Interference Suppression Using Adaptive Nulling Algorithm Without Calibration Sources

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INTERFERENCE SUPPRESSION USING ADAPTIVE NULLING ALGORITHM WITHOUT CALIBRATION SOURCES

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ABSTRACT

Interference suppression using adaptive nulling algorithm is an important array signal processing technique for radar/sonar sensing. However, in long term task, most of the arrays' parameters vary from time to time, which need known sources to re-calibrate. To be free of calibration sources, this paper presents an adaptive nulling algorithm using array observation data. We first establish the model of SV mismatches due to gain-phase error and sensor shifting. Then the angle-related bases of received signal subspace are estimated by applying a joint optimization method consists of Genetic algorithm and quasi-Newton method. In the end, the array weighting vector can be calculated, and the results of several numerical simulations are demonstrated, which shows that the proposed algorithm can significantly improve the interference suppression performance of sensor array.

***Index Terms*—** Adaptive nulling, Signal subspace, Steering vector estimation,Uncalibrated array

1. INTRODUCTION

Interference suppression is a major area of interest within the field of sensor array processing. Autonomous platform carrying sensor arrays may suffer from calibration error caused by platform vibrations, environment variations and error caused by time accumulation .

During the past 30 years, plenty of adaptive nulling methods had been proposed to decrease the effects of calibration error. By adding a scaled identity matrix to the sample covariance matrix (SCM), Cox proposed the diagonal loading beamformer [1]. Because of the precise selection of the diagonal level is hard to obtain, researches proposed robust Capon beamformer [2][3] and the worst-case optimization beamformer [4]. Based on this, convex optimization [5], linear constrains [6] and the iterative approaches [7] were applied to enhance the robustness of beamforming.

The aforementioned method mainly focused on maintaining main beam towards the desired direction in the case of calibration errors. However, when the SV mismatch is severe, these beamformers may suffer from performance degradation, especially in a high signal-to-noise ratio (SNR) environment [8].

Plenty of works show that the singal of interest (SOI) component in the SCM is the main cause of beamformer's performance degradation [8]. To create a covariance matrix that is free of the SOI component, an angular-sector-based covariance matrix reconstruction and estimation beamformer (REB) was proposed in [9]. Several categories of interference-plus-noise covariance matrix (INCM)-based beamformers were then proposed, such as the sparse reconstruction methods [10], and the low-complexity methods [11][12], subspace-based method [13]. In [14], an annulus-uncertainty-set-based method was proposed to alleviate the performance degradation due to random SV mismatch. In[15], the proposed algorithm corrected all SVs of possible interferences, and then corresponding interference power were estimated , which achieved satisfactory performance with high computational complexity. [10] demonstrated an INCM reconstruction-based adaptive beamformer for co-prime array, which shows effectiveness in suppressing interference.

Recently, weighted subspace fitting-based methods were proposed to overcome sensor position error [16][17]. However, when other kind of error exist, these methods may not correctly fit the signal subspace, which may cause failure in interference suppression.

In this paper, we propose a novel INCM-based adaptive nulling algorithm that only number of signals and performs robustly in the case of gain-phase error, sensor position error and unkonwn signal incident angle. By modeling the SV mismatches due to various causes, we estalish a joint optimization problem using the idea of signal subspace fitting. Next, instead of estimating powers of all signals, we reconstruct the INCM by extending the subspace bases transition to eigenspace bases transition. In the end, SV of the SOI is estimated and the results of simulations validate the performance of the proposed algorithm.

