G/M/1-type Markov Chain Model of Spread Spectrum (CDMA) Cognitive Radio Wireless Networks

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G/M/1-TYPE MARKOV CHAIN MODEL OF SPREAD SPECTURM (CDMA) COGNITIVE RADIO WIRELESS NETWORKS

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ABSTRACT

In this research paper, a G/M/1-type Markov Chain model (with two states at each level) of CDMA Cognitive Radio based wireless networks is developed. The equilibrium as well as transient performance evaluation of such networks is carried out efficiently.

1. Introduction:

Demand for Electro-Magnetic (EM) spectrum for wireless communication is constantly increasing. But it is observed that licensed Electro-Magnetic spectrum utilization is as low as 15% in some sprectrum bands. Thus, engineers are constantly searching for innovative ways of increasing spectrum utilization. One such innovative approach is the concept of COGNITIVE RADIO.

The licensed user of EM spectrum (in the licensed band) is called as Primary User (PU) and the other users trying to use the PU band are called as Secondary Users (SU). In the case of Cognitve Radio based spectrum utilization, if PU is not using its licensed band, Secondary User is allowed to access and utilize that band (using the concept of Spectrum Sensing). Once PU wants to use its licensed band, the secondary user must vacate the band.

Markov chains provide interesting stochastic models of various natural/artificial phenomena. Specifically, Continuous Time Markov Chains (CTMCs) are utilized extensively in queueing theory and its applications. These stochastic processes are studied extensively since they exhibit equilibrium behavior. Thus, efficient computation of equilibrium probability distribution of CTMC's is considered as an ever green open research problem. A class of two dimensional Markov chains, called Quasi-Birth-and-Death processes exhibit a matrix geometric solution for the equilibrium probability distribution.

Consider a Quasi-Birth-and-Death (QBD) process i.e. two dimensional Continuous Time Markov Chain (X_t , Y_t) with state space, E i.e. $E = \{ (i,j); i \geq 0, 1 \leq j \leq N \}$ $i.e.\ index\ \ 'i'\ indicates$

$$i^{th}\ level\ \ and\ \ index\ \ \ 'j'\ specifies\ the\ \ state\ at\ each\ level.$$

The generator matrix of such a CTMC is of the following form:

$$\bar{Q} = \begin{bmatrix} B_0 & B_1 & 0 & 0 & \cdots \\ B_2 & A_1 & A_0 & 0 & \dots \\ 0 & A_2 & A_1 & A_0 & \cdots \\ 0 & 0 & A_2 & A_1 & \cdots \\ 0 & 0 & 0 & A_2 & \cdots \end{bmatrix} \dots . (1.1) .$$

Thus, the equilibrium probability vector $\bar{\pi}$ is partitioned in the following manner:

$$\bar{\pi} = [\overline{\pi_0} : \overline{\pi_1} : \overline{\pi_2} : \dots \dots].$$

For such Continuous Time Markov Chains, we have that

$$\overline{\pi_{n+1}} = \overline{\pi_n} \ R \dots (1.2), \qquad \textit{where the rate matrix}, R \textit{ is}$$

the $\textit{minimal nonnegative solution of the matrix quadratic equation}$

$$R^2 A_2 + R A_1 + A_0 \equiv \bar{0}. \dots . (1.3)$$

We call such a solution as $\textit{matrix geometric solution}$.

G/M/1-type Markov chain (Continuous Time Markov Chain) is a generalization of QBD process in which, states at any level could receive transitions from states at every level above it (In QBD case, the transitions are allowed only from just one levelabove). In such case, matrix geometric recursive solution exists for the equilibrium probabilities. Also, the *rate matrix R* is the $\textit{minimal nonnegative solution of the}$ *following matrix power series equation*

$$\sum_{i=0}^{\infty} R^i \ A_i \equiv \bar{0} \ .$$

Traditionally, iterative procedures are employed to compute the rate matrix, R. In such procedures, the matrix power series equation is truncated into a matrix polynomial equation. Effectively, for sufficiently large values of 'i', A_i's are all assumed to be zero matrices. Hence, with such assumption, we have a structured class of G/M/1-type Continuous Time Markov chains.

Notation: In the following discussion, we denote such " structured G/M/1-type Markov processes" as "arbitrary G/M/1-type Markov chains". In effect, we assume that A_i's are all zero matrices for sufficiently large value of 'i' . Thus, in such a Markov chain, states at any level receive transitions only from states at levels that are finitely many levels (fixed finite number) above.

