EasyChair Preprint

№ 3453

Quantum Complementarity: Both Duality and Opposition

Vasil Penchev

May 21, 2020

Quantum Complementarity: Both Duality and Opposition

Vasil Penchev, vasildinev@gmail.com
Bulgarian Academy of Sciences: Institute of Philosophy and Sociology
Dept. of Logical Systems and Models

Abstract: *Quantum complementarity is interpreted in terms of duality and opposition. Any two conjugates are considered both as dual and opposite. Thus quantum mechanics introduces a mathematical model of them in an exact and experimental science. It is based on the complex Hilbert space, which coincides with the dual one. The two dual Hilbert spaces model both duality and opposition to resolve unifying the quantum and smooth motions. The model involves necessarily infinity even in any finitely dimensional subspace of the complex Hilbert space being due to the complex basis. Furthermore, infinity is what unifies duality and opposition, universality and openness, completeness and incompleteness in it. The deduced core of quantum complementarity in terms of infinity, duality and opposition allows of resolving a series of various problems in different branches of philosophy: the common structure of incompleteness in Gödel's (1931) theorems and Enstein, Podolsky, and Rosen's argument (1935); infinity as both complete and incomplete; grounding and self-grounding, metaphor and representation between language and reality, choice and information, the totality and an observer, the basic idea of philosophical phenomenology. The main conclusion is: Quantum complementarity unifies duality and opposition in a consistent way underlying the physical world.*

Introduction:

The paper investigates the concept of complementarity in quantum mechanics in terms of duality and opposition. The course of thought is the following:

Quantum mechanics is forced to involve infinity implicitly in its mathematical formalism unlike any other physical science to resolve the problem of how quantum and smooth motion to be described uniformly as no finite mathematical structure manages to do it. Only infinity is what can be both discrete and continuous at the same time, after which the axiom of choice turns out to be inevitably involved in one or other way. Furthermore, the concept of choice implies that of information as the quantity of choice and quantum mechanics is generalized to quantum information.

That structure of quantum mechanics is embedded in the mathematical formalism of Hilbert space, which it uses: Hilbert space is the complex and infinitely dimensional generalization of any real and finitely dimensional orthonormal vector space such as the usual three-dimensional Euclid space. However even any finitely dimensional subspace of the complex Hilbert space involves implicitly infinity just for its complex basis.

Hilbert space and its dual space are identical and complementary: They cannot be given simultaneously. Thus if any of both expresses either the discrete (quantum) or the continuous (smooth) aspect of uniform motion in quantum mechanics, the other the other. As the two dual spaces are identical, the two aspects of motion are united uniformly. As they are complementary, the two aspects though being opposite to each other do not contradict to each other for only one of them is actually given in any measurement.

Thus quantum mechanics introduces a model, which can be interpreted also as a new generalized model of duality and opposition to be united as complementarity. The core is the two fundamental, dual and opposite elements featuring a series of doctrines in philosophy to be considered both as identical as complementary, i.e. only one of them can be actually given.

The paper introduces a simple set-theory model of that philosophically generalized complementarity uniting duality and oppositeness in thus. Furthermore the same model is used to interpret the concept of incompleteness both in the foundation of mathematics (Gödel) and quantum mechanics (Einstein). One can demonstrate that completeness and incompleteness can be reconciled in infinity by that complementarity as two both dual and opposite fundamentals. Philosophical corollaries are deduced as to the totality, too.

Furthermore, the same set-theory model is utilized to explain the relation between language and reality as "image and simile" by means of representation and metaphor.

"Choice":

The role of the concept of choice and thus of information is also discussed being formalized in the model by dint of the axiom of choice. Both serve to describe quantitatively the relation of an inseparable whole such as a "much" and its well-ordered image as a "many". The theorems of the absence of "hidden variables" in quantum mechanics (John von Neumann 1932; Simon Kochen, Ernst Specker 1968) are thus generalized and included in the set-theory model.

At last, that set-theory model and its enumerated interpretations are utilized to describe the philosophical concept of complementarity in general as a kind of relation between duality and oppositeness in a relevant way consistent with vast areas of scientific knowledge.