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2. PROBLEM FORMULATION

Consider an array with *M* omnidirectional sensors that receives far-field narrowband signals from several sources. The array observation data at the k-th snapshot can be written as

$$\mathbf{x}(k) = \mathbf{a}_0 s_0(k) + \sum\nolimits_{1}^{Q} \mathbf{a}_q s_q(k) + \mathbf{n}(k) \ , \tag{1}$$

where $\mathbf{a}_0$ and $\mathbf{a}_q$ denote the actual SVs of the desired signal and the *q*-th interference, respectively. s_0, s_q, and $\mathbf{n}(k)$ denote the waveform of the desired signal, the *q*-th interference, and the additive white Gaussian noise vector, respectively. We assume that the desired signal, interferences, and noise to be uncorrelated with each another. To overcome the sensor displacement and gain-phase error, we start from modeling the mismatched SV from the nominal SV as

$$\begin{aligned} \mathbf{a}(\mathbf{d}_\mathrm{e}, \varphi_\mathrm{e}, \theta) &= \alpha \odot e^{j[k_\mathrm{w}(\bar{\mathbf{d}}+\mathbf{d}_\mathrm{e})\sin\theta + \varphi_\mathrm{e}]} \\ &= \bar{\mathbf{a}}(\theta) \odot \alpha \odot e^{j(k_\mathrm{w}\mathbf{d}_\mathrm{e}\sin\theta + \varphi_\mathrm{e})} \ , \end{aligned} \tag{2}$$

where $\bar{\mathbf{d}}$ is the assumed sensor position vector, $\mathbf{d}_\mathrm{e}$ denotes the sensor position error vector, $\odot$ denotes the Hadamard product, and k_w is the wavenumber. α and φ_e are the angle-independent sensor gain vector and phase error vector, respectively. Usually, the first sensor is the reference sensor in the array, which is assumed without sensor position error and phase error, and the sensor gain for the first sensor is 1. Therefore, $\mathbf{d}_\mathrm{e}$, α and φ_e can be expressed as

$$\begin{aligned} \mathbf{d}_\mathrm{e} &= [0, d_2, \cdots, d_M]^\mathrm{T} \in \mathbb{R}^{M\times 1} \\ \alpha &= [1, \alpha_2, \cdots, \alpha_M]^\mathrm{T} \in \mathbb{R}^{M\times 1} \\ \varphi_\mathrm{e} &= [0, \varphi_2, \cdots, \varphi_M]^\mathrm{T} \in \mathbb{R}^{M\times 1} \ . \end{aligned} \tag{3}$$

The SCM contains the information about the actual array calibration and the signals, we can eigen-decompose the SCM $\hat{\mathbf{R}}_\mathrm{x}$ as

$$\begin{aligned} \hat{\mathbf{R}}_\mathrm{x} &= \sum_{m=1}^{M} \lambda_m \mathbf{v}_m \mathbf{v}_m^\mathrm{H} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^\mathrm{H} \\ &= \mathbf{V}_\mathrm{S}\mathbf{\Lambda}_\mathrm{S}\mathbf{V}_\mathrm{S}^\mathrm{H} + \mathbf{V}_\mathrm{N}\mathbf{\Lambda}_\mathrm{N}\mathbf{V}_\mathrm{N}^\mathrm{H} \ , \end{aligned} \tag{4}$$

where λ_m and $\mathbf{v}_m$ are the m-th eigenvalue in descending order and the corresponding eigenvector, respectively. $\mathbf{\Lambda} = diag\{\lambda_1, \lambda_2, \cdots, \lambda_M\}$ is a diagonal matrix that consists of all eigenvalues in a descending order, $\mathbf{V}$ is the matrix that contains all eigenvectors. $\mathbf{\Lambda}_\mathrm{S} = diag\{\lambda_1, \lambda_2, \cdots, \lambda_L\}$ contains L dominant eigenvalues, and $\mathbf{V}_\mathrm{S}$ is the signal subspace that contains the corresponding eigenvectors. $\mathbf{\Lambda}_\mathrm{N} = diag\{\lambda_{L+1}, \cdots, \lambda_M\}$ consists of the remaining eigenvalues, and $\mathbf{V}_\mathrm{N}$ denotes the noise subspace that contains the corresponding eigenvectors.

3. PROPOSED ALGORITHM

In this section, we establish a hybrid optimization problem to estimate the angle-related bases consist of signal SVs using subspace fitting technique. Then we propose a novel INCM reconstruction method that directly eliminate the desired signal component from the sample covariance matrix.

