Syntactic Computation of Model Composites of Enactment Logic.

Frank Appiah

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SYNTACTIC COMPUTATION OF MODEL COMPOSITES OF ENACTMENT LOGIC.

FRANK APPIAH.

KING' COLLEGE LONDON, SCHOOL OF ENGINEERING, ENGLAND, UNITED KINGDOM.

frank.appiah@kcl.ac.uk

appiahnsiahfrank@gmail.com.

Extended Abstract[+]. This research is on provable forms based on syntactic theorem using Kleene Axiom schema. Enact model I and II of propositional formulas from enactment logic are proven in terms of theorems based on deductive rules. This work proves by deduction rules that Enact Model I and II are model theorems.

Keywords. model, composites, enactment, logic, proof, syntactic, theorem.

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1 ***AFFILIATE. UNIVERSITY OF LONDON, KING'S COLLEGE LONDON, DEPARTMENT OF ENGINEERING, LONDON, UK.**

1 INTRODUCTION

This research is based on axiomatical schemas[3] of Kleene that gives theorems based on proofs of enactment models[1]. Syntactic computing[3] in the propositional calculus is based on using axioms and deduction rules[3] to produce theorems. Kleene[3] give the following axiom schemas:

(1a) $\alpha \rightarrow (\beta \rightarrow \alpha)$.

(1b) $(\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow (\beta \rightarrow \gamma) \rightarrow (\alpha \rightarrow \gamma))$.

(2a) $\alpha \rightarrow (\beta \rightarrow (\alpha \wedge \beta))$,

(2b) $(\alpha \rightarrow \beta) \rightarrow \alpha$.

(2c) $(\alpha \wedge \beta) \rightarrow \beta$.

(3a) $\alpha \rightarrow (\alpha \vee \beta)$,

(3b) $\beta \rightarrow (\alpha \vee \beta)$,

(3c) $(\alpha \rightarrow \gamma) \rightarrow ((\beta \rightarrow \gamma) \rightarrow ((\alpha \vee \beta) \rightarrow \gamma))$.

(4a) $(\alpha \rightarrow \beta) \rightarrow ((\alpha \rightarrow \neg \beta) \rightarrow \neg \alpha)$,

(4b) $(\neg\neg \alpha) \rightarrow \alpha$.

Results of Work[1]:

The models of enactment logic are:

(1) $enact_E \rightarrow enact_L$

(2) $a_i \rightarrow a$

(3) $a_j \rightarrow l$

(4) $rank_i \rightarrow t$

(5) $enact_E \rightarrow (enact_E \rightarrow a_i)$

(6) $eanct_L \rightarrow (a_j \rightarrow l)$

(7) $a_i \rightarrow (a_j \rightarrow l)$

(8) $a_j \rightarrow (l \rightarrow rank_i)$

(9) $l \rightarrow (rank_i \rightarrow t)$

Axiom is produced by replacing the Greek variablee of an axiom schema by formula. Models (5) and (6) will be proved using Kleene expression axioms (1a) to (4b).

3 THEOREM PROVING

Let $Enact_E$ to be represented by E_E .

Let $Enact_L$ to be represented by E_L .

(5) $enact_E \rightarrow (enact_E \rightarrow a_i)$: **Enact Model I**

(5) is proven by theorem 5 as shown below:

Theorem 5:

An enact$_E$ implies consequence of $\rightarrow$ -composites of $enact_L \rightarrow a_i$, expressed as: $enact_E \rightarrow (enact_E \rightarrow a_i)$, then it is the consequence of $((enact_E \rightarrow enact_L) \rightarrow (enact_E \rightarrow a_i))$.

Proof. Theorem 5 is expressed as $((E_E \rightarrow E_L) \rightarrow (E_E \rightarrow a_i))$.

Axiom. (1) $((E_E \rightarrow E_L) \rightarrow (E_E \rightarrow a_i))$. given can be proved from Kleene's Axiom.

(2) $((E_E \rightarrow (E_L \rightarrow a_i)) \rightarrow (E_E \rightarrow E_L) \rightarrow (E_E \rightarrow (E_L \rightarrow a_i)))$, axiom of schema (1a).

(3) $((E_E \rightarrow E_L) \rightarrow (E_E \rightarrow (E_L \rightarrow a_i)))$ modus ponens of lines 1 and 2.