In this research paper, we design a Quasi-Birth-and-Death process (QBD) model of Cognitive Radio based wireless networks in which there are two states at each level. We generalize the model utilizing "Structured G/M/1-type Markov chains" for modeling Spread Spectrum (Code Division Multiple Access) Cognitive Radio Networks. Based on recent results of the authors [RaR1], [RaR2], an interesting explicit formula for the rate matrix, R is utilized for computing the equilibrium performance measures. Thus, design of Cognitive Radio wireless networks is studied.

This research paper is organized as follows. In Section 2, relevant research literature is reviewed. In Section 3, QBD model of cognitive radio based wireless networks is discussed in detail. Briefly, G/M/1-type Markov Chain model of CDMA Cogntive Radio networks is discussed. In Section 4, performance evalutation of cognitive radio based wireless networks is discussed. The research paper concludes in Section 5.

2. Review of Literature:

Researchers proposed various interesting stochastic models of cognitive radio based wireless networks [FCCM]. Some of these models utilize sophisticated stochastic processes such as Markov Decision processes. These detailed models require many modeling assumptions which cannot be justified in real world cognitive radio networks. Also, such models effectively carry out equilibrium performance evaluation. The "principle of parsimony" (Occam's Razor) states that " model should be chosen to be simple, but not any simpler". Using such principle, we propose a model based on Quasi-Birth-and- Death process. Our model employs the following simple but effective modeling assumptions:

(i) Channels are considered independent from the point of view of occupancy. Channels are decoupled from the point of view of occupancy. Hence, in the model we consider only one channel and determine the performance of the wireless network employing cognitive radio.

(ii) The successive packet arrivals for primary users as well as secondary users are independent and identically distributed.

(iii) The channel occupancy times of primary and secondary users are independent random variables

Our effective goal is to arrive at a Markovian queueing model. Such a simple but effective queueing model enables "real time computation" of performance measures. In effect, we propose a model that enables performance evaluation of cognitive radio based networks in real time.

3. QBD Model of Cognitive Radio based Wireless Networks: Generalization to Spread Spectrum (CDMA) Cognitive Radio Networks:

Continuing our discussion in section 2, we explicitly state the modeling assumptions:

(i) Packet arrivals occur for Primary User (PU) and Secondary User (SU) according to a Poisson process. Specifically, the arrival rates are μ_p and μ_s respectively.

(ii) The Packet Service times (i.e. channel occupancy times) of primary user and secondary user are independent exponential random variables with means $\frac{1}{\mu_d}$, $\frac{1}{\mu_r}$ respectively.

(iii) There are two channel states, namely { PU present, PU absent }

(iv) The secondary users utilize direct sequence spread spectrum (CDMA) multiple access (i.e orthogonal signature sequences are utilized by all the secondary users). Thus, in effect, the probability that multiple secondary users can successfully transmit their packets (without collision) is strictly positive (not zero).

- With the above modeling assumptions, we choose the state of the Continuous Time Markov Chain (CTMC) as $\{ \ (X_t, Y_t \ ): \ X_t \geq 0, \ Y_t \in \{ 1,2 \} \ \}$, where

$$X_t \ is \ Number \ of \ Secondary \ Users \ waiting \ to \ snatch \ the \ Primary \ User \ band \ and$$

$$Y_t \ is \ the \ Primary \ User \ Band \ Occupancy \ States \ i.e.$$

$$\{ Y_t = 1 \ if \ band \ is \ empty \ and \ Y_t = 2 \ if \ band \ is \ occupied \}.$$

With the above modeling assumptions, it readily follows that (X_t, Y_t) forms a Quasi-Birth-and-Death process with the generator matrix Q. We now determine the sub-matrices in the generator matrix of QBD process i.e. sub-matrices in Q in equation (1.1). The sub-matrices are given by

$$B_0 = \begin{bmatrix} -2\ (\mu_p + \mu_s) & \mu_p \\ \mu_d & -2\ (\mu_p + \mu_d) \end{bmatrix} \ , \ B_1 = A_0 \ \ and \ \ B_2 = A_2 \ldots\ldots (3.1)$$

Also, the other sub-matrices are given by

$$A_0 = \begin{bmatrix} \mu_s & (\mu_p + \mu_s) \\ (\mu_p + \mu_d) & \mu_p \end{bmatrix} \ , \ A_2 = \begin{bmatrix} \mu_r & (\mu_p + \mu_r) \\ (\mu_r + \mu_d) & \mu_r \end{bmatrix} \ldots (3.2) \quad and$$

$$A_1 = \begin{bmatrix} -\ (\ 3\ \mu_p + 2\mu_s + 2\mu_r\ ) & \mu_p \\ \mu_d & -\ (\ 3\ \mu_d + 2\mu_p + 2\mu_r\ ) \end{bmatrix} \ldots . (3.3)$$