3.1. Angle-related bases estimation

When the precise information about the array and the signals are exactly known, the signal subspace equals to the space spanned by the actual SVs of signals, which is

$$\text{span}\{\mathbf{V}_\mathrm{S}\} = \text{span}\{\mathbf{A}\} \ , \tag{5}$$

where $\mathbf{A} = [\mathbf{a}(\theta_0), \mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_0)]$ denote the actual SV set consists of all Q+1 signal SVs. It is worth noticing that when the interference is coherent with desired signal, or the INR is extremely higher than the SNR, the number of dominate eigenvalues L does not equal the number of signals. However, the number of signals can be estimated using various algorithms.

According to (2), the gain errors is independent of phase errors, then the gain error of the mth sensor can be estimated as

$$\hat{\alpha}_m = \sqrt{\frac{\hat{\mathbf{R}}_\mathrm{x}(m,m) - \lambda_M}{\hat{\mathbf{R}}_\mathrm{x}(1,1) - \lambda_M}} \ , \tag{6}$$

where $\hat{\mathbf{R}}_\mathrm{x}(m,m)$ denotes the mth diagonal elements of the SCM, and λ_M is the smallest eigenvalue of the SCM. It can be seen that the sensor position error and the phase error only influence the phase of the SV. It is difficult to accurately estimate the precise directions, sensor position errors and the phase errors separately because $k_\mathrm{w}\mathbf{d}_\mathrm{e}\sin\theta$ in (2) can be treated as angle-related phase errors, which is coupled with the angle-independent phase errors φ_e. However, we can estimate the mismatched SV set by minimizing the difference of the signal subspace and the space spanned by the possible mismatched SV set as

$$\hat{\mathbf{A}}(\hat{\mathbf{d}}_\mathrm{e}, \hat{\varphi}_\mathrm{e}, \hat{\Theta}) = \min_{\mathbf{d}_\mathrm{e}, \varphi_\mathrm{e}, \Theta} \text{tr}\{\ \mathbf{P}^\perp \mathbf{V}_\mathrm{S}\mathbf{W}\mathbf{V}_\mathrm{S}^\mathrm{H}\} \ , \tag{7}$$

where $\mathbf{W}$ is a positive definite weighting matrix, which equals to $(\mathbf{\Lambda}_\mathrm{S} - \lambda_M\mathbf{I})^2\mathbf{\Lambda}_\mathrm{S}^{-1}$ under the condition of lowest asymptotic variance and $\mathbf{P}^\perp$ is the orthogonal projection matrix, which is formed as $\mathbf{P}^\perp = \mathbf{I} - \mathbf{A}(\mathbf{d}_\mathrm{e}, \varphi_\mathrm{e}, \Theta)\mathbf{A}^+(\mathbf{d}_\mathrm{e}, \varphi_\mathrm{e}, \Theta)$, where $\mathbf{A}(\mathbf{d}_\mathrm{e}, \varphi_\mathrm{e}, \Theta) \in \mathbb{C}^{M\times(Q+1)}$ represents the possible mismatched SV set, $(\cdot)^+$ denotes the Moore-Penrose inversion, and $\mathbb{C}$ is the complex number field. Θ consist of possible directions of all signals. The i-th column in $\mathbf{A}(\mathbf{d}_\mathrm{e}, \varphi_\mathrm{e}, \Theta)$ represents the possible SV of the ith signal, which can be formed as

$$\mathbf{a}(\mathbf{d}_\mathrm{e}, \varphi_\mathrm{e}, \theta_i) = \hat{\alpha} \odot e^{j[k_\mathrm{w}(\mathbf{d}+\mathbf{d}_\mathrm{e})\sin\theta_i + \varphi_\mathrm{e}]} \ , \tag{8}$$

