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Estimation of State Variable Absolute Values Maxima for Feedback Linearizable Dynamical Systems

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Abstract

The problem of stabilizing the origin is considered for nonlinear dynamical systems, taking into account constraints on absolute values of the state variables. For a stabilizing feedback constructed using the state feedback linearization approach in combination with subsequent use of linear modal control, guaranteed estimates of the maximum absolute values of the closed-loop system's state variables are obtained. As an illustrative example third-order chain of integrators is used.

1 Introduction and problem statement

Control of dynamical systems subject to various state, output and input constraints remains a popular research topic during the last decades. One of the effective analytical approaches to synthesizing stabilizing feedback controls which allows to account for constraints explicitly is integrator backstepping [6]. The backstepping considerations are based on a step-by-step analysis of the interrelations of the equations of a dynamical system. A special positive definite function of the system's state is constructed step-by-step and at the last stage it becomes the Lyapunov function of the closed-loop system under proper choice of the control input [6]. Combined with using the barrier Lyapunov functions [7] backstepping proved to be a powerful tool to design stabilizing feedbacks under state and output constraints [1–4, 7–9, 11, 12].

In [2] it is shown that applying the integrator backstepping approach when constructing stabilizing state feedback for a chain of integrators of arbitrary order results in a linear modal control. Then, it has been proved that fulfillment of the specified constraints on the state variables during transient processes can be ensured by selecting roots of the characteristic polynomial of the closed-loop system to satisfy the nonlinear inequalities obtained in [2, 3] based on a logarithmic barrier Lyapunov function.

This paper is focused on further refinement of the estimates for the state variables of a closed-loop control system obtained in [2, 3]. We consider nonlinear dynamical systems that

can be written as

$$\begin{aligned} \dot{x}_1 &= x_2, \\ &\vdots \\ \dot{x}_{n-1} &= x_n, \\ \dot{x}_n &= f(x) + g(x)u, \end{aligned} \tag{1}$$

where $x = (x_1, \ldots, x_n)^{\mathrm{T}} \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the control input, $f(x)$ and $g(x)$ are locally Lipschitz, $g(x) \neq 0$ for all $x \in \mathbb{R}^n$.

The control problem in question is to find a state feedback control law $u = u(x)$ such that for any given positive constants $M_j > 0$, $j \in \overline{1,n}$, and initial values $x(0)$, $|x_j(0)| < M_j$, $j \in \overline{1,n}$, the condition $\|x(t)\| \to 0$ as $t \to +\infty$ holds, with the constraints

$$|x_j(t)| < M_j,\ j \in \overline{1,n},\ t \geq 0, \tag{2}$$

being satisfied. Inequalities (2) are supposed to be held by providing relevant guaranteed estimates of the maximum absolute values of the state variables of the closed-loop system as time functions. Here, $\|\cdot\|$ stands for the Euclidean norm on $\mathbb{R}^n$.

2 Some preliminaries

For any given positive constants $\bar{c}_j > 0$, $j \in \overline{1,n}$, which later appear to be absolute values of the closed-loop system's characteristic polynomial prescribed roots, consider the following relations [2, 3]:

$$\begin{aligned} &z_1 = x_1,\ \alpha_1(x_1) = -\bar{c}_1 z_1, \\ &z_j = x_j - \alpha_{j-1}(x_1, \ldots, x_{j-1}),\ \alpha_j(x_1, \ldots, x_j) = \sum_{s=1}^{j-1} \frac{\partial \alpha_{j-1}}{\partial x_s} x_{s+1} - \bar{c}_j z_j = -\sum_{i=1}^{j} \kappa_{ij} x_i, \\ &\kappa_{jj} = \sum_{s=1}^{j} \bar{c}_s,\ \kappa_{j-s+1,j} = \sum_{1 \leq i_1 < i_2 < \ldots < i_s \leq j} \bar{c}_{i_1} \bar{c}_{i_2} \ldots \bar{c}_{i_s},\ s = \overline{2, j-1},\ \kappa_{1j} = \prod_{s=1}^{j} \bar{c}_s,\ j \in \overline{2,n}. \end{aligned} \tag{3}$$

Note that the variables z_j, $j \in \overline{1,n}$, defined by (3) are introduced when applying the integrator backstepping [6] considerations in [2, 3] to stabilize a dynamical system of the form (1) in view of the state constraints (2).

