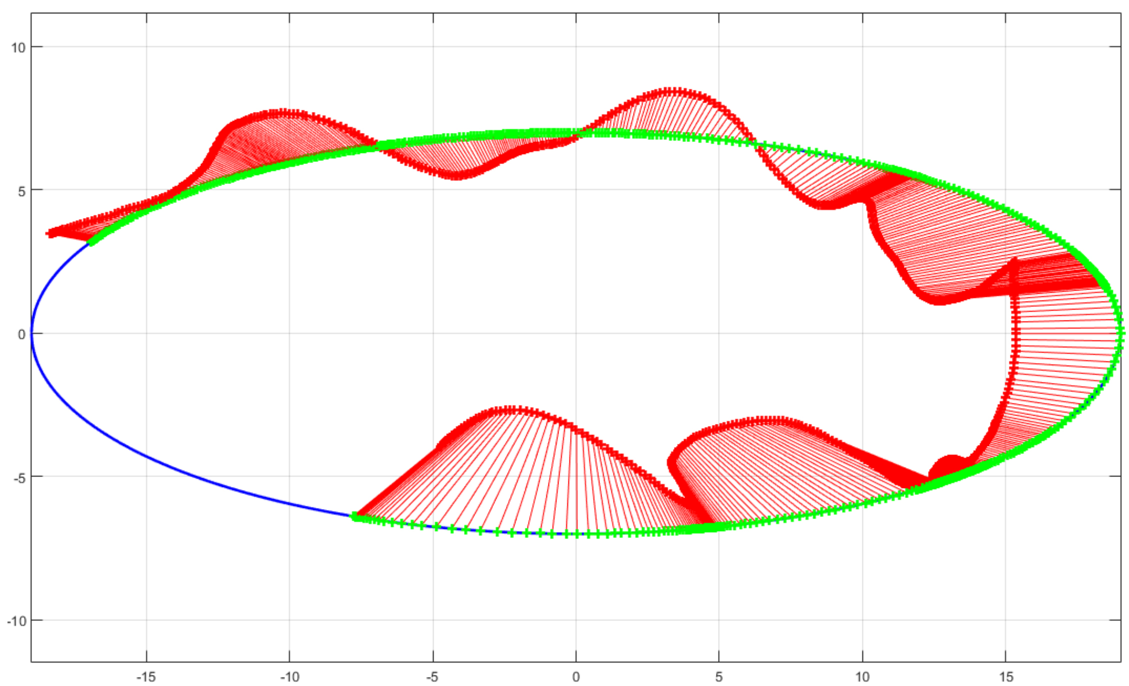


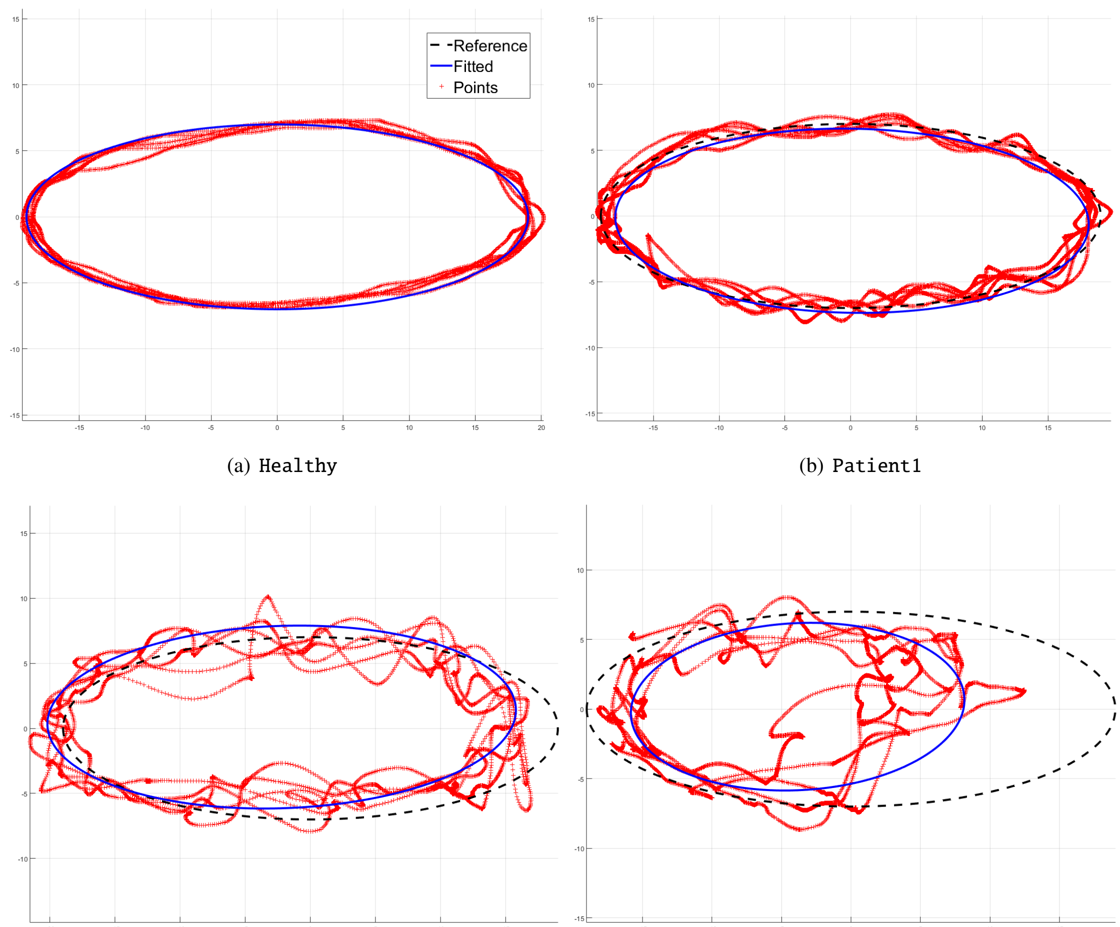
Figure 2: Example of distance calculated with the Radial metrics.

2.3. *Complexity Estimation*

The algorithm for the distance estimation with the Radial metrics features a linear complexity. The entire procedure depends only on a single parameter: the number n of points stored in P. All the calculations are repeated for each point, which results in a time complexity $O(n)$.

3. Results

This section reports the result obtained using the Radial metrics as a fitting function of a Genetic Algorithm in the experiment defined in [1]. The method has been used to gather information about subjects' performance during the execution of a specific task, where they were asked to draw an ellipse based on a



(a) Healthy (b) Patient1

(c) Patient2 (d) Patient3

Figure 3: Shape fitting results on different human trajectories.

given virtual reference (size 19×7 *cm*). The movements of the subjects were recorded using the passive arm-support device Trackhold [12]. The participants were three ischaemic post-stroke patients (males, 62 ± 12 yrs old), and a healthy subject (male, 28 yrs old) with no known neuromuscular disorder affecting his upper limb.

The graphical results are shown in Fig. 3 having:

- the reference shape in the black dotted line;
- the movements of the participants, sampled at a frequency of 60 *Hz* and represented by the red dots; and
- the fitted ellipse in the blue line.

As as shown in the image, the algorithm correctly retrieve the best fit from different degree of performance (given by the participants' degree of impairment).

4. Conclusion

This work describes the "Radial metrics", a simple approach to evaluate the distance between one or more points and an ellipse in the bidimensional space. The metrics is currently used as a perform evaluator for a real-time assistance tuning algorithm in a robotic-based neurorehabilitation scenario; however, it can be used in any other kind of geometric problem involving curve fitting or distance evaluation.

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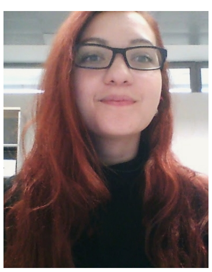

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