Probabilistic Symbolic Execution
A New Hammer

Willem Visser
Stellenbosch University

Joint work with Matt Dwyer, Jaco Geldenhuys, Corina Pasareanu, Antonio Filieri, …

Probabilistic Symbolic Execution

Symbolic Execution

+

Model Counting


Mabalingwe
Nature Reserve
200 m
13/05/2011


Zimbabwe
Namibia
Botswana
Windhoek
Mabalingwe Nature Reserve
Pretoria
Mbabane
Swaziland
Maseru
South Africa
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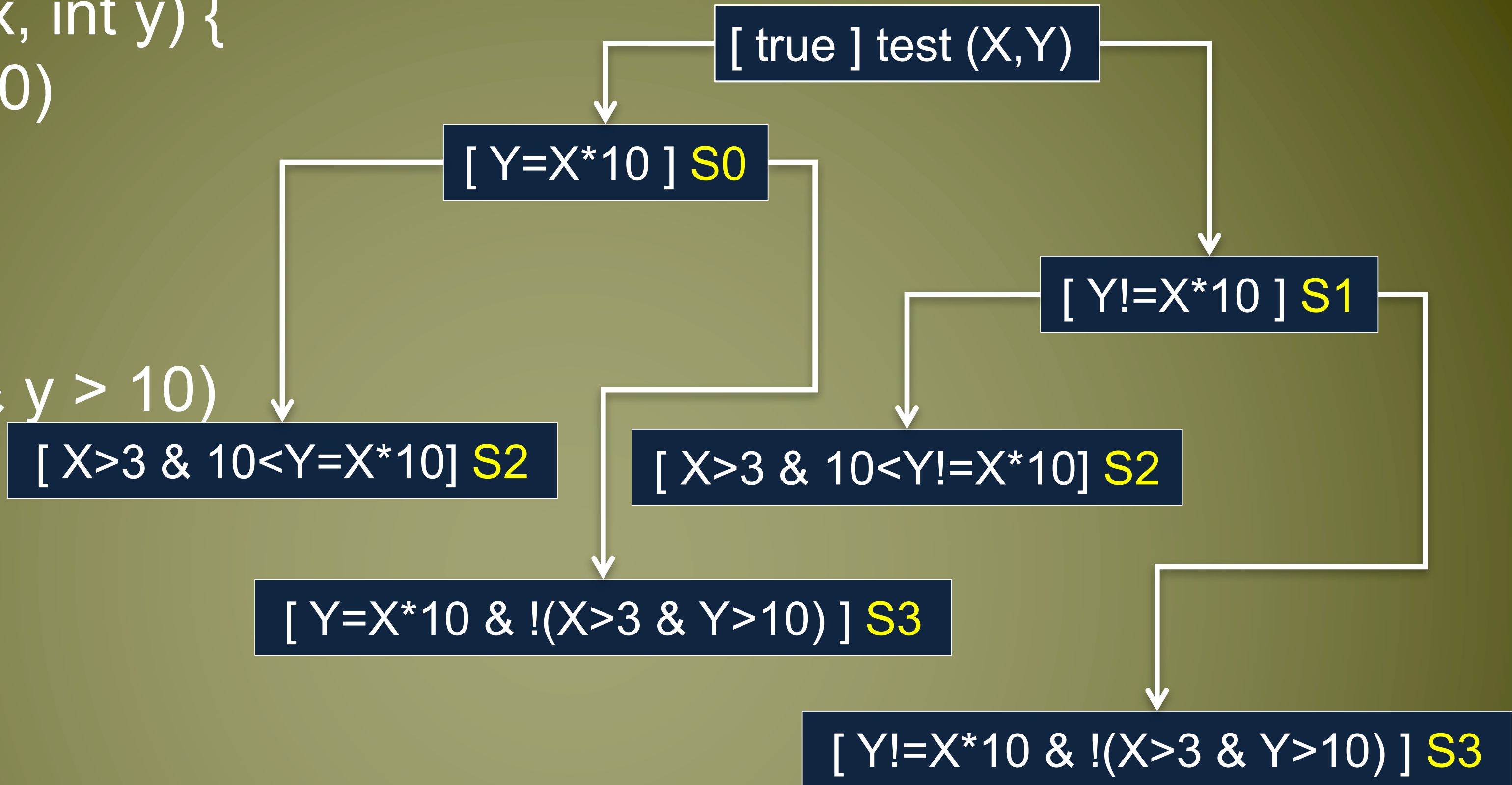
Saving the Whooping Crane