where $\hat{\alpha} = [1, \hat{\alpha}_2, \cdots, \hat{\alpha}_2]^\mathrm{T}$ is the estimated gain error vectors in (5). The minimization problem in (6) is obvious a nonlinear optimization problem with $2M + Q - 1$ variables. The previous work in [17] use genetic algorithm to tackle this problem. However, with large number of variables, the genetic algorithm requires large number of generations or iterations to present a satisfactory result. Therefore, we use a joint optimization method that initialize with a few generations of Genetic algorithm, then we use a quasi-Newton Method called the BFGS method to tackle this optimization problem. First, we need to construct the solution vector of the minimization problem as

$$\delta = [d_2, \cdots, d_M, \varphi_2, \cdots, \varphi_M, \theta'_0, \cdots, \theta'_Q]^{\mathrm{T}} \tag{9}$$

where $\theta'_q, q = 0, 1, \cdots, Q$ is the possible DOAs of all signal in an ascending order, and θ'_0 is not necessarily the DOA of the SOI. (6) can be rewritten as

$$\hat{\mathbf{A}}(\hat{\delta}) = \min_{\delta} F(\delta) \tag{10}$$

where $F(\delta)$ is the objective function in (6). Hence, the iteration algorithm of the BFGS method can be formed as

$$\hat{\delta}_{(l+1)} = \hat{\delta}_{(l)} - \beta_{(l)}[F''(\hat{\delta}_{(l)})]^{-1}F'(\hat{\delta}_{(l)}) \tag{11}$$

where $\hat{\delta}_{(l)}$ and $\beta_{(l)}$ are the solution vector and step length at lth iteration, respectively. $F'(\delta)$ and $F''(\delta)$ indicate the gradient and Hessian of $F(\delta)$, respectively. and the gradient can be obtained as

$$F'(\delta) = [\frac{\partial F}{\partial d_m}, \cdots, \frac{\partial F}{\partial \varphi_m}, \cdots, \frac{\partial F}{\partial \theta'_0}, \cdots, \frac{\partial F}{\partial \theta'_Q}]^{\mathrm{T}}, \tag{12}$$

where the $\partial F/\partial d_m$ denotes the partial derivative of variable d_m. The close-form of partial derivative is difficult to obtain, we can approximate the partial derivative by central difference. For example, $\partial F/\partial d_2$ can be approximated as

$$\frac{F(\hat{\delta}_{(l)})}{\partial d_2} \approx \frac{F(\hat{\delta}_{(l)} + \Delta\delta_{d_2}) - F(\hat{\delta}_{(l)} - \Delta\delta_{d_2})}{2\Delta d_2}, \tag{13}$$

where $\Delta\delta_{d_2} = [\Delta d_2, \mathbf{0}]^{\mathrm{T}}$, and Δd_2 denotes a very small positive value. By using the central difference method, the gradient of $F(\delta)$ can be efficiently calculated. Moreover, the close-form of Hessian matrix is difficult to obtain. By utilizing the BFGS method, the inversion of the Hessian matrix $[F''(\hat{\delta}_{(l)})]^{-1}$ can be obtained. However, the BFGS method is sensitive to the initial value $\hat{\delta}_0$, when $\hat{\delta}_0$ is far from the real values, the BFGS may fail to converge. Considering the sensitivity of BFGS, we can estimate initial values of BFGS by using a global optimization method such as the genetic algorithm with small number of generations. By combining the genetic algorithm and the BFGS method, the mismatched SV set in (6) can be estimated as

$$\hat{\mathbf{A}}_{\mathrm{S}} = \hat{\mathbf{A}}(\hat{\mathbf{d}}_{\mathrm{e}}, \hat{\varphi}_{\mathrm{e}}, \hat{\Theta}) = F(\hat{\delta}). \tag{14}$$

It is worth noticing that though the difference between span$\{\hat{\mathbf{A}}_{\mathrm{S}}\}$ and span$\{\mathbf{V}_{\mathrm{S}}\}$ is minimized, the estimated parameters $\hat{\mathbf{d}}_{\mathrm{e}}$, $\hat{\varphi}_{\mathrm{e}}$, $\hat{\Theta}$ is not necessarily accurate because these parameters are coupled together and cannot be precisely and separately estimated in this method.