The argument of Einstein, Podolsky, and Rosen:

Einstein, Podolsky and Rosen's argument (1935) can be considered as another interpretation of the famous Gödel incompleteness theorems (1931) in terms of quantum mechanics. The close friendship between the Princeton refugees Einstein and Gödel might address that fact. However the outlines of a common set-theory structure interpretable in both ways are much more essential concerning the incompleteness of infinity:

An arbitrary infinite countable set "A" and another set "B" so that their intersection is empty are given. One constitutes their union "C", which will be an infinite set whatever B is. Utilizing the axiom of choice, a one-to-one mapping "f " exists. One designates the image of B into A through f by "B(f)" so that B(f) is a true subset of A. If the axiom of choice holds, there is always an internal and equivalent image as B(f) for any external set as B. Thus, if one accepts that B(f) coincides with B, whether an element b of B belongs or not to A is an undecidable problem as far as b(f) coincides with b. However if the axiom of choice is not valid, one cannot guarantee that f exists and should display how a constructive analog of "f" can be built. If one shows how f to be constructed at least in one case, this will be a constructive proof of undecidablity as what Gödel's is. The same consideration is valid also as to its interpretation in terms of duality and opposition: Indeed any b and its image b(f) cam be considered both as dual and opposite and thus as complementary. If there is any duality and opposition (complementarity), they can be modeled as above involving infinity, some procedure of choice (either constructive or by the axiom of choice) and thus information.

The argument of Einstein, Podolsky, and Rosen interprets the same structure: There is an initial quantum system Q, which is divided to two other systems P and S moving with some relative speed to each other in space-time. For Q, P, and S are quantum systems and they are represented by three infinite-dimensional Hilbert spaces, the EPR argument can be bared to the following set-theory core: There is an initial infinity Q, which is divided into two infinities P and S, each of which suggests an external viewpoint to the other. So each of the two pairs (P, S) and (S, P) models the above structure and can serve to demonstrate the "incompleteness of quantum mechanics", which produces that description of reality. However the cause is the paradoxical properties of infinity rather than the description of quantum mechanics once forced to introduce infinity in itself. P and S can be interpreted partly both as dual and opposite and thus complementary, partly as entangled. Consequently, entanglement can be generalized even to a philosophical property implied by a form of "reconciliation" of duality and opposition by restricting the degrees of freedom both of P and S (i.e. both dual and opposite fundamental elements).

Entanglement:

The concept of entanglement is coined by the theory of quantum information to designate that special correlation of two or more quantum entities. Furthermore, it means an exactly defined mathematical structure grounded on Hilbert spaces and underlying all phenomena of entanglement studied by quantum mechanics.That same structure can be utilized for a mathematical model of metaphor as a special kind

correlation between the meanings and senses of two or more words. The philosophical core of the model can be described so. Metaphor restricts the meaning of a term by the meaning of another term in a probabilistic, loose way calling for interpretation.

The introduction of that underlying mathematical structure allows of establishing unambiguous correspondence between metaphor and entanglement in an absolutely exact, mathematical way, after which measurement in quantum mechanics corresponds to interpretation in language: This determines some interpretations of a given metaphor as more probable, but no one can be excluded.

The term utilized as metaphor restricts the area of meaning of its object to a small true subset of it. That set can ground the essential features, properties or relations of the object of the metaphor pioneering the scientific or even formal definition of the term serving as the object of the metaphor at issue. Thus some metaphor founds any scientific notion therefore "erasing" the grounding metaphor and the rest interpretations except one of them. The corresponding phenomena in quantum information is the process of de-coherence, after which the interacted object is cut off from its environment just as a rigorously defined notion is cut off from its context to designate one and the same in any context.

The opposed process can be observed both by the theory of metaphor and that of quantum information: A notion begins to lose its clear outlines coined in everyday speech and media accumulating new and new interpretations and uses. A quantum entity analogically starts to lose the measured values of the quantities as if dissolving in the common and inseparable whole of the universe. The suggested mathematical structure describes equally well both processes representing its interpretations.

The outlined approach allows a common philosophical viewpoint to the physical world, language and some mathematical structures therefore calling for the universe to be understood as a joint physical, linguistic and mathematical *universum*, in which physical motion and metaphor are one and the same rather than only similar in a sense.

The same viewpoint is consistent with the pair of duality and opposition as complementarity and entanglement generalized in a philosophical way as above. Thus metaphor can be considered ontologically in the same terms as a way to be represented the complicated relation between language and reality as duality and opposition. Furthermore language and reality unified by metaphor can be deduced from duality and opposition:

Reality and language:

Reality as if is doubled in relation to language: The one counterpart of reality is within the language as the representation of the other counterpart of reality being outside the language and existing by itself. Both representation and metaphor are called to support the correspondence between the two twins as an "image and simile".

The mechanism of that correspondence and its formal conditions are investigated by the construction already described above in detail:

Language is reduced to an infinite countable set (A) of its units of meaning, either words or propositions, or whatever others. It includes all possible meanings, which can be ever expressed in the language rather than the existing till now, which would always a finite set. The external twin of reality is introduced by another set (B) such that its intersection with the above set of language to be empty.