Symbolic Execution

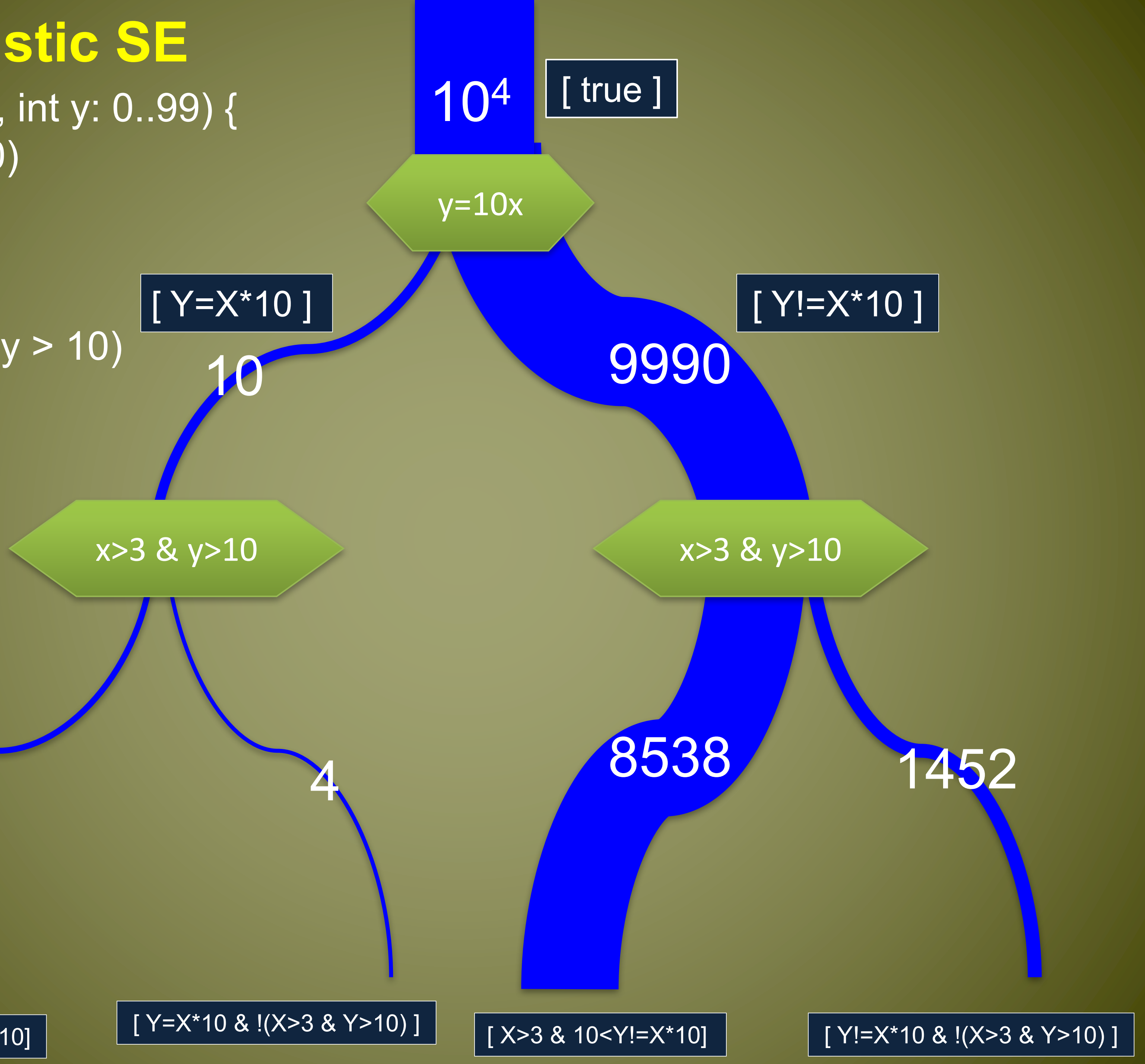
```
void test(int x, int y) {
   if (y == x*10)
      S0;
   else
      S1;
   if (x > 3 && y > 10)
      S2;
   else
      S3;
}
```



Test(1,10) reaches S0,S3
Test(0,1) reaches S1,S3
Test(4,11) reaches S1,S2

Probabilistic SE

```
void test(int x, int y: 0..99) {
  if (y == x*10)
    S0;
  else
    S1;
  if (x > 3 && y > 10)
    S2;
  else
    S3;
}
```



LattE Model Counter

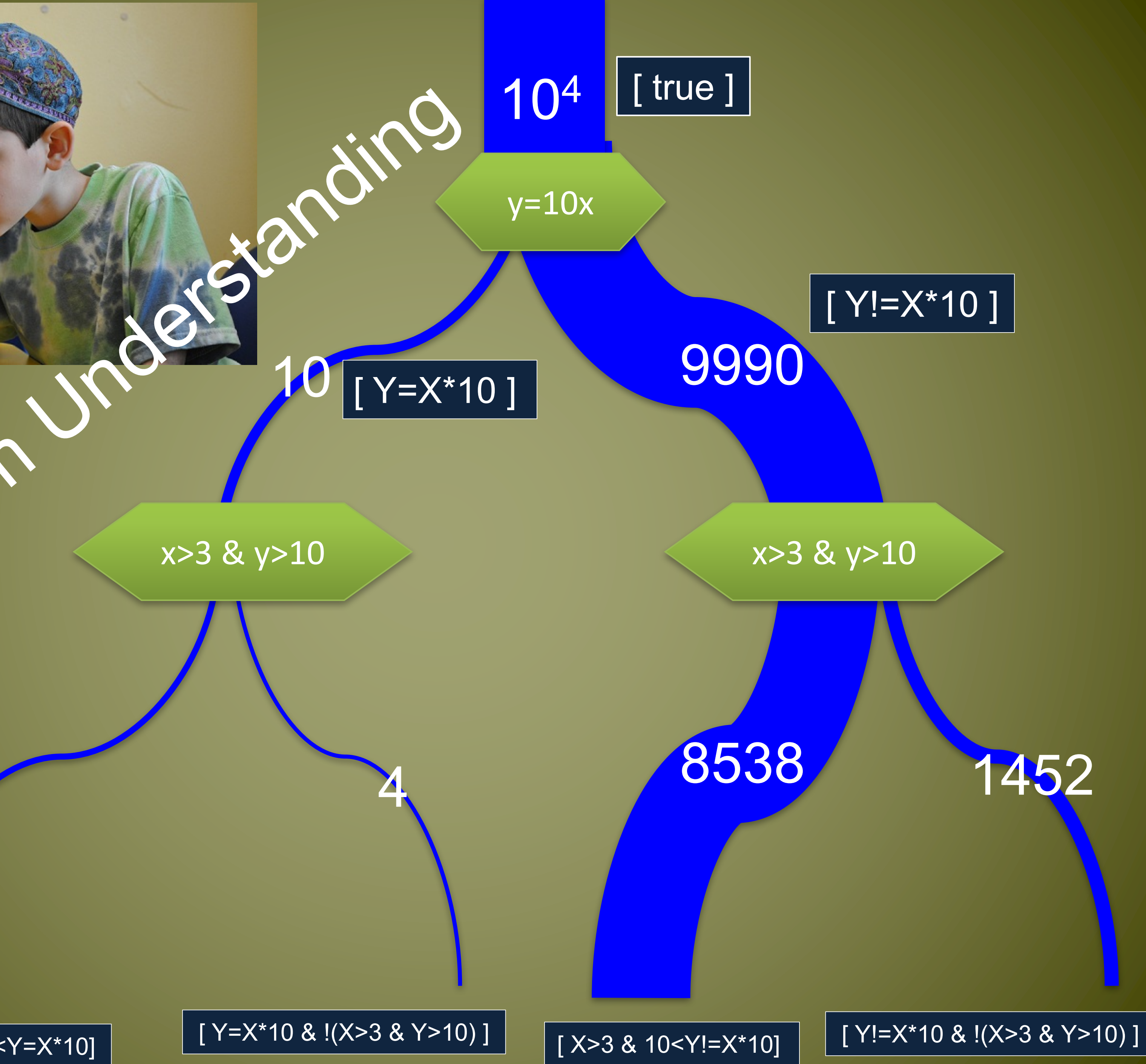
http://www.math.ucdavis.edu/~latte/

Count solutions for conjunction of Linear Inequalities

Things we can handle…

- Usage profiles (ICSE 2013)
- Domains
  - Linear Integer Arithmetic (ISSTA2012)
  - Floating point and non-linear (PLDI2014)
    - approximate
  - Data structures (SPIN2015)
  - Strings (CAV2015 by Tevfik Bultan)