3.2. Covariance matrix reconstruction

To avoid using the estimated power of interference and noise, we can reconstruct the INCM by eliminating the SOI component directly from the SCM using subspace techniques. With well estimated mismatched SV set $\hat{\mathbf{A}}_{\mathrm{S}}$, (4) can be rewritten as span$\{\hat{\mathbf{A}}_{\mathrm{S}}\} \approx$ span$\{\mathbf{V}_{\mathrm{S}}\}$, where $\mathbf{V}_{\mathrm{S}}$ can be seen as a set of orthogonal bases of the signal subspace, and $\hat{\mathbf{A}}_{\mathrm{S}}$ can be regarded as a set of angle-related non-orthogonal bases of the signal subspace. Because the signal subspace is orthogonal to the noise subspace, each column in $\hat{\mathbf{A}}_{\mathrm{S}}$ is orthogonal to the noise subspace, which can be expressed as span$\{\hat{\mathbf{A}}_{\mathrm{S}}\}\perp$span$\{\mathbf{V}_{\mathrm{N}}\}$.

Therefore, span$\{\hat{\mathbf{A}}_{\mathrm{S}}\}$ is the orthogonal complement of span$\{\mathbf{V}_{\mathrm{N}}\}$ in span$\{\mathbf{V}\}$, which indicates that

$$\text{span}\{[\hat{\mathbf{A}}_{\mathrm{S}}\,\mathbf{V}_{\mathrm{N}}]\} \approx \text{span}\{[\mathbf{V}_{\mathrm{S}}\,\mathbf{V}_{\mathrm{N}}]\} = \text{span}\{\ \mathbf{V}\}, \tag{15}$$

where $[\hat{\mathbf{A}}_{\mathrm{S}}\,\mathbf{V}_{\mathrm{N}}]$ is an $M \times M$ matrix, and each column in $[\hat{\mathbf{A}}_{\mathrm{S}}\,\mathbf{V}_{\mathrm{N}}]$ can be seen as a basis of the observation space span$\{\mathbf{V}\}$. Hence, we define a bases transition matrix from $[\hat{\mathbf{A}}_{\mathrm{S}}\,\mathbf{V}_{\mathrm{N}}]$ to $\mathbf{V}$, and $\mathbf{V}$ is

$$\mathbf{T} = [\hat{\mathbf{A}}_{\mathrm{S}}\,\mathbf{V}_{\mathrm{N}}]^{+}\mathbf{V},\ \mathbf{V} = [\hat{\mathbf{A}}_{\mathrm{S}}\,\mathbf{V}_{\mathrm{N}}]\mathbf{T}. \tag{16}$$

Then the SCM in (4) can be rewritten as

$$\hat{\mathbf{R}}_{\mathrm{x}} = \mathbf{V}\boldsymbol{\Lambda}\mathbf{V}^{\mathrm{H}} = [\hat{\mathbf{A}}_{\mathrm{S}}\,\mathbf{V}_{\mathrm{N}}]\mathbf{T}\boldsymbol{\Lambda}\mathbf{T}^{\mathrm{H}}[\hat{\mathbf{A}}_{\mathrm{S}}\,\mathbf{V}_{\mathrm{N}}]^{\mathrm{H}}. \tag{17}$$