Deduction Rule. $\frac{\alpha,\ (\alpha\rightarrow\beta)}{\beta}$ *,where* β *is a consequence of* α *and* $(\alpha\rightarrow\beta)$.

$$\frac{(E_E\rightarrow(E_L\rightarrow a_i))=\alpha,\ (\alpha\rightarrow(E_E\rightarrow E_L)\rightarrow(E_E\rightarrow(E_L\rightarrow a_i))=\beta)}{\beta}$$

$(E_E\rightarrow E_L)\rightarrow(E_E\rightarrow(E_L\rightarrow a_i))=\beta$ is the consequence of

$(E_E\rightarrow(E_L\rightarrow a_i))=\alpha$ and $(E_E\rightarrow E_L)\rightarrow(E_E\rightarrow(E_L\rightarrow a_i))=\beta$.

(4) $$((E_E\rightarrow E_L)\rightarrow(E_E\rightarrow(E_L\rightarrow a_i))) \\ \rightarrow(((E_E\rightarrow E_L)\rightarrow((E_E\rightarrow(E_L\rightarrow a_i))\rightarrow(E_E\rightarrow a_i)))\rightarrow((E_E\rightarrow E_L)\rightarrow(E_E\rightarrow a_i)))$$

axiom of schema (1b).

(5) $((E_E\rightarrow E_L)\rightarrow((E_E\rightarrow(E_L\rightarrow a_i))\rightarrow(E_E\rightarrow a_i)))\rightarrow((E_E\rightarrow E_L)\rightarrow(E_E\rightarrow a_i))$

modus ponens of lines 3 and 4.

Deduction Rule. $\frac{\alpha,\ (\alpha\rightarrow\beta)}{\beta}$, modus ponens 3 and 4 gives,

$$\frac{\alpha_4=\alpha_{3,}\ \ (\alpha_4=\alpha_3\rightarrow\beta_4)}{\beta_4}\ .$$

β_4 is a consequence of $\alpha_3=\alpha_4$ and $(\alpha_3=\alpha_4\rightarrow\beta_4)$.

$$\frac{(E_E\rightarrow E_L)\rightarrow(E_E\rightarrow(E_L\rightarrow a_i))=\alpha_3=\alpha_4,(\alpha_3=\alpha_4\rightarrow,\beta_4)}{((E_E\rightarrow E_L)\rightarrow((E_E\rightarrow(E_L\rightarrow a_i))\rightarrow(E_E\rightarrow a_i)))\rightarrow((E_E\rightarrow E_L)\rightarrow(E_E\rightarrow a_i))=\beta_4}$$

(6) An axiom of schema (1b) gives ,

$(E_E\rightarrow E_L)\rightarrow((E_E\rightarrow(E_L\rightarrow a_i))\rightarrow(E_E\rightarrow a_i))$.

(7) Modus ponens on line 5 and 6 gives,

$$\frac{\alpha_5=\alpha_{6,}\ \ (\alpha_5=\alpha_6\rightarrow\beta_5)}{\beta_5}$$

$$\frac{(E_E\rightarrow E_L)\rightarrow((E_E\rightarrow(E_L\rightarrow a_i))\rightarrow(E_E\rightarrow a_i))=\alpha_5=\alpha_{6,}(\alpha_5=\alpha_6\rightarrow\beta_5)}{((E_E\rightarrow E_L)\rightarrow(E_E\rightarrow a_i))=\beta_5}$$

β_5 is the consequence of $\alpha_5=\alpha_6$ and $(\alpha_5=\alpha_6\rightarrow\beta_5)$. Thus

$(E_E\rightarrow E_L)\rightarrow(E_E\rightarrow a_i)$. Hence

$E_E\rightarrow(E_L\rightarrow E_L)\vdash (E_E\rightarrow E_L)\rightarrow(E_E\rightarrow a_i)$.

$E_E,\ E_L\ ,E_L\ \vdash (E_E\rightarrow E_L)\rightarrow(E_E\rightarrow a_i)$

$(E_E\rightarrow E_L)\rightarrow(E_E\rightarrow a_i)$ is a theorem.

(6) $eanct_L\rightarrow(a_j\rightarrow l)$: **Enact Model II**

(6) is proven by theorem 6 as shown below:

Theorem 6:

An enact$_E$ implies consequence of $\rightarrow$ -composites of $a_j\rightarrow l$, expressed as: $enact_L\rightarrow(a_j\rightarrow l)$, then it is the consequence of $((enact_L\rightarrow a_j)\rightarrow(enact_L\rightarrow l))$.