Program Understanding



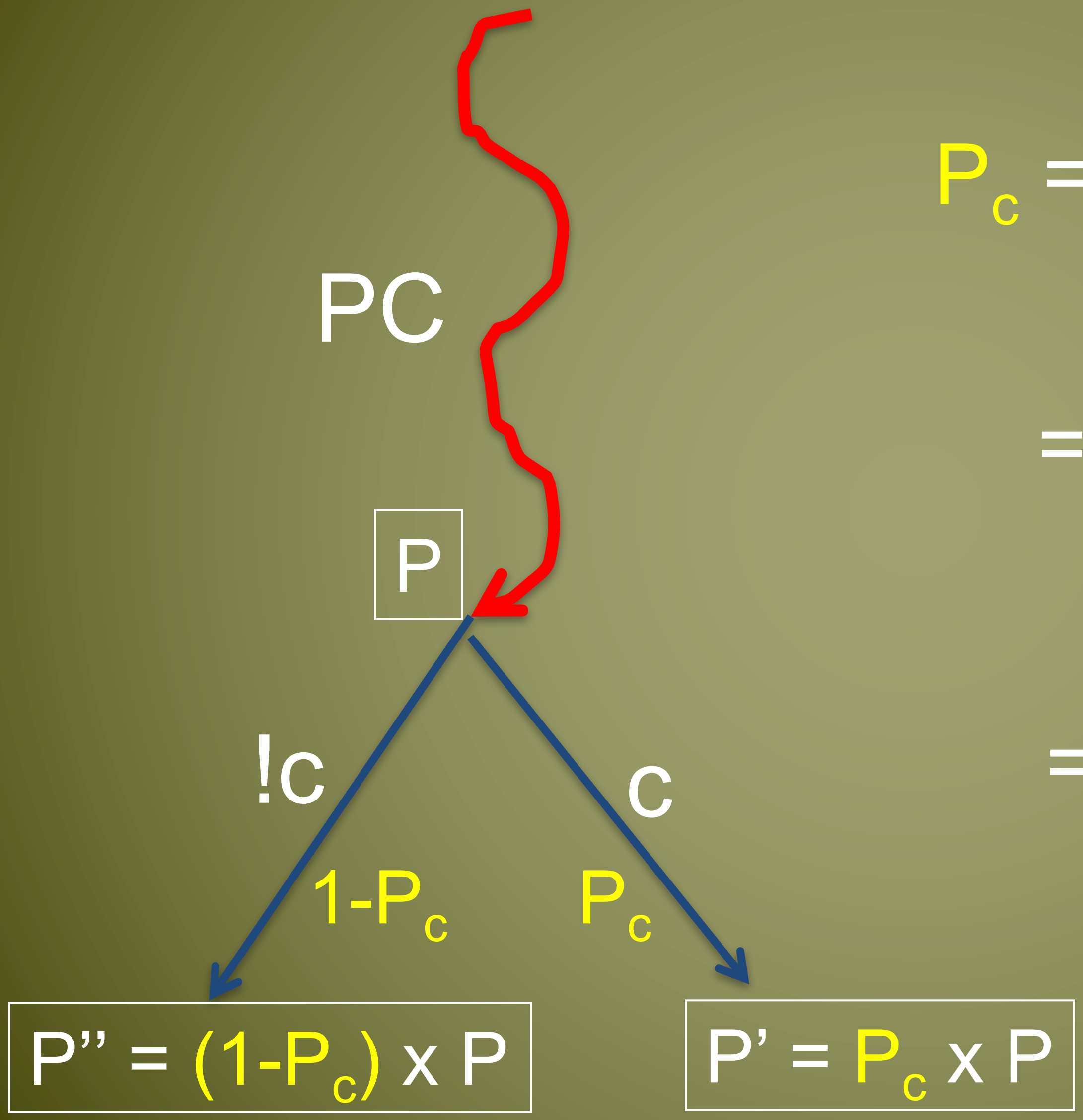
A Path Condition defines the constraints on the inputs to execute a path

How likely is a PC to be satisfied?