Next, the control law

$$u = \frac{1}{g(x)}\Big(-f(x) + \alpha_n(x)\Big) = \frac{1}{g(x)}\Big(-f(x) - \sum_{i=1}^{n} \kappa_{in} x_i\Big) \tag{4}$$

results in the dynamical system (1) having the form

$$\dot{x}_1 = x_2,\ \ldots,\ \dot{x}_{n-1} = x_n,\ \dot{x}_n = -\sum_{i=1}^{n} \kappa_{in} x_i. \tag{5}$$

Let us notice that the state feedback (4) obtained in [2, 3] as a result of using integrator backstepping to solve the control problem in question coincides with conventional state feedback

linearization control [5] law applicable to dynamical systems of the form (1). In view of (3) the roots of the system (5) characteristic polynomial are $-\bar{c}_j < 0$, $j \in \overline{1,n}$. Hence, the origin $x = 0$ is globally asymptotically stable for any positive constants $\bar{c}_j > 0$, $j \in \overline{1,n}$, appearing in the formulae (3).

It is well known that for a linear dynamical system of the form (5) it is not obvious how to explicitly guarantee prescribed bounds (2) on the state vector components due to possible peaking phenomenon [10]. We rewrite the closed-loop system (5) in the new backstepping connected state variables $z = (z_1, \ldots, z_n)^{\mathrm{T}} \in \mathbb{R}^n$ defined by (3) as below

$$\begin{aligned} \dot{z}_1 &= z_2 - \bar{c}_1 z_1, \\ \dot{z}_2 &= z_3 - \bar{c}_2 z_2, \\ &\vdots \\ \dot{z}_{n-1} &= z_n - \bar{c}_{n-1} z_{n-1}, \\ \dot{z}_n &= -\bar{c}_n z_n. \end{aligned} \tag{6}$$

In [2,3] it is shown that representing linear dynamics (5) as (6) allows to obtain explicit estimates of the z state variables absolute value maxima for all $t \geq 0$ using a logarithmic barrier Lyapunov function [7]. Then, taking into account the equalities (3) that define a linear non-degenerate change of variables $z = \Phi(x)$, with the origin $x = 0$ being translated to the origin $z = 0$, the relevant estimates (2) can be obtained [2,3].

In particular, under arbitrary initial conditions of the form

$$x_1(0) \neq 0, \; x_j(0) = 0, \; j \in \overline{2,n}, \tag{7}$$

the following guaranteed absolute value estimates are derived in [3] for the system (5) state variables time behavior:

$$\begin{aligned} &|x_1(t)| < |x_1(0)| = \hat{M}_1, \; |x_2(t)| < \bar{c}_1|x_1(0)| = \hat{M}_2, \\ &|x_3(t)| < \bar{c}_2\bar{c}_1|x_1(0)| = \hat{M}_3, \; |x_4(t)| < (\bar{c}_3 + \bar{c}_1)\bar{c}_2\bar{c}_1|x_1(0)| = \hat{M}_4, \\ &|x_j(t)| < |x_1(0)| \prod_{s=1}^{j-1} \bar{c}_s + \sum_{s=1}^{j-2} \kappa_{j-s-1,j-2}\hat{M}_{j-s} = \hat{M}_j, \; j \in \overline{5,n}, \; t > 0. \end{aligned} \tag{8}$$

3 Main results

In this paper, we refine the estimates (8) obtained in [3] for solutions of the system (1) in closed-loop form with the state feedback (4). The following theorem can be proved.