Proof. Theorem 6 is expressed as $((E_L\rightarrow a_j)\rightarrow(E_L\rightarrow l))$.

Axiom. (1) $((E_L\rightarrow a_j)\rightarrow(E_L\rightarrow l))$. given can be proved from Kleene's Axiom.

(2) $((E_L\rightarrow(a_j\rightarrow l))\rightarrow(E_L\rightarrow a_j)\rightarrow(E_L\rightarrow(a_j\rightarrow l)))$, axiom of schema (1a).

(3) $((E_L\rightarrow a_j)\rightarrow(E_L\rightarrow(a_j\rightarrow l)))$ modus ponens of lines 1 and 2.

Deduction Rule. $\frac{\alpha,\ (\alpha \to \beta)}{\beta}$ *,where* β *is a consequence of* α *and* $(\alpha \to \beta)$.

$$\frac{(E_L \to (a_j \to l)) = \alpha,\ \ (\alpha \to (E_L \to a_j) \to (E_L \to (a_j \to l)) = \beta)}{\beta}$$

$(E_L \to a_j) \to (E_L \to (a_j \to l)) = \beta$ is the consequence of

$(E_L \to (a_j \to l)) = \alpha$ and $(E_L \to a_j) \to (E_L \to (a_j \to l)) = \beta$.

(4) $((E_L \to a_j) \to (E_L \to (a_j \to l))) \to (((E_L \to a_j) \to ((E_L \to (a_j \to l)) \to (E_L \to l))) \to ((E_L \to a_j) \to (E_L \to l)))$

axiom of schema (1b).

(5) $((E_L \to a_j) \to ((E_L \to (a_j \to l)) \to (E_L \to l))) \to ((E_L \to a_j) \to (E_L \to l))$

modus ponens of lines 3 and 4.

Deduction Rule. $\frac{\alpha,\ (\alpha \to \beta)}{\beta}$, modus ponens 3 and 4 gives,

$$\frac{\alpha_4 = \alpha_{3,}\ \ (\alpha_4 = \alpha_3 \to \beta_4)}{\beta_4} \quad .$$

β_4 is a consequence of $\alpha_3 = \alpha_4$ and $(\alpha_3 = \alpha_4 \to \beta_4)$.

$$\frac{(E_L \to a_j) \to (E_L \to (a_j \to l)) = \alpha_3 = \alpha_4, (\alpha_3 = \alpha_4 \to , \beta_4)}{((E_L \to a_j) \to ((E_L \to (a_j \to l)) \to (E_L \to l))) \to ((E_L \to a_j) \to (E_E \to l)) = \beta_4}$$

(6) An axiom of schema (1b) gives ,

$(E_L \to a_j) \to ((E_L \to (a_j \to l)) \to (E_L \to l))$.

(7) Modus ponens on line 5 and 6 gives,

$$\frac{\alpha_5 = \alpha_{6,}\ \ (\alpha_5 = \alpha_6 \to \beta_5)}{\beta_5}$$

$$\frac{(E_L \to a_j) \to ((E_L \to (a_j \to l)) \to (E_L \to a_j)) = \alpha_5 = \alpha_{6,} (\alpha_5 = \alpha_6 \to \beta_5)}{((E_L \to a_j) \to (E_L \to l)) = \beta_5}$$

β_5 is the consequence of $\alpha_5 = \alpha_6$ and $(\alpha_5 = \alpha_6 \to \beta_5)$. Thus $(E_L \to a_j) \to (E_L \to l)$.

Hence $E_L \to (a_j \to l) |- (E_L \to a_j) \to (E_L \to l)$.

$E_L, a_j, l \; |- (E_L \to a_j) \to (E_L \to l)$

$(E_L \to a_j) \to (E_L \to l)$ is a theorem.

5 CONCLUSION

This concludes work on a deduction theorem proving on Enact model I and II of enactment logic in [1,2]. Futherwork will discuss the three remaining enact models (III), (IV) and (V) of enactment logic.

Compliance with Ethical Standards

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Conflict of Interest:

Author, Dr. Frank Appiah declares that he has no conflict of interest .

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