$$\frac{\text{\# solutions to the PC}}{\text{Domain Size}}$$

Assuming uniform distribution of values

Conditional and Path Probabilities

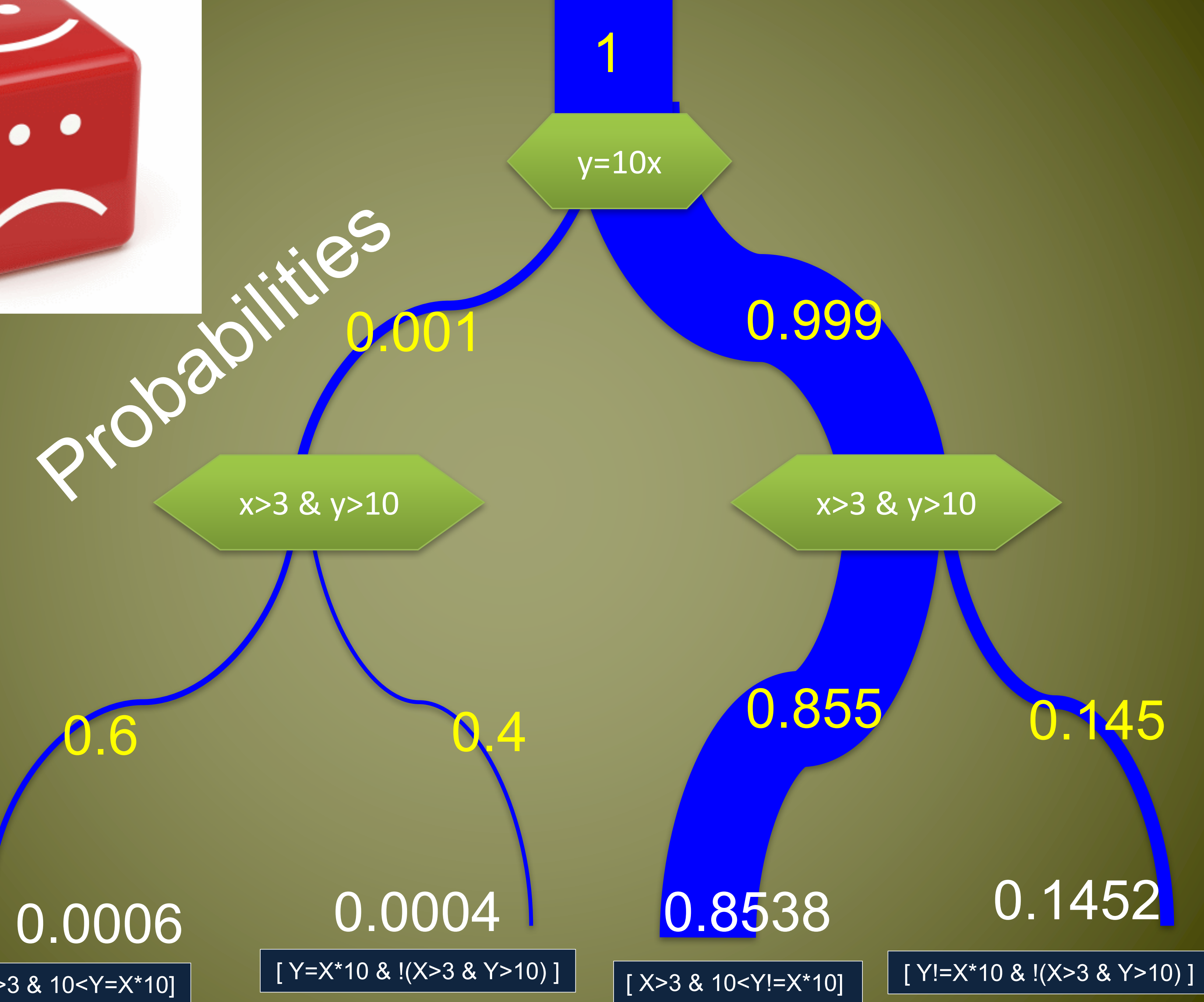


$$P_c = \text{Prob}\ (c \mid PC)$$

$$= \frac{\text{Prob}\ (c\ \&\ PC)}{\text{Prob}\ (PC)}$$

$$= \frac{\text{Prob}\ (c\ \&\ PC)}{\boxed{P}}$$

Probabilities



Reliability

0.9996 Reliable

Information Leakage via Side Channels
Pasareanu and Bultan

- Side channels produce a set of observables that partition a secret
  - Classically: execution time

$$\mathcal{O} = \{o_1, o_2, \ldots o_m\},$$

- Shannon Entropy
  - Expected amount of information gain in terms of bits

$$\mathcal{H}(P) = -\sum_{i=1,m} p(o_i)\log_2(p(o_i))$$

- Probabilistic Symbolic Execution

the probability of observing o_i is:

$$p(o_i) = \frac{\sum_{cost(\pi_j)=o_i} \sharp(PC_j(h,l))}{\sharp D}$$

Information Leakage Example
from slides by Tevfik Bultan

```
// binary 4-digit pin, D = 256
bool checkPIN(guess[]) {
  for (int i = 0; i < 4; i++)
    if (guess[i] != PIN[i])
      return false;
    return true;
  }
```

```
bool checkPIN(guess[]) {
  matched = true;
  for(int i=0; i<4; i++)
    if (guess[i] != PIN[i])
      matched = false;
    else
      matched = matched;
  return matched;
}
```

PATHS:

1. Return false; 128 values
2. Return false; 64 values
3. Return false; 32 values
4. Return false; 16 values
5. Return true; 16 values

Assuming observable is time
H = 1.875

Assuming observable is output
H = 0.33729

(Java) Probabilistic Programming

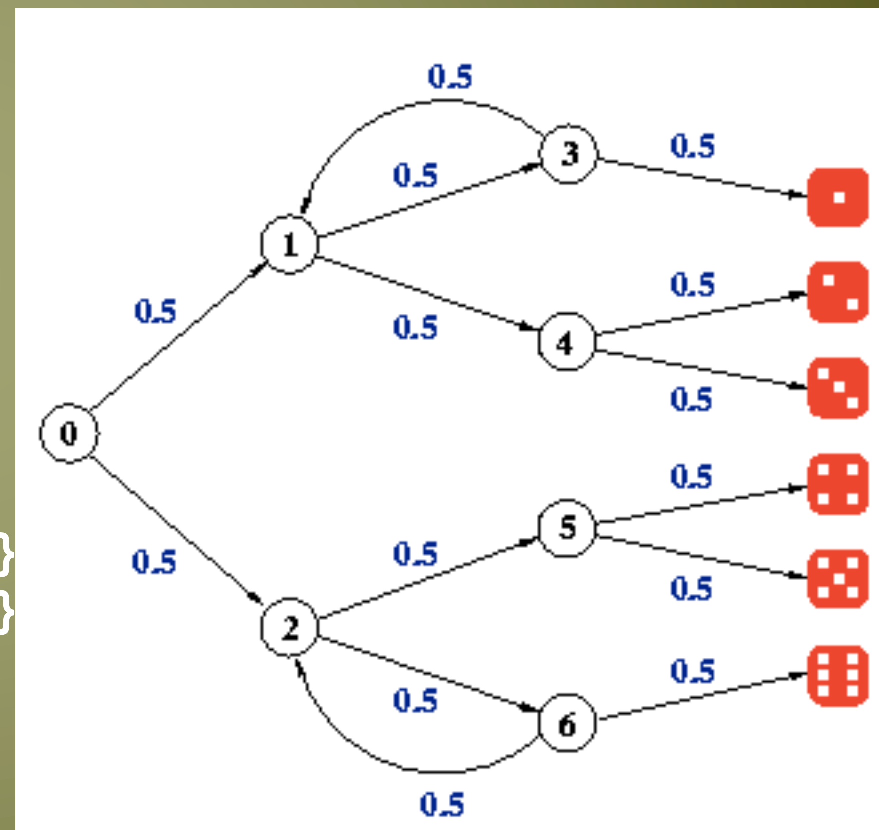
- Combine general purpose programming with probability distributions to answer interesting questions.
  - (Easily) encode Bayesian Networks, Hidden Markov Models, etc. as a (Java) program with a few special keywords
  - probability(loc), observe(cond), flip(ratio)
- Using Probabilistic Symbolic Execution for inference

```
public static void FOSE() {
    boolean c1 = flip(0.5);
    boolean c2 = flip(0.5);
    observe(c1 || c2);
    if (c1) probability(1);
}
```