Theorem 1. *Let us fix arbitrary positive constants $\bar{c}_j > 0$, $j \in \overline{1,n}$. Then, for solutions of the system (1) with control (4) and initial values of the form (7) the condition $\|x(t)\| \to 0$ as $t \to +\infty$ is satisfied and the inequalities*

$$\begin{aligned} &|x_1(t)| < |x_1(0)|, \\ &|x_2(t)| < \min\{\bar{c}_1, \bar{c}_2, \ldots, \bar{c}_{n-1}, \bar{c}_n\}|x_1(0)|, \\ &|x_s(t)| < \min\big\{\bar{c}_{i_1}\bar{c}_{i_2}\ldots\bar{c}_{i_{s-1}}, \; i_1 > i_2 > \ldots > i_{s-1}, \\ &i_1 = \overline{s-1,n}, \; i_2 = \overline{s-2,n-1}, \; \ldots, \; i_{s-1} = \overline{1,n-s+2}\big\}|x_1(0)|, \; s \in \overline{3,n}, \; t > 0, \end{aligned} \tag{9}$$

hold.

The proof of Theorem 1 is outlined below in the Appendix A section.

Let us notice that in view of the estimates (9) the given constraints (2) can always be fulfilled by a proper choice of the system's characteristic polynomial prescribed roots $-\bar{c}_j < 0$, $j \in \overline{1,n}$. Due to a factorial structure of the inequalities (9) for each fixed index value $j = s$, $s \in \overline{2,n}$, the constraint $|x_s(t)| < M_s$, $t > 0$, can be readily fulfilled e.g. by the $\bar{c}_{s-1}$ value selection to satisfy the condition

$$\bar{c}_1\bar{c}_2 \dots \bar{c}_{s-1}|x_1(0)| < M_s. \tag{10}$$

4 Illustrative example

As an example, consider an arbitrary mechanical system whose dynamics with respect to one of the generalized coordinates, e.g. as a result of feedback linearization, can be represented as a third-order chain of integrators

$$\dot{x}_1 = x_2,\ \dot{x}_2 = x_3,\ \dot{x}_3 = u, \tag{11}$$

where $x = (x_1, x_2, x_3)^{\mathrm{T}} \in \mathbb{R}^3$ is the state vector of the generalized coordinate, velocity and acceleration, respectively; $u \in \mathbb{R}$ is the control input.

Let us present a solution to the constrained regulation problem: $x_1(t) \to 0$ as $t \to +\infty$ using state feedback $u = u(x)$, taking into account the constraints for all $t \geq 0$ on the values of the generalized coordinate $|x_1(t)| < M_1$, velocity $|x_2(t)| < M_2$ and acceleration $|x_3(t)| < M_3$. We will consider arbitrary initial values $x(0)$ of the form (7) in the case of $n = 3$.

As a solution to the regulation problem in question the linear state feedback can be proposed

$$u = -\sum_{i=1}^{3} \kappa_{i3} x_i, \tag{12}$$

where the gain coefficients $\kappa_{i3} > 0$, $i = \overline{1,3}$, satisfy the formulae (3). Here, the absolute values $\bar{c}_j > 0$, $j \in \overline{1,3}$, of the characteristic polynomial roots are chosen to fulfill the inequalities (10) which take the form

$$\bar{c}_1|x_1(0)| < M_2,\ \bar{c}_1\bar{c}_2|x_1(0)| < M_3.$$

It should be noted that for the state feedback (12) considered on the solutions of the closed-loop system (11) with initial values $x(0)$ of the form (7) due to the relations (6), (A.2), (A.3) and (A.5) the following estimate holds:

$$|u(t)| = |\dot{x}_3(t)| = |z_1^{(3)}(t)| = |\ddot{z}_2(t) - \bar{c}_1\ddot{z}_1(t)| < \bar{c}_1\bar{c}_2\bar{c}_3|x_1(0)| = u(0) = \hat{u},\ t > 0.$$

