Dynamics of a Multibody Vertical Transportation System Under Seismic Excitation

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Abstract

The frequencies of long period seismic excitations fall within the range of the fundamental frequencies of tall structures. This results in large resonance responses of the structures. Their resonance motions affect the modular vertical transportation systems (VTS) operating in tall buildings. The main objective of this work is to develop an improved, computationally efficient dynamic model to predict the effects of resonances in VTS [1].

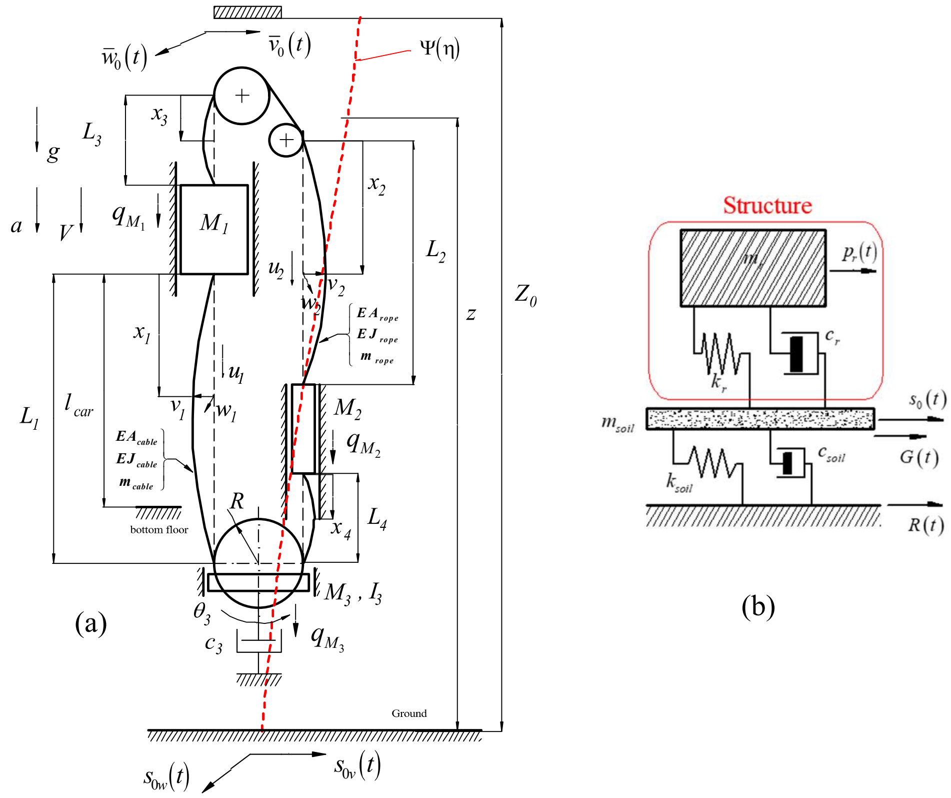

Figure 1: Simplified diagrams of the VTS and of the structure – soil – bedrock model.

Fig. 1(a) shows a model of a VTS installed in a cantilever vertical host structure subjected to seismic ground motions s_{0v} and s_{0w}. The structure is subject to the fundamental bending mode resonance conditions. The fundamentad mode shape function is expressed by a polynomial shape function $\Psi = 3\eta^2 - 2\eta^3$ where $\eta = z/Z_0$. A model of the structure is shown in Fig. 1(b) [2]. In this model $R(t)$ represents the absolute motion of the bedrock, $G(t)$ is the motion of the ground layer (represented by the mass m_{soil}) relative to the bedrock, $s_0(t) = R(t) + G(t)$ is the absolute motion of the ground. The properties of the ground layer are represented by the coefficient of stiffness k_{soil} and the coefficient of viscous damping c_{soil}. The parameters of the structure are the modal mass m_r, modal stiffness coefficient k_r, and modal damping coefficient c_r. The displacements of the structure relative to the ground are $\bar{v}_0 = \Psi(\eta_0)p_r(t), \eta_0 = \eta(Z_0)$, where $p_r(t)$ denotes the modal coordinate. The motion of the structure is then desribed by the ordinary differential equations (ODE) given by Eq. (1):

$$
\begin{aligned}
&\ddot{p}_r(t) + 2\zeta_r\omega_r\dot{p}_r(t) + \omega_r^2 p_r(t) = P_r(t) \\
&\ddot{G}(t) + 2\zeta_s\omega_{soil}\dot{G}(t) - \frac{c_r}{m_{soil}}\dot{p}_r + \omega_{soil}^2 G(t) - \frac{k_r}{m_{soil}}p_r = -\ddot{R}(t)
\end{aligned}
\tag{1}
$$

where $\omega_{soil} = \sqrt{\frac{k_{soil}}{m_{soil}}}$ and ζ_s is the layer damping ratio. The model depicted in Fig. 1(a) represents a multibody system with rigid bodies (RB) shown as the masses M_1, M_2 and M_3, respectively. These correspond to the car, counterweight (CWT) and compensating sheave assembly (CSA) of the VTS, respectively, and are constrained by elastic long slender continua (LSC) of time-varying length $L_i(t)$, where i = 1,4 are referred to as compensating cables, and i = 2,3 as suspension ropes, at the car and CWT sides, respectively. They have small bending stiffness E_iJ_i and are of mass per unit length m_i, where i = 1,2. The dynamic vertical displacements of RB are denoted by q_{M_1}, $q_{M_{12}}$ and q_{M_3}, respectively. I_3 is the second moment of inertia of the CSA, with its rotational degree of freedom represented by the angular coordinate θ_3. A nonlinear viscous damping (tie-down) element of the coefficient of damping c_3 is applied to constrain vertical motions of the CSA. The equations of motion of RB are given by Eq. (2):