0.6667

Classic Examples

```
public static void PRISMDiceExample() {
  int s = 0;
  int d = 0; // dice value
  while (true) {
    if (s==0) { s = flip(0.5) ? 1 : 2; }
    else if (s == 1) { s = flip(0.5) ? 3 : 4;}
    else if (s == 2) { s = flip(0.5) ? 5 : 6;}
    else if (s == 3) { if (flip(0.5)) { s = 1;}
                        else { s = 7; d = 1; }}
    else if (s == 4) { s = 7; d = flip(0.5) ? 2 : 3;}
    else if (s == 5) { s = 7; d = flip(0.5) ? 4 : 5;}
    else if (s == 6) { if (flip(0.5)) { s = 2;}
                        else { s = 7; d = 6;}}
    else { /* s = 7 */ break; }
  }
  probability(d); // probability of seeing each value for d
}
```



0.16667 for all d

“Semantic” Difference Between Programs

On what percentage of the input space does P and P’ give different outputs?

```
public static void check(int a, int b, int c) {
        assert P(a, b, c) == P’(a, b, c);
}
```

Record path conditions when assertion fails and count their sizes then divide by total domain size to get % difference

Difference Example

```
Boolean PP(int i, int j) {
   return i > j;
}
```

100% different

```
Boolean PP(int i, int j) {
   return i >= j;
}
```

99% different

```
Boolean PP(int i, int j) {
   return i != j;
}
```

50.5% different

```
Boolean PP(int i, int j) {
   return i == j;
}
```

49.5% different

```
Boolean PP(int i, int j) {
   return i < j;
}
```

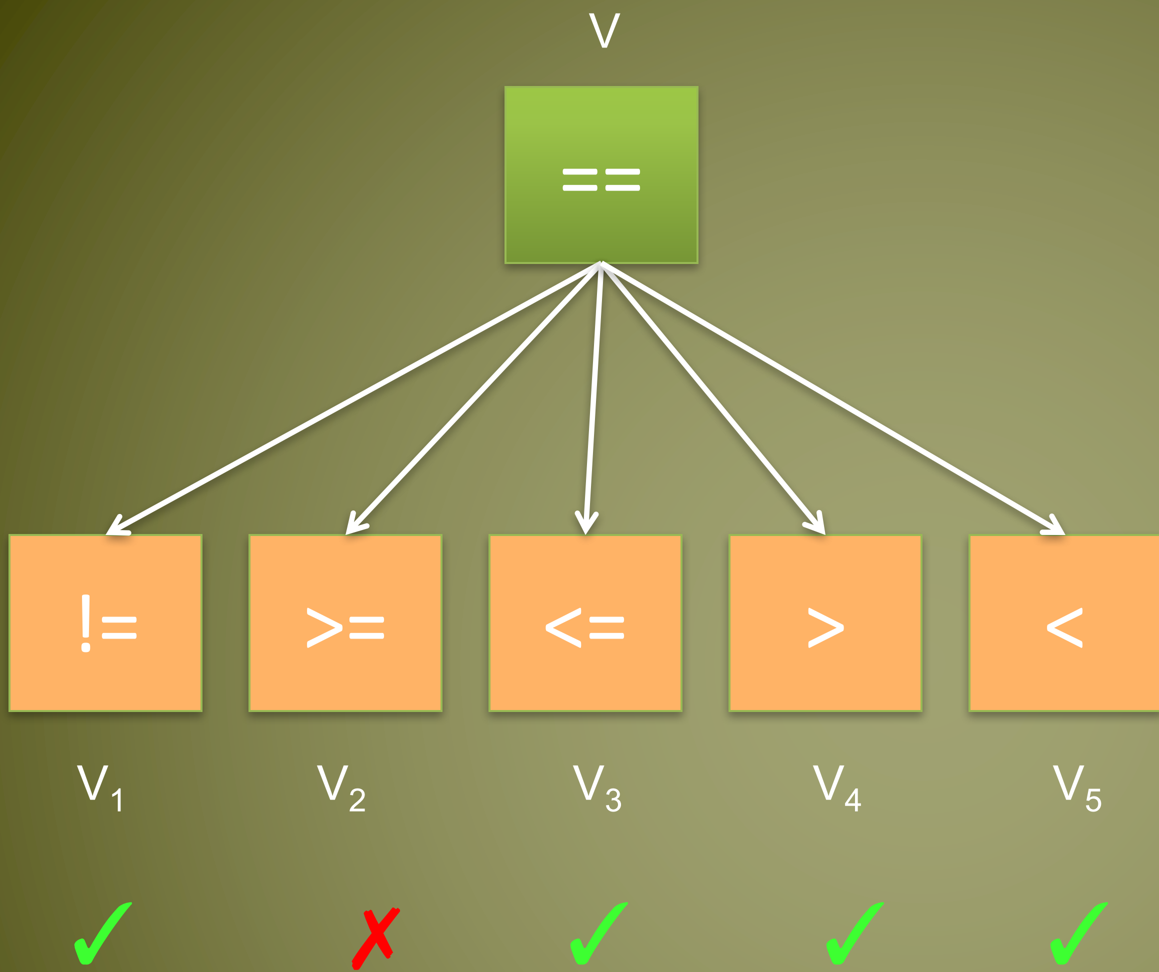
1% different

```
Boolean P(int i, int j : 0..99) {
   return i <= j;
}
```

Mutations

Taking an analytical look

especially when used to seed faults



Ref. Version

Apply Mutation Ops

Mutations

Are mutations *killed* using ref's tests

Mutation is Killed if there exist a test that fails on it

$$\text{Mutation Score} = \frac{\#\ \text{Killed}}{\#\ \text{Mutations}}$$


Killing Mutations == Finding real errors?

Assuming the answer is yes…

Mutations have found another use

FAULT SEEDING

How good is my super-duper new bug finding tool at finding seeded faults?

How hard is it to kill a mutant?

Previous work: fixed the test suite

We consider ***ALL*** test inputs
and show
the influence of varying the oracle

How hard is it to kill a mutant?

Spoiler Alert

Not hard at all

What

How easy or hard is it to kill a mutant?