The simulation results of the system (11) with control (12) are shown in Figs. 1–4. The following initial values $x(0) = (10, 0, 0)^{\mathrm{T}}$ of the state variables and control parameters $\kappa_{13} = 0.3$, $\kappa_{23} = 1.4$, $\kappa_{33} = 2.1$, $\bar{c}_1 = 1$, $\bar{c}_2 = 0.6$, $\bar{c}_3 = 0.5$ were used. The resultant state and control variables boundary values are $M_1 = 10$, $M_2 = 5$, $M_3 = 3$ and $\hat{u} = 3$.

5 Conclusion

The problem of stabilizing the origin is solved for state feedback linearizable nonlinear dynamical systems under full state constraints. The constraints are satisfied by providing relevant guaranteed estimates of the maximum absolute values of the state variables of the closed-loop system. A special set of the system's state initial values is employed. The future research can be focused on extending the obtained estimates for a wider class of initial conditions.

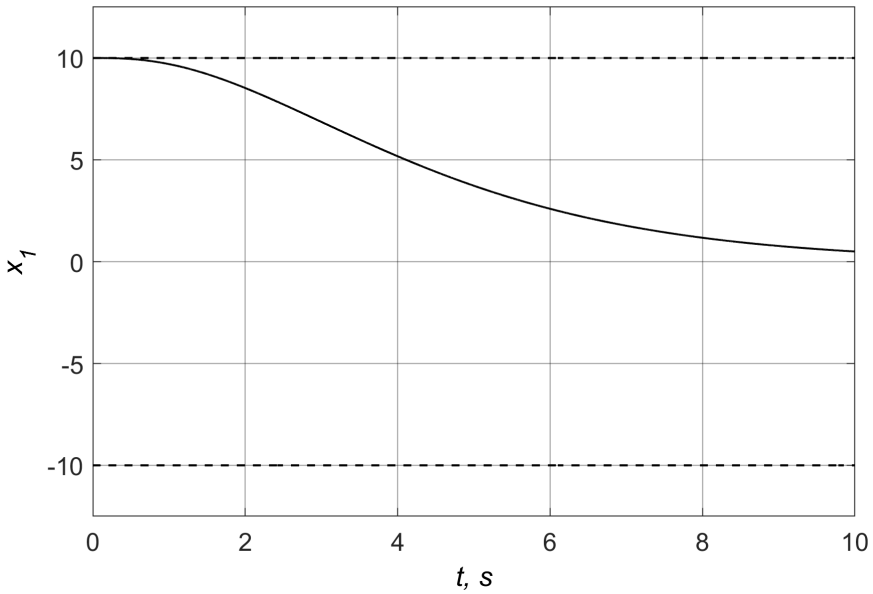

Figure 1: Generalized coordinate $x_1(t)$ time behavior (solid line) and the guaranteed boundary values $\pm M_1$ (dashed lines).

