Title

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ABSTRACT

The implications of electro-osmosis with interfacial slip on electro-hydrodynamic transport in micro-channels having complex (yet symmetric) cross sectional shapes, by employing a generic semi-analytical approach, been presented in this poster. This study is motivated by the fact that in reality, the cross sectional shapes of microchannels may often deviate from circular or rectangular ones, primarily as an artifact of the underlying fabrication process. We also devise an approximate technique of flow rate prediction under these conditions, using a combined consideration of electroosmotic slip (under thin electrical double layer limits) and Navier slip conditions (originating out of confinement induced hydrophobic interactions) at the fluid-solid interface. We further assess the effectiveness of the approximate solutions in perspective of the exact solutions, as a parametric function of the relative thickness of the electrical double layer. We illustrate the underlying consequences through examples of elliptic, polygonal, point star-shaped and annular microchannel cross sectional shapes.

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INTRODUCTION

- Electrokinetic flow actuation mechanism utilizes favorable interfacial phenomena over small scales. This phenomenon is completely rely on the formation of EDL adjacent to the charged surface.
- The slipping hydrodynamics (may originate from hydrophobic interactions in a fluid-solid interface or formation of rarefied gas laeyr or a direct slippage because of characteristically high shear rates prevailing in the interfacial region) intrinsically coupled with interfacial electrohydrodynamics , make an apparent deviation from the classical no-slip boundary condition based conceptual paradigm.
- The interfacial slip velocity due to electrokinetic effects given by Helmholtz-Smoluchowski velocity. But the interfacial slip due to hydrophobic interaction , represented by Navier slip condition.
- Different cross-sectional shapes (Point-star, elliptic, annular, polygonal) of microchannel, may originated from natural artifacts of fabrication process or produced.

MATHEMATICAL FORMULATION

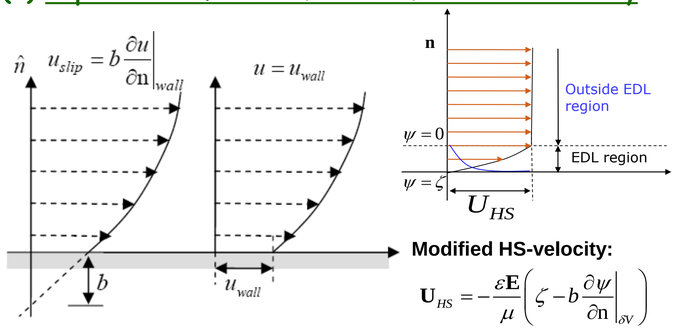
(A) Potential distribution:

Poisson equation	Ionic charge density	Boltzmann distribution
$\nabla^2\psi = -\frac{\rho_e}{\varepsilon}$	$\rho_e = ez\left(n^+ - n^-\right)$	$n^{\pm} = n_0 exp\left(\mp\frac{ez\psi}{k_B T}\right)$

Poisson-Boltzmann

$$\nabla^2\psi = \frac{2n_0 ez}{\varepsilon}\sinh\left(\frac{ez\psi}{k_B T}\right) \Rightarrow \psi(\mathrm{n}) = \frac{4k_B T}{ez}\tanh^{-1}\left[e^{-\kappa \mathrm{n}}\tanh\left(\frac{ez\zeta}{4k_B T}\right)\right]$$

(B) Slip and Modified Helmholtz-Smoluchowski velocity:



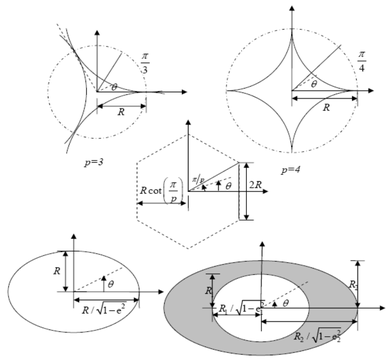
Modified HS-velocity:

$$\mathbf{U}_{HS} = -\frac{\varepsilon \mathbf{E}}{\mu}\left(\zeta - b\left.\frac{\partial \psi}{\partial \mathrm{n}}\right|_{\delta V}\right)$$

(C) Problem formulation:

Actual	Approximate
$\nabla \bullet \mathbf{u} = 0$	$0 = \mu\nabla^2\mathbf{u} + \rho_e\mathbf{E}$
$\mathbf{0} = -\nabla p + \mu\nabla^2\mathbf{u} + \mathbf{F}_{EK}$	$\mathbf{u} = b\left.\frac{\partial \mathbf{u}}{\partial \mathrm{n}}\right\|_{boundary}$ (Inside EDL)
$\mathbf{F}_{EK} = \rho_e\mathbf{E}$	$\mathbf{0} = -\nabla p + \mu\nabla^2\mathbf{u}_{app}$
$\mathbf{u} = b\left.\frac{\partial \mathbf{u}}{\partial \mathrm{n}}\right\|_{boundary}$	$\mathbf{u} = \mathbf{U}_{HS}$
Q	Q_{app}

(D) Different Geometries:



p-point star:

$$r = \frac{R}{p}\sqrt{\left(1+(p-1)^2+2(p-1)\cos(p\alpha)\right)}$$

$$\theta = \tan^{-1}\left(\frac{(p-1)\sin(\alpha)-\sin((p-1)\alpha)}{(p-1)\cos(\alpha)+\cos((p-1)\alpha)}\right)$$

$$for \quad 0 \le \alpha \le \frac{\pi}{p}$$

Polygonal:

$$r = R\cot\left(\frac{\pi}{p}\right)\sec(\theta); 0 \le \theta \le \frac{\pi}{p}$$

(E) Velocity:

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right]\left(u(r,\theta) - \frac{\varepsilon E}{\mu}\psi(r,\theta)\right) = \frac{1}{\mu}\frac{dp}{dz}$$

Momentum equation

$$u(r,\theta)\big|_{r=g(\theta)} = b(\nabla u \cdot \hat{\mathbf{n}})\big|_{r=g(\theta)}$$

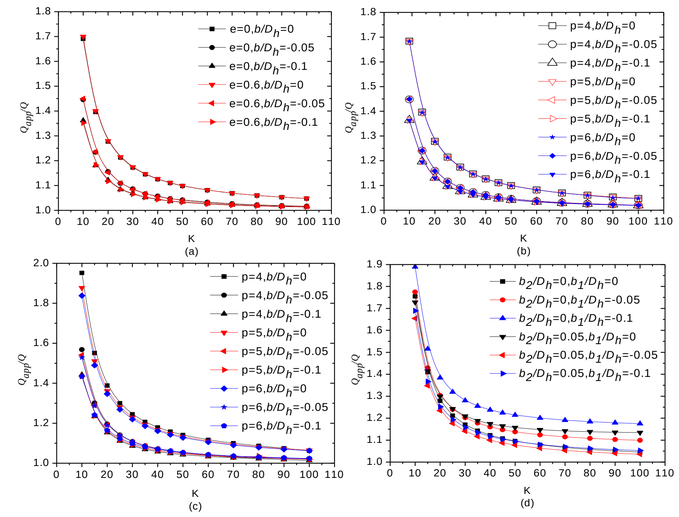
Boundary cond.

Non-dimensional variables:

$$\bar{r} = \frac{r}{D_h},\ \bar{g} = \frac{g}{D_h},\ \bar{b} = \frac{b}{D_h},\ \bar{u} = \frac{u}{U_P},\ \bar{\psi} = \frac{\psi}{\zeta},\ K = \kappa D_h,\ \bar{U}_E = -\frac{\varepsilon\zeta E}{\mu U_P},\ U_p = \frac{-D_h^2}{4\mu}\frac{dp}{dz}$$

RESULT

Ratio of volume flow rate as predicted by the approximate and exact solutions for (a) circular (e=0) and elliptic (e=0.6) geometry, (b) polygonal geometry, (c) point-star geometry and (d) annular geometry ($R_1 = \frac{1}{2}R_2$) , taking $\bar{U}_E = -1$



CONCLUSIONS

- Exact semi-analytical solutions for EOF in microchannels with interfacial slip has been obtained by two different approaches, for complex geometries.
- Flow rate prediction in approximate case are in good agreement with the actual case for non-interacting EDLs, i.e.,

$$Q_{app}/Q \to 1 \ as \ K \to \infty$$

- For small values of K, the error in the prediction is magnified, but as the slip length increases, closure turn out to be the agreement.

REFERENCES