How

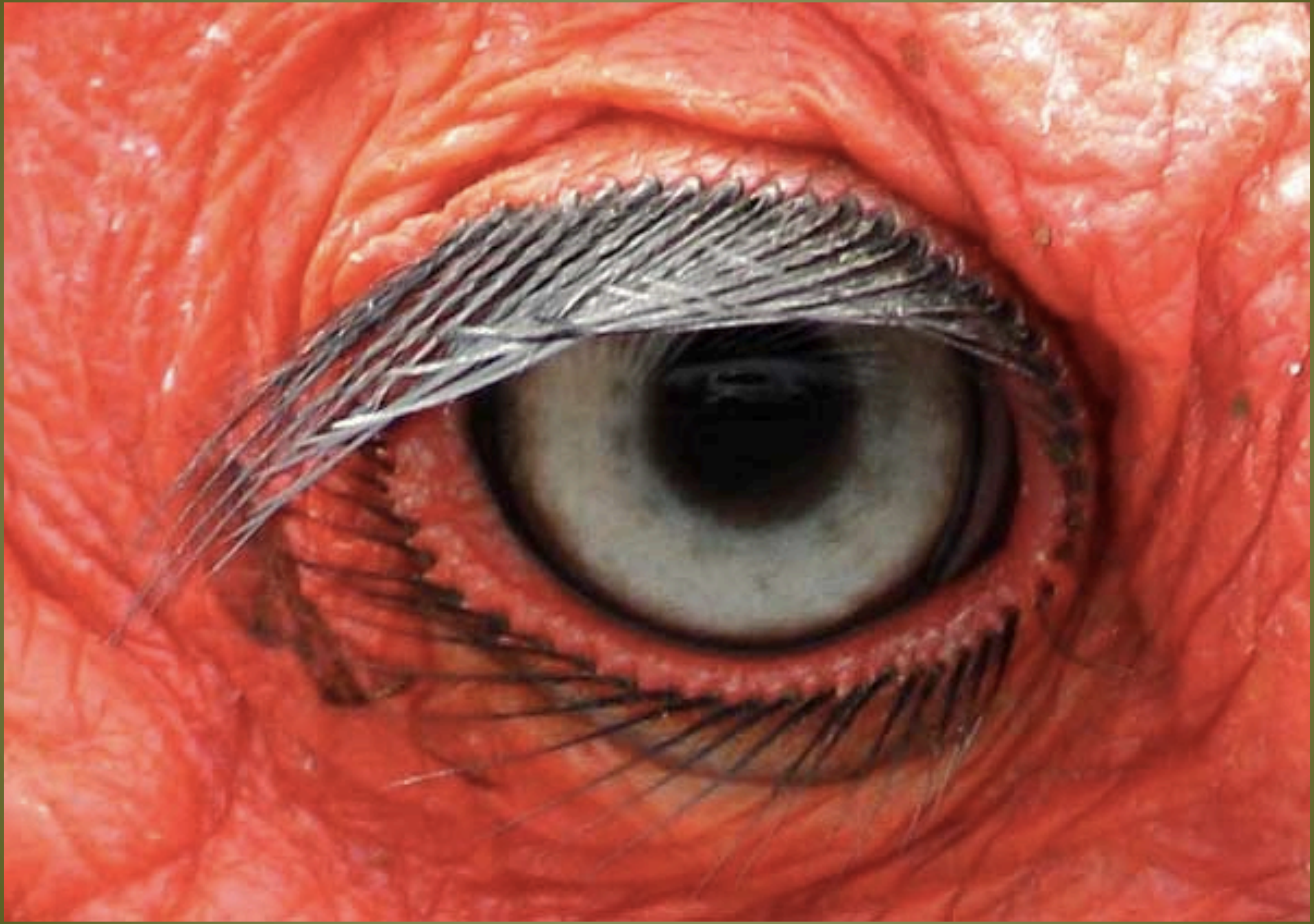
On what percentage of the input space does the oracle for the reference version and mutated version give different outputs?

diff == 0% => Equivalent Mutant
diff < threshold% => Stubborn Mutant

Implementation

- Listener for Symbolic PathFinder (SPF)
  - Traps calls to every bytecode instruction executed
- Collects path conditions when oracle differs
- Count the solutions to these with Green and Barvinok
- Also collects path conditions at the point of mutation and counts the sizes
  - Special NOP bytecode is pushed at this point
- Dumps a CSV file with the output
- Dockerfile to recreate image to run experiments

In the initial results



We saw something interesting

What did we find?

```
public static int classify(int i, int j, int k) {
  if ((i <= 0) || (j <= 0) || (k <= 0))
     return 4;
  int type = 0;
  if (i == j) type = type + 1;
  if (i == k) type = type + 2;
  if (j == k) type = type + 3;
  if (type == 0) {
    if ((i + j <= k) || (j + k <= i) || (i + k <= j))  type = 4;
    else type = 1;
    return type;
  }
```

Stubborn Barrier

```
  if (type > 3) type = 3;
  else if ((type == 1) && (i + j > k))  type = 2;
  else if ((type == 2) && (i + k > j))  type = 2;
  else if ((type == 3) && (j + k > i))  type = 2;
  else type = 4;
  return type;
}
```

Almost all Mutations are Stubborn (<1%)

Why?

```
public static int classify(int i, int j, int k) {
  if ((i <= 0) || (j <= 0) || (k <= 0))
     return 4;
  int type = 0;
  if (i == j) type = type + 1;
  if (i == k) type = type + 2;
  if (j == k) type = type + 3;
  if (type == 0) {
    if ((i + j <= k) || (j + k <= i) || (i + k <= j))  type = 4;
    else type = 1;
    return type;
  }

  if (type > 3) type = 3;
  else if ((type == 1) && (i + j > k))  type = 2;
  else if ((type == 2) && (i + k > j))  type = 2;
  else if ((type == 3) && (j + k > i))  type = 2;
  else type = 4;
  return type;
}
```

Only 3% of inputs pass here

Results with Reachability
Arithmetic + Constant Replacement

Programs	Muts	Stubborn < 0.1%	Really < 0.1%	Always 100%	Easy > 33%
TRI-YHJ	5	0	0	**4**	5
TRI-V1	19	1	0	**8**	18
TRI-V2	8	1	0	**5**	7
TCAS	38	8	4	**9**	28

Reach it … kill it

Results with Reachability

Relational Operators

Programs	Muts	Stubborn < 0.1%	Really < 0.1%	Always 100%	Easy > 33%
TRI-YHJ	40	0	0	5	24
TRI-V1	85	6	3	4	61
TRI-V2	55	0	0	3	38
TCAS	185	32	24	12	46

Reach it …good chance of killing it

Luckily
not all relational operators
behave the same

Results by Relational Operator

Operator	Muts	Equiv	Stubborn	Always	Easy
!=,==	17	0.00%	5.88%	**23.53%**	64.71%
<,>=	5	80.00%	0.00%	**20.00%**	20.00%
<=,>	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
==,!=	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
~~==,>~~	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
>, <=	6	0.00%	0.00%	**50.00%**	83.33%
>=,<	3	0.00%	0.00%	**33.33%**	100.00%
<,<=	5	80.00%	**20.00%**	0.00%	0.00%
<=,<	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
>,>=	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
>=,>	3	0.00%	**100.00%**	0.00%	0.00%