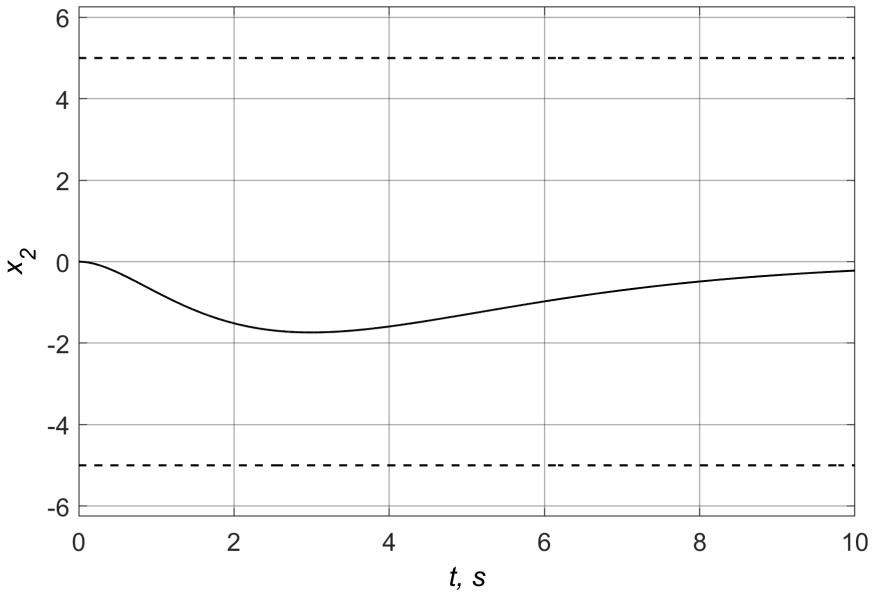

Figure 2: Generalized velocity $x_2(t)$ time behavior (solid line) and the guaranteed boundary values $\pm M_2$ (dashed lines).

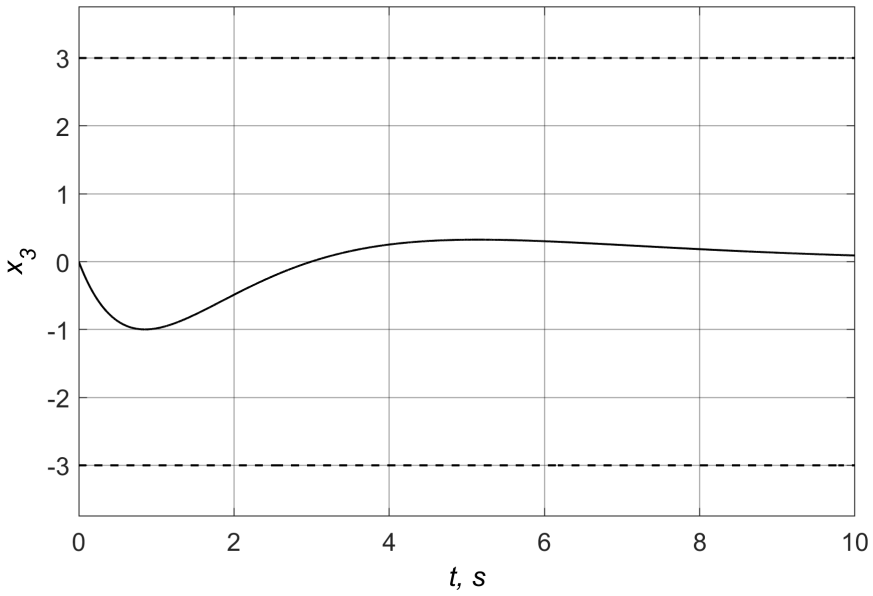

Figure 3: Generalized acceleration $x_3(t)$ time behavior (solid line) and the guaranteed boundary values $\pm M_3$ (dashed lines).