- **Generalization to Spread Spectrum (CDMA) Cognitive Radio Networks**: The secondary users utilize orthogonal signature sequences as in direct sequence spread spectrum wireless communication systems. Thus, there is a positive probability that multiple secondary users can successfully transmit their packets without collision (sharing the channels efficiently). Let

$$p_j \ be \ the \ probability \ that \ 'j-1' \ secondary \ users \ successfully \ transmit \ their \ packets \ using \ a \ single \ channel \ (\ without \ collision\ ).$$

In such case, A_j matrices are non-zero for $3 \leq j \leq (m + 1)$, where a maximum of 'm' secondary users can successfully transmit their packets on a channel (without collision).

Specifically, for $3 \leq j \leq (m + 1)$, we have $A_j = \begin{bmatrix} p_j \, \mu_r & (\mu_p + p_j \, \mu_r) \\ (p_j \mu_r + \mu_d) & p_j \mu_r \end{bmatrix}$.

Now, from the above discussion, it is clear that we have a G/M/1-type Makov Chain model with two states at each level. In [RaR1], we studied the efficient computation of equilibrium probability distribution of such a Continuous Time Markov Chain. In the following section, we utilize the results in [RaR1] in the equilibrium/Transient performance evaluation of Cognitive Radio based Wireless networks.

4. Equilibrium and Transient Performance Evaluation:

In the case of QBD process with two states at each level, utilizing the Cayley-Hamilton Theorem, it is shown that the rate matrix can be computed by an explicit formula [RaR1]. We briefly summarize the result.

Let the characteristic polynomial of R be given by

$$Determinant(\mu \; I - R) = (\; \mu^2 + b_1 \, \mu + b_0 \;) \ldots . . (4.1).$$

Also, it readily follows that

$$(\mu^2 A_2 + \mu A_1 + A_0) = \; (\mu \; I - R)(\mu \, A_2 + R A_2 + A_1) \ldots (4.2)$$

Hence, eigenvalues of rate matrix R are the zeroes of $g(\mu) = Det(\mu^2 A_2 + \mu A_1 + A_0)$ which are strictly within the unit circle (as the spectral radius of R is strictly less than one). Also, $(A_2 + A_1 + A_0) \, \bar{e} = \overline{0}$, *where $\bar{e}$ is a column vector of all ones*. Hence $\mu = 1$ is a zero of $g(\mu)$. Thus, using the formula for zeroes of a cubic polynomial, the zeroes of $g(\mu)$ which are within the unit circle can be computed by a closed form formula. Thus, it is shown in [RaR1] that

$$R = [\, b_0 A_2 - A_0 \,] \, [A_1 - b_1 A_2 \,]^{-1} \, .$$

Using the above formula, the equilibrium performance measures can be efficiently computed. The 2 x 2 matrices are those specified in the QBD model in Section 2.

It is also shown in [ZaC] that the transient probability distribution of an arbitrary QBD can be computed by a matrix geometric recursion. Using the ideas in [RaR1], we reason that the recursion matrix can be computed efficiently.

We now provide some numerical results for transient probability mass function of the QBD

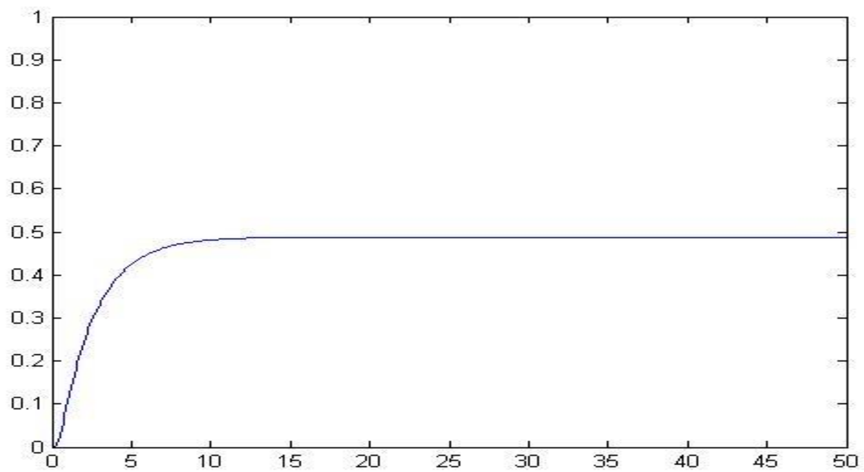


Figure 1. Transient Probability of CTMC being in State 10, as a function of time, given that it starts out in state '1'