NEGATION operators are good at creating easy to kill mutants

OFF BY ONE operators are good at creating hard to kill mutants

Unfortunately so far
we were looking at an ideal situation:
we used a “perfect” oracle that can
reliably detect mutations

Lets see what happens if we vary
the precision of the oracle

The tale of 2 Oracles for BinTree

```
public boolean repOK() {
  return checkTree(root,0,9);
}

private boolean checkTree(Node n,
                          int min,
                          int max) {
  if (n == null) return true;
  if (n.value < min || n.value > max)
    return false;
  boolean resL = checkTree(n.left,
                           min,
                           n.value-1);
  if(!resL) return false;
  else
    return checkTree(n.right,
                     n.value+1,
                     max);
}
```

```
public String linearize() {
  if (!repOK()) return "NotABST”;
  return linearize(root);
}

private String linearize(Node n) {
  StringBuilder b = new StringBuilder();
  b.append("(");
  if (n != null) {
    b.append(n.value).append(' ');
    b.append(linearize(n.left));
    b.append(' ’);
    b.append(linearize(n.right));
  }
  b.append(")");
  return b.toString();
}
```

Linearize vs repOK for BST

Operator	Muts	Equiv Linearize	Equiv repOK	Easy Linearize	Easy repOK	Always Linearize	Always repOK
All	67	30%	66%	57%	31%	21%	15%
AOR+Const	12	83%	83%	0%	0%	0%	0%
ROR	55	18%	62%	69%	38%	25%	18%
Negation	23	4%	47%	78%	52%	48%	34%

Precise Oracle, less Equivalent, but more easily killed

Imprecise Oracle, more Equivalent, but less easily killed

A Study of Equivalent and Stubborn Mutation Operators using Human Analysis of Equivalence

Xiangjuan Yao
College of Science, China University of Mining and Technology, China

Mark Harman
CREST Centre, University College London, UK

Yue Jia
CREST Centre, University College London, UK

- ~~They found for the Relational Operators you get stubborn and equivalent mutants in almost equal amounts (other classes had no such connection)~~
- ~~They also found that more mutations implied more equivalent mutations, but no such correlation with stubborn mutations~~

Beware of Empirical Software Engineering!

WARNING!!!

Can we find an analytical link between coverage and fault detection?

If we assume we know nothing about the distribution of test inputs, then…

For a given program P, calculate the probability of achieving X% coverage with a test suite of size k

For a faulty program P, calculate the probability of observing the bug with a test suite of size k ✓

Step 1: Probabilistic Symbolic Execution

```
public int simple(int x, int y) {
    int a = 0;
    if (x < 4) { // 25
        a = 0;
    } else {
        a = x;
    }
    if (y < 4) { // 30
       return a + y;
    } else {
      return x + y;
    }
}
```

Collect all paths with coverage and probability (x,y:0..9):

[30T, 25T] 0.36
[30T, 25F] 0.24
[30F, 25T] 0.24
[30F, 25F] 0.16

For 100% coverage:
30T, 30F, 25T and 25F

Step 2: Sample and Calculate

1. Sample k-paths M times based on the probability (with replacement)
2. For these k-paths calculate coverage, based on number of samples that gets the coverage, lets say c
3. c/M gives the probability

Assume k=2 & 100% coverage

[30T, 25T] 0.36
[30T, 25F] 0.24
[30F, 25T] 0.24
[30F, 25F] 0.16

Pick 10^6 2-tests, see on how many do you cover all 4 options, if 230k times, then probability is 23%.

Probability of getting full coverage with 2-tests, is 23%

Step 3: Calculate Probability of Bug

1. Use previous stuff to calculate on what percentage of inputs can an oracle observe the bug, call this probability p
2. Prob(bug | for a given k) = $1 - (1 - p)^k$

//spec simple(x,y) = x+y

```
Public int simple(int x,  int y) {
   int a = 0;
   if (x < 4) { // 25
      a = 0;
   } else {
      a = x;
   }
   if (y < 4) { // 30
     return a + y;
   } else {
    return x + y;
   }
}
```