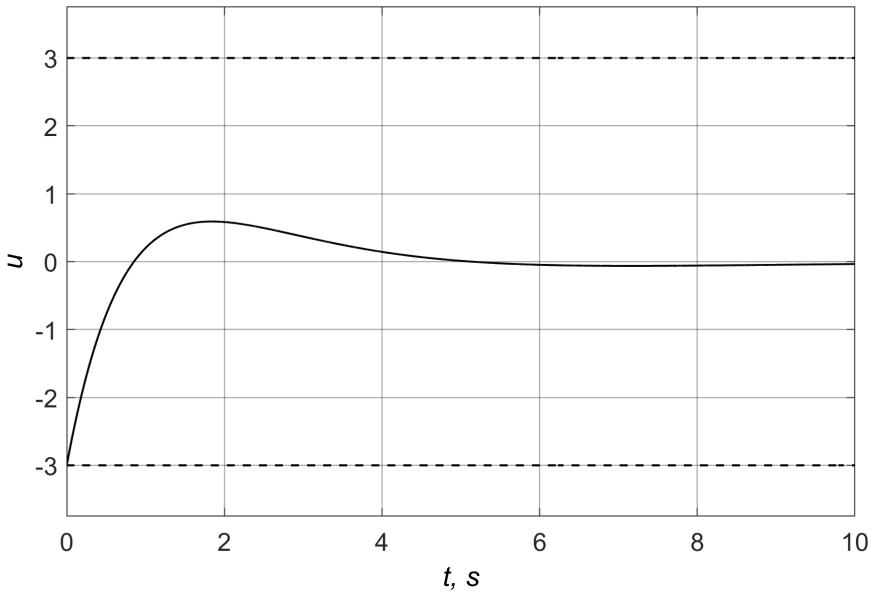

Figure 4: Control $u(t)$ as time function (solid line) and the guaranteed boundary values $\pm\hat{u}$ (dashed lines).

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A Proof of Theorem 1 (sketch)

Due to the formulae (3) and (7) the corresponding initial conditions for the system (6) will be written as

$$z_1(0) = x_1(0),\ z_j(0) = x_1(0)\prod_{s=1}^{j-1}\bar{c}_s,\ j \in \overline{2,n}. \tag{A.1}$$

In [3] it is shown that absolute values of the state variables $z_j(t)$, $j \in \overline{1,n}$, of the system (6) with arbitrary positive coefficients $\bar{c}_j > 0$, $j \in \overline{1,n}$, and the initial conditions (A.1) are decreasing functions of the independent variable $t \in (0,+\infty)$, and the following equalities hold:

$$z_j^{(s)}(0) = 0,\ s \in \overline{1,n-j},\ z_j^{(n-j+1)}(0) = -\bar{c}_n z_n(0) = -x_1(0)\prod_{s=1}^{n}\bar{c}_s \neq 0,\ j \in \overline{1,n-1}. \tag{A.2}$$

For the purpose of further use in constructing estimates for the state variables of system (5), we will estimate maxima of absolute values of the derivatives $z_j^{(s)}(t)$, $j \in \overline{1,n-1}$, $s \in \overline{1,n-j}$, for all $t \geq 0$. Taking into account equations of the system (6), the conditions for zeroing the derivatives $z_j^{(s+1)}(t)$ to find potential extrema points $t = t_{js} > 0$ for the corresponding functions $z_j^{(s)}(t)$ will be written as

$$z_j^{(s+1)}(t_{js}) = z_{j+1}^{(s)}(t_{js}) - \bar{c}_j z_j^{(s)}(t_{js}) = 0,\ j \in \overline{1,n-1},\ s \geq 1.$$

Hence, the candidates for extremal values of the derivatives $z_j^{(s)}(t_{js})$ satisfy the following equalities:

$$z_j^{(s)}(t_{js}) = z_{j+1}^{(s)}(t_{js})/\bar{c}_j,\ j \in \overline{1,n-1},\ s \geq 1. \tag{A.3}$$

It should be noted that extreme values of the derivatives $z_j^{(s)}(t)$, $j \in \overline{1,n-1}$, $s \in \overline{1,n-j}$, as functions of the independent variable $t > 0$, exist, since there is an alternation of intervals of increase and decrease in their absolute values for $t \in (0,+\infty)$. Indeed, from (A.2) it follows that there exists a value of the independent variable $t = t_1$ such that for all $t \in (0,t_1)$ the derivatives $z_j^{(s)}(t)$, $j \in \overline{1,n}$, $s \in \overline{1,n-j+1}$, are nonzero and have the same sign opposite to the sign of $x_1(0)$. Finally, due to global asymptotic stability of the equilibrium position $z = 0$ of the system (6) absolute values of the derivatives $z_j^{(s)}(t)$, $j \in \overline{1,n-1}$, $s \in \overline{1,n-j}$, must decrease and tend to zero. Without loss of generality, assume that $t = t_{js} > 0$ represent the points of global maxima of absolute values of the corresponding derivatives $z_j^{(s)}(t)$, $j \in \overline{1,n-1}$, $s \in \overline{1,n-j}$, for all $t \geq 0$.