If the axiom of choice does not hold, the relation between the sets B(f) and B cannot be defined rigorously as an exact representation but rather as some simile and the vehicle between the two twins of reality can be only metaphor.

Furthermore the metaphor can be anyway defined to a set of one-to-one representations of the only similar external twin into a set of internal "twins", each of which is a different interpretation of the external "twin" so that a different metaphor is generated in each case. The representation seems to be vague,

defocussed, after which the image is bifurcate and necessary described by some metaphors within the language.

Consequently, reality is in an indefinite, bifurcate position to language according to the choice formalized in the axiom of choice. If that choice is granted, the language generates an exact image of reality in itself; if not, only some simile can exist expressible within it only by metaphors. Thus the "image and simile" of a metaphor is another way both complementarity (duality and opposition) and entanglement in language phenomena.

"Phenomenon":

The fundamental concept of phenomenology, that of phenomenon can be interpreted as an original and initial invariance in relation to the fundamental opposition of classical philosophy, that of subject and object. Consequently phenomenon can be thought as an initial structure or "eidos" of any entity, which is as single as plural. In particular, its plurality requires at least doubling, just which classical philosophy designates and studies as that opposition of subject and object.

Furthermore that concept of phenomenon correlates with that of the totality generating and/ or generated by a counterpart in terms of the totality. That "plural singularity" of the phenomenon can be directly deduced from the definitively necessary properties of the totality: any externality of the totality should be within it just being total. Thus the totality generates infinity in itself by itself and can be thought as equivalent to infinity in a restricted sense.

An "observer of the totality":

The observer can be involved as the correlative counterpart of the totality: An observer opposed to the totality externalizes an internal part outside. Thus the phenomena of an observer and the totality turn out to converge to each other or to be one and the same. In other words, the phenomenon of an observer includes as the necessity singularity of the solipsistic Self, which (or "who") is the same as that of the totality.

Furthermore, observation can be thought as that primary and initial action underlain by the phenomenon of an observer as above. That action of observation consists in the externalization of the solipsistic Self outside as some external reality. It is both a zero action and the singularity of the phenomenon of action.

As a zero action, it serves as a reference frame, in which any other action can be situated and thus its phenomenon can be yielded. The essence of that zero action consists in the totality to be ordered in a "zero" way so that to be the same or to remain the same after being ordered as reality. That condition constitutes observation as that, to which the totality is invariant in the two hypostases: (1) a primary and initial "chaos" unorderable in principle and (2) the reality, which is already somehow well-ordered within it and by itself.

As the singularity of the phenomenon of action, the "action" of observation is the common ground of all actions and therefore making them both possible and juxtaposable. Thus it can be "bracketed" so that as if only reality and real actions take place and so grounding the natural attitude to the world, commonly shared by people.

Furthermore, observation as if generating reality should be discussed in terms of ordering. The phenomenon of ordering is choice: Ordering represents a primary choice between the ordered and (the) unordered i.e. between at least two alternatives, among which only one should be chosen as ordering rejecting all the rest. Consequently ordering is equivalently representable as a series of choices or even as a single choice in a philosophical sense: the choice to be chosen the choice itself and therefore choosing ordering and reality. However reality hides that choice, which generates it. The mechanism for the choice underlying reality to be hidden can be found in observation as follows:

The observation itself is invariant to the choice. Observation is just the zero action: that action, which is "not" yet an action, but a reference frame or benchmark, to which any other, "real" action can be constituted as adequate to reality somehow already existing "in advance". Thus observation needs only some pure existence to remain from the choice after it is invariant to that choice and does not need it properly but only

the abstract, “pure” existence (of it or at all). After the choice turns out to be hidden, only the result of the choice, i.e. reality is only what can be endowed with that unconditional existence. The phenomenon of observation as a “zero action” is what can constitute reality as somehow already ordered ostensibly by itself and thus ground the natural attitude to the world as properly natural hiding all process of the phenomenological genesis of it.

Thus duality and opposition can ground a series of basic concepts of philosophical phenomenology.

The self-grounding of quantum mechanics vs “hidden variables”

Quantum mechanics offers a model of self-grounding by means of complementarity, duality and opposition. That model has the advantage to be used by nature for the self-foundation of the physical world by itself on quantum ground. Furthermore, the ability of the self-grounding of that quantum model is the sufficient condition for it to be what takes place into the physical world.