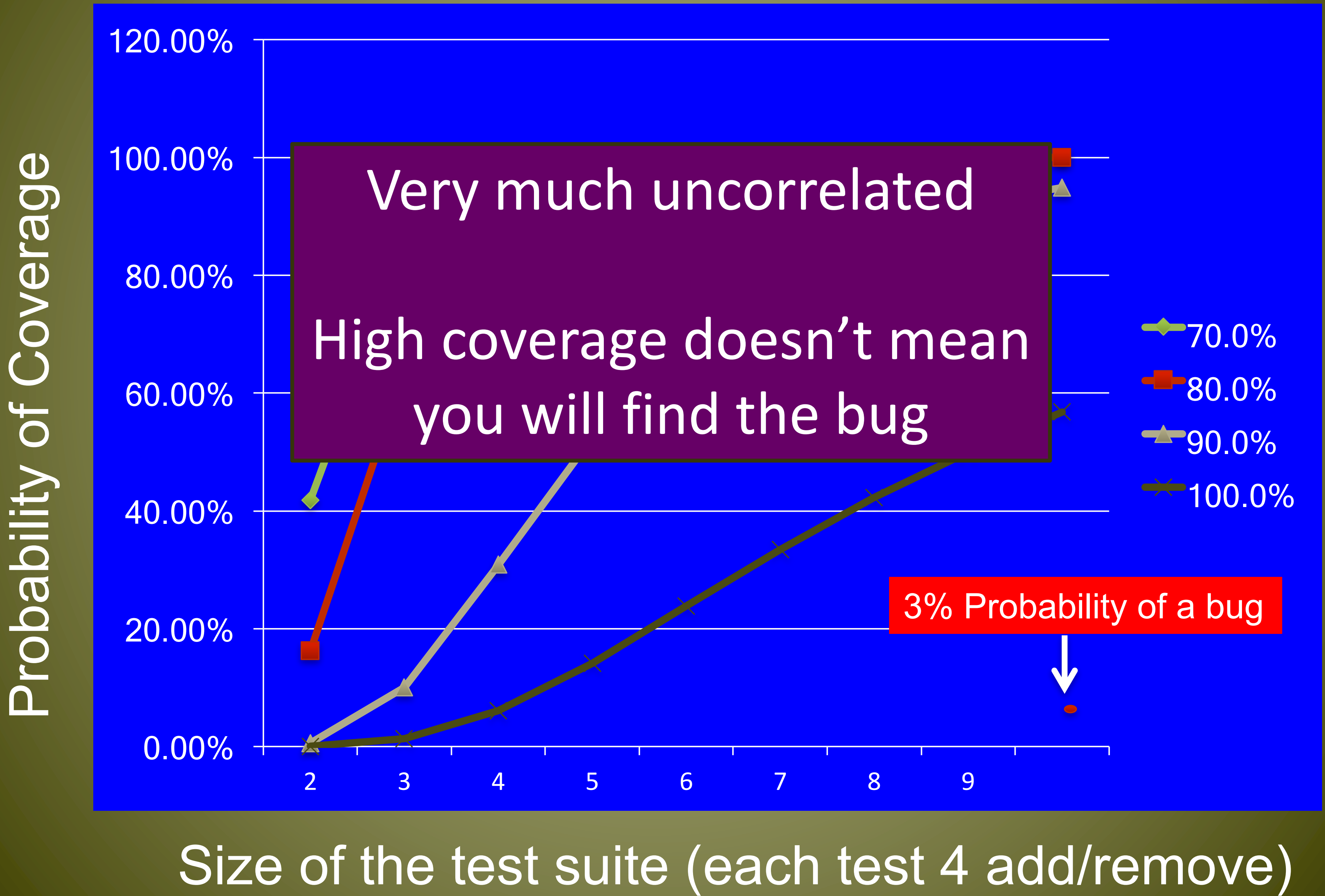
Prob(bug) = 12/100

PC for bug: y!=y+x /\ y<4 /\ x<4

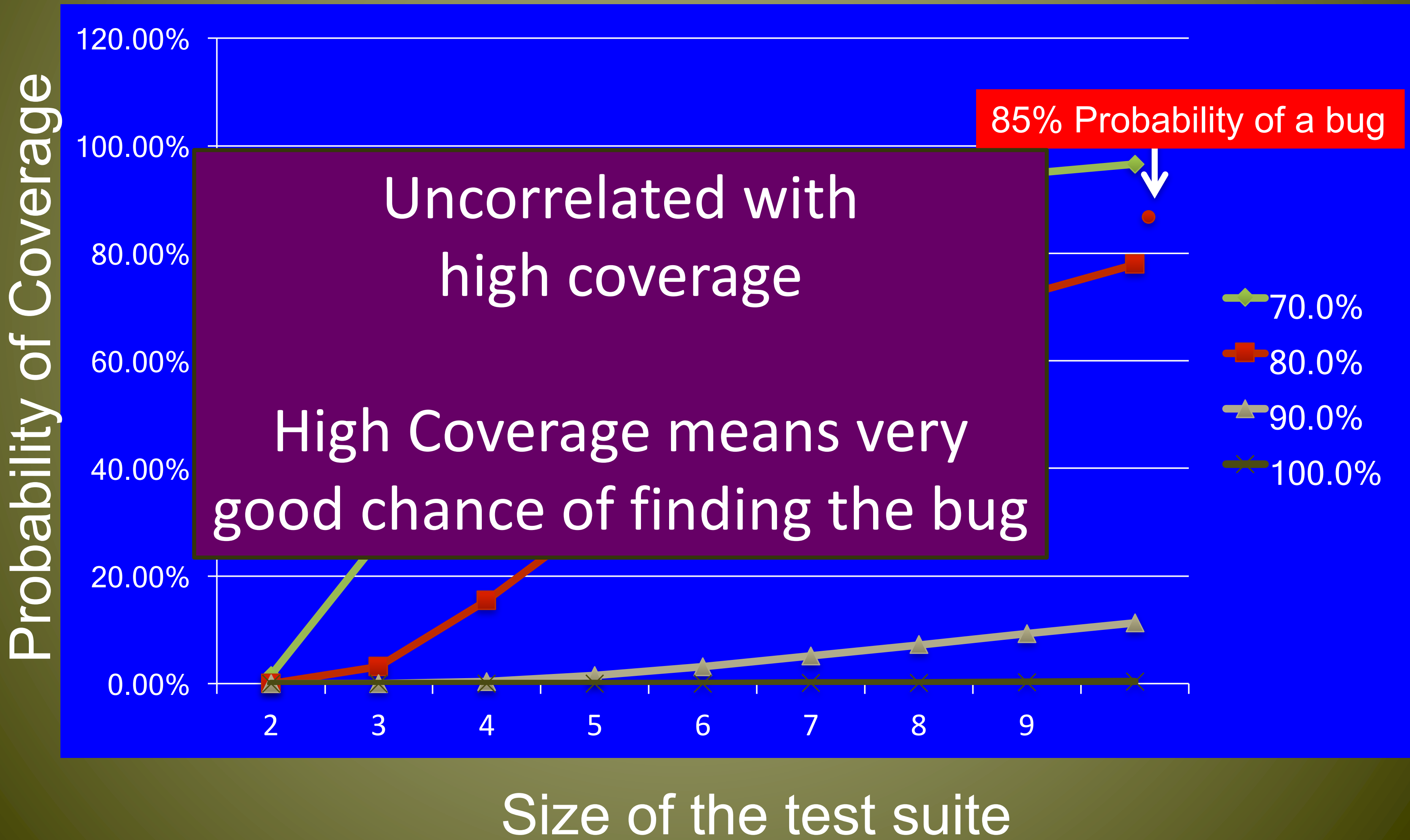
then Prog(bug| k = 2) = 22.6%

Probability of seeing the bug and obtaining coverage is therefore about the same, and thus one can argue they will correlate

Broken BinaryTree Example



TRI-YHJ, i.e. broken TriangleClassify



Still working on this…

- Need more faults, the two shown were real errors not mutations
- Can create mutations and repeat all of this
- Need to see if we can find real examples from literature and analyze them
- Note that empirical work in this setting can easily be skewed to show whatever you want; only if you analyze truly large datasets with very good tests can you say something useful
- Even though this will probably only work for small programs it might give some interesting insights

Other ongoing work

- Probabilistic Java Programming
  - Including parametric analysis
  - Add sampling to scale to larger examples

Monte-Carlo Tree Search for WCET
  - Works much better than Monte-Carlo or Reinforcement Learning
- Whitebox Fuzzing revisited
  - Infer input grammars by iterative symbolic execution, i.e. derive seed-file structure on-the-fly?