Let us first consider the case, when $s = 1$. Due to the strictly monotonic time behavior of the state variables $z_j(t)$, $j \in \overline{1,n}$, of the system (6) in view of the equalities (A.2) the following relations hold for all $t > 0$

$$|\dot{z}_j(t)| = |z_{j+1}(t) - \bar{c}_j z_j(t)| < \bar{c}_j|z_j(t)| < \bar{c}_j|z_j(0)| = |z_{j+1}(0)|,\ j \in \overline{1,n-1}.$$

Then, as a result of iterative application of the formulae (A.3) it follows that

$$
\begin{aligned}
&|\dot{z}_j(t)| < \min\{|z_{j+1}(0)|, |z_{j+2}(0)|/\bar{c}_j, |z_{j+3}(0)|/(\bar{c}_j\bar{c}_{j+1}),\\
&\qquad \ldots, |z_n(0)|/(\bar{c}_j\bar{c}_{j+1}\ldots\bar{c}_{n-2}), |\dot{z}_n(0)|/(\bar{c}_j\bar{c}_{j+1}\ldots\bar{c}_{n-1})\},\ j \in \overline{1,n-2},\\
&|\dot{z}_{n-1}(t)| < \min\{|z_n(0)|, |\dot{z}_n(0)|/\bar{c}_{n-1}\},\ |\dot{z}_n(t)| = |-\bar{c}_n z_n(t)| < |\dot{z}_n(0)| = \bar{c}_n|z_n(0)|,\ t > 0.
\end{aligned}
$$

Thus, taking into account the inequalities (A.1), one gets the estimates

$$
\begin{aligned}
&|\dot{z}_1(t)| < \min\{\bar{c}_1, \bar{c}_2, \ldots, \bar{c}_{n-1}, \bar{c}_n\}|x_1(0)|,\\
&|\dot{z}_j(t)| < \min\{\bar{c}_j, \bar{c}_{j+1}, \ldots, \bar{c}_{n-1}, \bar{c}_n\}\prod_{s=1}^{j-1}\bar{c}_s|x_1(0)|,\ j \in \overline{2,n-1},\\
&|\dot{z}_n(t)| < \bar{c}_n\prod_{s=1}^{n-1}\bar{c}_s|x_1(0)|,\ t > 0.
\end{aligned}
\tag{A.4}
$$

Further, for the case of $s = 2$ due to the equalities (A.2), (A.3) and the same sign of the derivatives $\dot{z}_j(t)$, $j \in \overline{1,n}$, for all $t > 0$, which holds due to the strictly monotonic time behavior of the system (6) state variables, we can write

$$
\begin{aligned}
&|\ddot{z}_j(t)| = |\dot{z}_{j+1}(t) - \bar{c}_j\dot{z}_j(t)| < |\dot{z}_{j+1}(t_{j+1,1})|,\ j \in \overline{1,n-2},\\
&|\ddot{z}_{n-1}(t)| = |\dot{z}_n(t) - \bar{c}_{n-1}\dot{z}_{n-1}(t)| < |\dot{z}_n(0)|,\ t > 0.
\end{aligned}
$$