To eliminate the component of the SOI from the SCM, the INCM can be directly reconstructed as

$$\hat{\mathbf{R}}_{\mathrm{i+n}} = [\hat{\mathbf{A}}_{\mathrm{S}}\,\mathbf{V}_{\mathrm{N}}]\mathbf{D}\mathbf{T}\boldsymbol{\Lambda}\mathbf{T}^{\mathrm{H}}\mathbf{D}^{\mathrm{H}}[\hat{\mathbf{A}}_{\mathrm{S}}\,\mathbf{V}_{\mathrm{N}}]^{\mathrm{H}}, \tag{18}$$

where $\mathbf{D}$ denotes a $M \times M$ diagonal matrix as

$$\mathbf{D} = diag\{[\mu, \mathbf{1}_{1\times M-1}]\}. \tag{19}$$

where μ is a very small positive constant that ensure the SOI component can be eliminated. $\mu = 0$ is not recommended because it may result in zero eigenvalue or extremely small positive eigenvalues of $\hat{\mathbf{R}}_{\mathrm{i+n}}$, which may cause the INCM noninvertible.

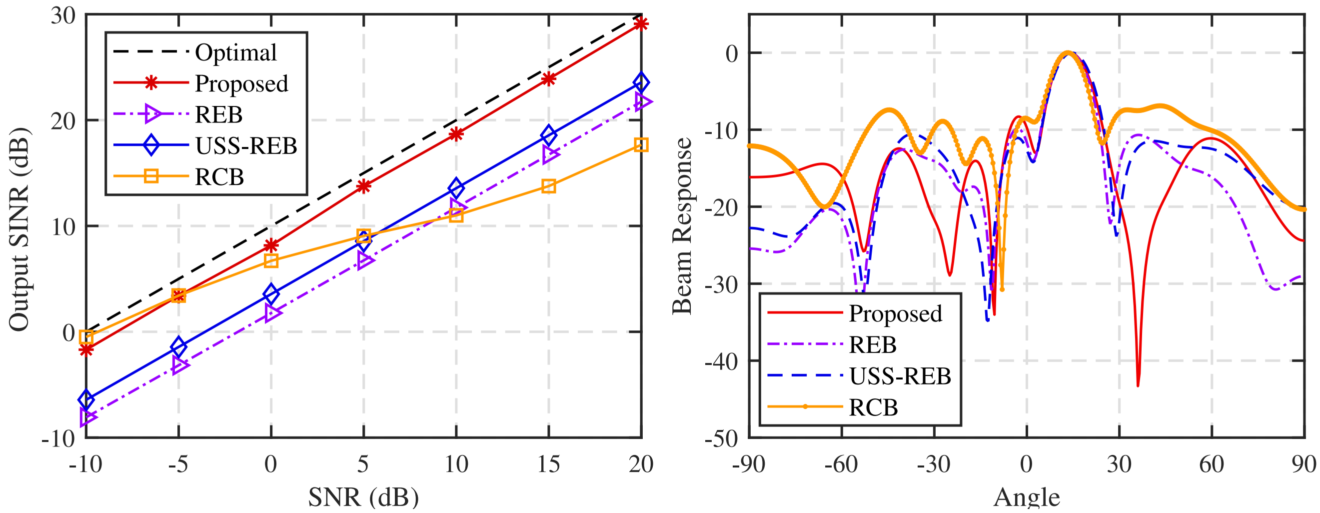
3.3. SOI SV estimation

The actual SV of the SOI is usually unavailable in practical applications, which needs to be estimated or corrected. In this subsection, the desire signal covariance matrix (DSCM) will be reconstructed using the idea similar to the INCM reconstruction, and the SV of the SOI can be estimated from the reconstructed DSCM.

$$\hat{\mathbf{R}}_{\mathrm{s}}(f_{\mathrm{r}}) = \int_{\Theta_{\mathrm{s}}} \frac{\mathbf{a}(\theta, f_{\mathrm{r}})\mathbf{a}^{\mathrm{H}}(\theta, f_{\mathrm{r}})}{\mathbf{a}^{\mathrm{H}}(\theta, f_{\mathrm{r}})\hat{\mathbf{R}}_{\mathrm{F}}^{-1}\mathbf{a}(\theta, f_{\mathrm{r}})} d\theta, \tag{20}$$

where Θ_{s} is the angular sector of the SOI. Unlike $\hat{\mathbf{R}}_{\mathrm{i+n}}$, $\hat{\mathbf{R}}_{\mathrm{s}}$ is supposed to contain only the SOI that originates from $\theta_0 \in \Theta_{\mathrm{s}}$, and the SV of the SOI can be estimated as

$$\hat{\mathbf{a}}_0 = \sqrt{M}\mathcal{P}\{\hat{\mathbf{R}}_{\mathrm{x}} - \hat{\mathbf{R}}_{\mathrm{i+n}}\} \tag{21}$$



(a) Output SINR versus input SNR.

