Lebesgue Constants and Optimal Node Systems via Symbolic Computations

Short Paper

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Abstract

Polynomial interpolation is a classical method to approximate continuous functions by polynomials. To measure the correctness of the approximation, Lebesgue constants are introduced. For a given node system \( X^{(n+1)} = \{x_1 < \ldots < x_{n+1}\} (x_j \in [a,b]) \), the Lebesgue function \( \lambda_n(x) \) is the sum of the modulus of the Lagrange basis polynomials built on \( X^{(n+1)} \). The Lebesgue constant \( \Lambda_n \) assigned to the function \( \lambda_n(x) \) is its maximum over \([a,b]\). The Lebesgue constant bounds the interpolation error, i.e., the interpolation polynomial is at most \((1 + \Lambda_n)\) times worse than the best approximation. The minimum of the \( \Lambda_n \)'s for fixed \( n \) and interval \([a,b]\) is called the optimal Lebesgue constant \( \Lambda^*_n \). For specific interpolation node systems such as the equidistant system, numerical results for the Lebesgue constants \( \Lambda_n \) and their asymptotic behavior are known \([3, 7]\). However, to give explicit symbolic expression for the minimal Lebesgue constant \( \Lambda^*_n \) is computationally difficult. In this work, motivated by Rack \([5, 6]\), we are interested for expressing the minimal Lebesgue constants symbolically on \([-1, 1]\) and we are also looking for the characterization of the those node systems which realize the minimal Lebesgue constants. We exploited the equioscillation property of the Lebesgue function \([4]\) and used quantifier elimination and Groebner Basis as tools \([1, 2]\). Most of the computation is done in Mathematica \([8]\).

Acknowledgement. The research of the author was partially supported by the HSRF (OTKA), grant number K83219.

References