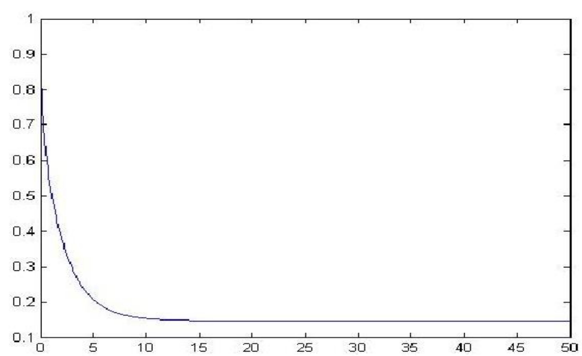
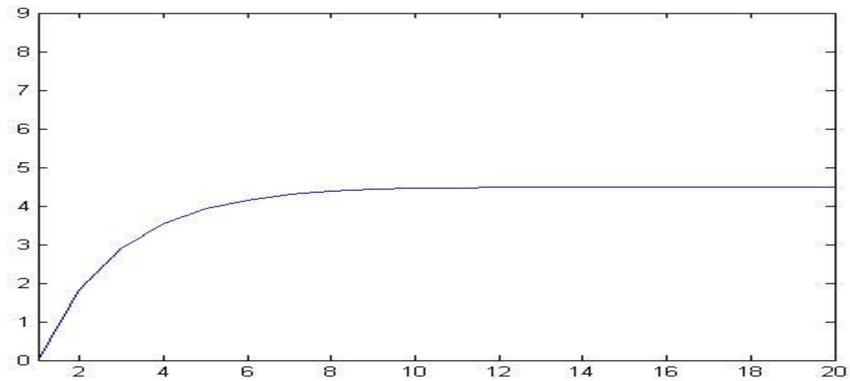
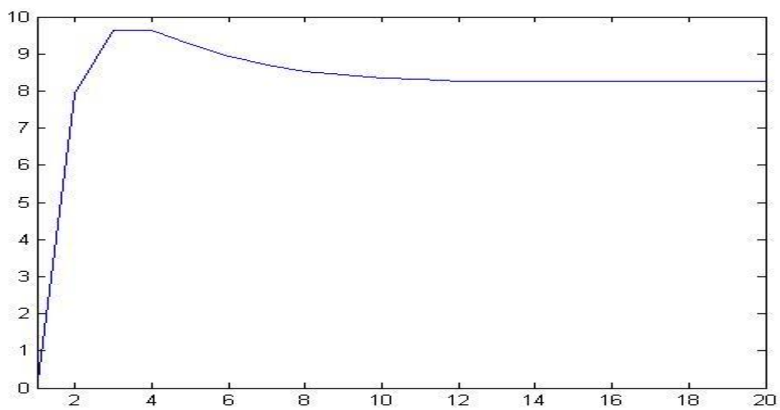


Figure 2: Transient Probability of the CTMC being in state '1' as a function of time, given that the CTMC starts out in state '1'.

- We can plot the expected value of the transient probability distribution and variance of the transient probability distribution (as a function of time) respectively , as follows:





5. Conclusions:

In this research paper, a QBD model of Cognitive Radio based Wireless networks is designed. The equilibrium as well as transient performance evaluation of such networks is efficiently determined.

REFERENCES:

[FCCM] S. Filippi, O.Cappe, F.Clerot and E. Moulines," A Near Optimal Policy for Channel Allocation in Cognitive Radio, " European Workshop on Reinforcement Learning, 2008 [EWRL 2008], pp. 69-81, Lecture Notes in Computer Science (LNCS), pp.69-81, Springer

[Rama1] G. Rama Murthy," Transient and Equilibrium Analysis of Computer Networks: Finite Memory and Matrix Geometric Recursions," Ph.D thesis, 1989, Purdue University, West Lafayette, USA

[RaC**]** G. Rama Murthy and E.J Coyle, " Finite Memory Recursive Solutions in Stochastic Models: Equilibrium and Transient Analysis," Technical Rept. no.29, Purdue University, West Lafayette, May 1989.

[RaR1] G. Rama Murthy and Alexander Rumyantsev**,"** On an exact solution of the rate matrix of Quasi-Birth-Death process with small number of phases" **Proceedings of 31st European Conference on Modelling and Simulation (ECMS 2017), Budapest, Hungary .**

[RaR2] G. Rama Murthy and Alexander Rumyantsev," On an exact solution of the rate matrix of G/M/1-type Markov Process with small number of phases," **Journal of Parallel and Distributed Computing, Elsevier Science, Vol. 119, pp. 172-178, September 2018**