(b) Comparision of normalized beampattern.

Fig. 1: Performance comparison of different adaptive beamformers.

where $\mathcal{P}\{\cdot\}$ denotes the eigenvector corresponding to the largest eigenvalue of a Hermitian matrix. Therefore, the weighting vector of the proposed focused wideband beamformer can be written as

$$\mathbf{w} = \frac{\hat{\mathbf{R}}_{\mathrm{i+n}}^{-1}\hat{\mathbf{a}}_0}{\mathbf{a}_0^{\mathrm{H}}\hat{\mathbf{R}}_{\mathrm{i+n}}^{-1}\hat{\mathbf{a}}_0} \tag{22}$$

4. NUMERICAL SIMULATION

In this section, we consider an non-ideal scenario to evaluate the robustness of the proposed beamformer. We assume that an uliform linear array with 10 omnidirectional sensors receive signals from three far-field sources. The sensors are assumed evenly spaced at half wavelength. Two interferences with interference-to-noise ratio at 20 dB impinge from -25° and 35°, and the desired signal impinges from 15°. The desired signal and interferences are generated from zero means complex Gaussian noises and therefore spatially and temporally independent. In the case of SINR versus SNR, the number of snapshots is fixed at $K = 30$, and in the case of SINR versus snapshots number, the SNR is fixed at 10 dB.

The results are averages of 200 Monte-Carlo simulations. In these simulations, We assumed that the calibration error is partially caused by gain and phase perturbations in each sensor, which distributed in $\mathcal{N}(0, 0.1^2)$ and $\mathcal{N}(0, (0.1\pi)^2)$. Besides, the calibration error contains sensor position error, which is a normal distribution in $\mathcal{N}(0, 0.1^2)$ except for the reference sensor. The proposed beamformer is compared with 3 beamformers, namely the RCB [2], REB [9] and USS-REB [18]. All tested beamformers are compared in the scale of output SINR.

Fig. 1a shows output SINR of four different beamformer. In this case, signal direction error is subject to uniform distribution in $[-2^\circ, 2^\circ]$. With the increase of SNR, the performance of RCB degrades severely due to the SOI component in the SCM. For REB and USS-REB, the output SINRs are lower than RCB when the SNR is smaller than 5 dB. Although proposed beamformer performs worse than RCB when the SNR is -10 dB, it can avoid self-cancellation at high SNR and efficiently suppress interferences in the case of multiple calibration error.

Fig. 1b demonstrate the normalized beam-pattern when SNR = 20 dB. It is obvious that all tested beamformer can steer the main lobe to $\theta_0 = 15^\circ$. However, RCB fails to form nulls around the actual direction of two interferences, the nulls of REB and USS-REB deviate from actual direction of interferences. The proposed beamformer can form two nulls precisely at $\theta_0 = -25^\circ$ and $\theta_0 = 35^\circ$. In other words, the proposed beamformer can suppress interferences in the case of multiple calibration error without known calibration sources.

5. CONCLUSION

This paper proposes an adaptive beamforming and nulling method, which aims to suppress interferences based on the array observation data, and calculate the beamformer's weighting vector using covariance matrix reconstruction. A set of angle-related bases of signal subspace are estimated by applying a joint optimization method. By applying eigen-space bases transition method, the INCM is reconstructed by eliminating SOI component in the SCM, and then the SV of SOI is estimated. The results of several numerical simulations show the performance of proposed beamformer.

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