Therefore, after iterative application of the formulae (A.3) one gets

$$
\begin{aligned}
&|\ddot{z}_j(t)| < \min\{|\dot{z}_{j+1}(t_{j+1,1})|, |\dot{z}_{j+2}(t_{j+2,1})|/\bar{c}_j, |\dot{z}_{j+3}(t_{j+3,1})|/(\bar{c}_j\bar{c}_{j+1}),\\
&\qquad \ldots, |\dot{z}_{n-1}(t_{n-1,1})|/(\bar{c}_j\bar{c}_{j+1}\ldots\bar{c}_{n-3}), |\dot{z}_n(0)|/(\bar{c}_j\bar{c}_{j+1}\ldots\bar{c}_{n-2}),\ j \in \overline{1,n-2},\\
&|\ddot{z}_{n-1}(t)| < |\dot{z}_n(0)|,\ |\ddot{z}_n(t)| < |\ddot{z}_n(0)| = \bar{c}_n^2|z_n(0)|,\ t > 0,
\end{aligned}
$$

and the inequalities (A.4) yield the estimates

$$
\begin{aligned}
&|\ddot{z}_1(t)| < \min\big\{\bar{c}_k\bar{c}_l, k > l,\ k = \overline{2,n},\ l = \overline{1,n-1}\big\}|x_1(0)|,\\
&|\ddot{z}_j(t)| < \min\big\{\bar{c}_k\bar{c}_l, k > l,\ k = \overline{j+1,n},\ l = \overline{j,n-1}\big\}\prod_{s=1}^{j-1}\bar{c}_s|x_1(0)|,\ j \in \overline{2,n-2},\\
&|\ddot{z}_{n-1}(t)| < \bar{c}_n\bar{c}_{n-1}\prod_{s=1}^{n-2}\bar{c}_s|x_1(0)|,\ |\ddot{z}_n(t)| < \bar{c}_n^2\prod_{s=1}^{n-1}\bar{c}_s|x_1(0)|,\ t > 0.
\end{aligned}
\tag{A.5}
$$

Furthermore, reasoning in a similar way one can show that the following relations hold for all $s \geq 3$:

$$
\begin{aligned}
&|z_1^{(s)}(t)| < \min\big\{\bar{c}_{i_1}\bar{c}_{i_2}\ldots\bar{c}_{i_s},\ i_1 > i_2 > \ldots > i_s,\\
&\qquad i_1 = \overline{s,n},\ i_2 = \overline{s-1,n-1},\ \ldots,\ i_s = \overline{1,n-s+1}\big\}|x_1(0)|,\ s \in \overline{3,n-1},\\
&|z_j^{(s)}(t)| < \min\big\{\bar{c}_{i_1}\bar{c}_{i_2}\ldots\bar{c}_{i_s},\ i_1 > i_2 > \ldots > i_s,\ i_1 = \overline{j+s-1,n},\ i_2 = \overline{j+s-2,n-1},\\
&\qquad \ldots,\ i_s = \overline{j,n-s+1}\big\}\prod_{s=1}^{j-1}\bar{c}_s|x_1(0)|,\ j \in \overline{2,n-s},\ s \in \overline{3,n-2},\ t > 0.
\end{aligned}
\tag{A.6}
$$

Thus, by virtue of the equalities $x_s = z_1^{(s-1)}$, $s \in \overline{1,n}$, from (A.4)–(A.6) it follows that absolute values of the system (1) state variables with the control law (4) in case of initial conditions (7) satisfy the estimates

$$
\begin{aligned}
&|x_1(t)| = |z_1(t)| < |x_1(0)|,\\
&|x_2(t)| = |\dot{z}_1(t)| < \min\{\bar{c}_1, \bar{c}_2, \ldots, \bar{c}_{n-1}, \bar{c}_n\}|x_1(0)|,\\
&|x_s(t)| = |z_1^{(s-1)}(t)| < \min\big\{\bar{c}_{i_1}\bar{c}_{i_2}\ldots\bar{c}_{i_{s-1}},\ i_1 > i_2 > \ldots > i_{s-1},\\
&i_1 = \overline{s-1,n},\ i_2 = \overline{s-2,n-1},\ \ldots,\ i_{s-1} = \overline{1,n-s+2}\big\}|x_1(0)|,\\
&s \in \overline{3,n},\ t > 0.
\end{aligned}
$$

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