$$\begin{gathered} M_1\ddot{q}_{M_1} - E_1A_1e_1 + E_2A_2e_3 = 0;\ M_2\ddot{q}_{M_2} - E_1A_1e_4 + E_2A_2e_2 = 0; \\ M_3\ddot{q}_{M_3} + E_1A_1e_1 + E_1A_1e_4 + F_d = 0;\ I_3\ddot{\theta} - D(E_1A_1e_1 + E_1A_1e_4)/2 = 0 \end{gathered} \tag{2}$$

where E_i, A_i i = 1,2 denote the moduli of elasticity and cross-sectional areas of LSC, and e_i, i = 1,2,..,4, denote the quasi-static axial strains in the LSC components. D is the diameter of the compensating sheave and F_d represents the damping force in the tie-down element. The kinematic constraint $2q_3 - u_1 - u_4 = 0$, where $u_1 = q_{M_3} + R\theta_3, u_4 = q_{M_3} - R\theta_3$, is applied to complement Eq. (2). The equations of motion of LSC are given by the nonlinear partial differential equations (PDE) presented in Eq. (3):

$$\begin{gathered} m_i\bar{v}_{itt} + E_iJ_i\bar{v}_{ixxxx} - \{T_i - m_i[V^2 + (g - a_i)x_i] + E_iA_ie_i\}\bar{v}_{ixx} + m_ig\bar{v}_{ix} \\ + 2m_iV\bar{v}_{ixt}, = F_i^v[t, L_i(t)], \\ m_i\bar{w}_{itt} + E_iJ_i\bar{w}_{ixxxx} - \{T_i - m_i[V^2 + (g - a_i)x_i] + E_iA_ie_i\}\bar{w}_{ixx} + m_ig\bar{w}_{ix} \\ + 2m_iV\bar{w}_{ixt} = F_i^w[t, L_i(t)] \end{gathered} \tag{3}$$

where g is the acceleration of gravity, $\bar{v}_i(x_i, t), \bar{w}_i(x_i, t)$ i = 1,2,...,4, represent the lateral dynamic displacements of LSC relative to the host structure, and F_i^v, F_i^w are the excitation terms. The response of the structure is determined from Eq. (1). The Galerkin method is applied to discretize Eq. (3). The resulting nonlinear ODE system is then solved numerically by using a stiff solver. The dynamic interactions when the frequency of the building is tuned to the natural frequencies of the VTS are investigated. In Fig. 2 (a-c) the frequency plots are shown, and vertical responses of the RBs are presented in (d). The scenario is when the car is moving upwards at speed 5 m/s. The plots demonstrate interactions of the RB responses with the LSC resonance conditions when the frequency of the excitation (represented by horizontal red lines) is tuned to the natural frequencies of the LSC system.

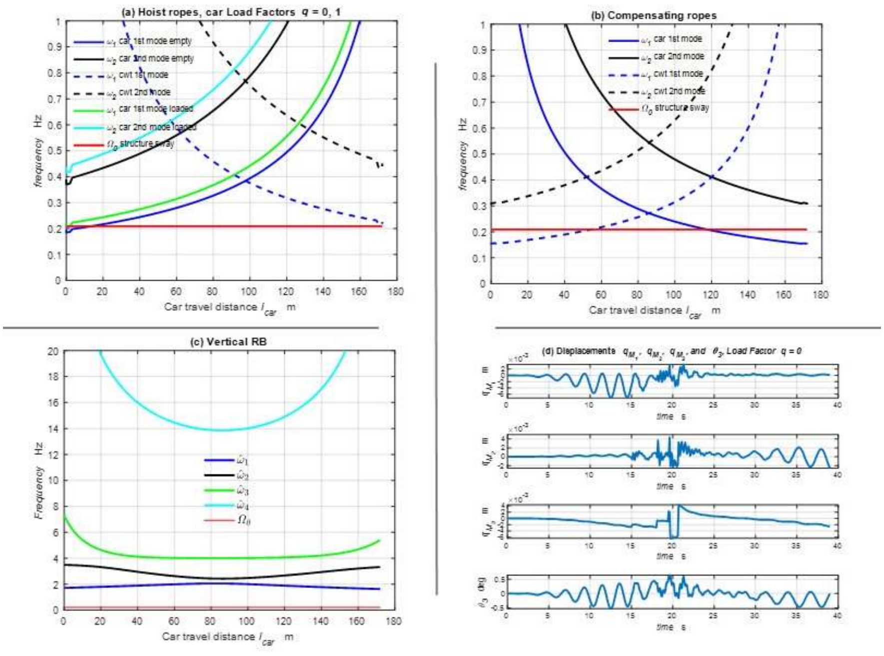

Figure 2: The frequency plots: (a) suspension LSC (b) compensating LSC (c) vertical RB, and (d) the dynamic displacements of RB.

References

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[2] Weber, H.; Kaczmarczyk, S.; Iwankiewicz, R.: Non-liner Response of Cable-Mass-Spring System in High-Rise Buildings under Stochastic Seismic Excitation. Materials, Vol. 14, 2021.