The essence of that model is complementarity at all as well as that between the grounding and the grounded in particular, and thus their symmetry and identity. The strategy of quantum grounding is as follows:

What is to be founded is divided into two parts called complementary to each other. This means that both cannot be given together or simultaneously. They can be also designated as incommeasurable to each other. Furthermore, the hidden part is interpreted as what founds, and the demonstrated part as what is founded by the former, and thus an epistemologist can interpreted them as correspondingly meta-theory and theory in our cognition.

Those two parts is necessary to be identical. If one accepts that they are identical in any relation, their mutual exchange as the grounding and grounded is just the same as the initial situation. In other words, those two parts should be considered as invariant to the relation of grounding: Any of both parts can be any of both members of this relation. However the relation is not accepted as symmetric and it only allows symmetry as a particular case realized by the two parts at issue. After that is the case any of both parts self-founds itself by the hidden aid of its identical twin. Even more, both self-foundations are as indistinguishable as both twin parts are such. That special case of the relation of grounding can be called self-grounding.

The theorems about the absence of hidden variables in quantum mechanics (Neumann 1932; Kochen and Specker 1968) exclude any well-ordering of any quantum state before measurement for any well-ordering implies a hidden variable equivalent to it. However, the corresponding quantum state after measurement is necessarily well-ordered for it is adequately represented by a statistical ensemble of measured results well-ordered in time necessarily. If the quantum state before measurement and the corresponding one after measurement are postulated to be mapped one-to-one (for the cognoscibility of the quantum world), this requires the well-ordering theorem equivalent to the axiom of choice. To be reconciled any quantum state before measurement being coherent and the corresponding state after measurement being a statistical ensemble, an initial invariance to the axiom of choice is necessary. It can be designated as quantum invariance.

That invariance is founded on a general logical or ontological structure described above. That structure allows of the quantum invariance to be interpreted directly in terms of the grounding and grounded as follows: The coherent quantum state before measurement is for the grounding, the statistical ensemble after measurement is for the grounded, and quantum measurement is for that relation of grounding, which is self-grounding. Thus the invariance to the axiom of choice defined as above means for the grounding and grounded to be equated. Furthermore, that invariance to the axiom of choice implies at the same time another invariance between any relations of grounding, because of which the equation of the grounding and grounded is equivalent to any other relation between them and thus it represents exhaustively all the class of the relations of grounding. In other words, that strategy of grounding consists in the following: the conditions, under which the relation of grounding is equivalent to that of self-grounding to be investigated, to be shown that these conditions are equivalent to that equivalence of the relation of grounding and that of self-

grounding. Consequently the ultimate statement is: The statement that any relation of grounding is equivalent to a relation of self-grounding is a tautology.

The so-called invariance to the axiom of choice restricts that result so: It is a statement of pure existence once the axiom of choice is involved. However, it does not depend on it directly as just the invariance to it is required.

Interpretations and conclusions:

Any consistent grounding is self-justified for there is a self-grounding equivalent to it. However this is pure existence and there is no constructive procedure in general for that equivalent self-grounding to be expressed explicitly. Any entity is self-grounded in itself by itself. This can be explicated by doubling the entity by a complementary and identical counterpart and following the pathway described above.

Remote interpretations can address Husserl's phenomenology or Heidegger's doctrine as well as any philosophical criticism (e.g. those of Wittgenstein or of positivism) to the problem of grounding and self-grounding as redundant or pointless.

An open problem is the relation to infinity: One can state that any finite entity cannot be founded in the above way as it can be always enumerated, thus well-ordered, and the strategy of a complementary grounding twin is inapplicable.

1. Quantum complementarity unifies duality and opposition in a consistent way underlying the physical world.

2. It can serve to elucidate a series of philosophical problems, a few of which are enumerated above: grounding and self-grounding, language and reality, metaphor and representation, incompleteness, infinity as both complete and incomplete, choice and information, the totality and an observer, the idea of philosophical phenomenology.

References:

Gödel, Kurt 1931, "Über formal unentscheidbare Sätze der Principia mathematica und verwandter Systeme I," *Monatshefte der Mathematik und Physik*. **38** (1): 173-198.

Einstein, Albert, Boris **Podolsky**, and Nathan Rosen 1935 "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" *Physical Review*. 1935, **47** (10): 777-780.

Kochen, Simon and Ernst **Specker** 1968. "The problem of hidden variables in quantum mechanics," *Journal of Mathematics and Mechanics*. 17 (1): 59-87.

Neumann, Johan von 1932. *Mathematische Grundlagen der Quantenmechanik*, Berlin: Verlag von Julius Springer.