TacticToe
Learning to Reason with HOL4 Tactics

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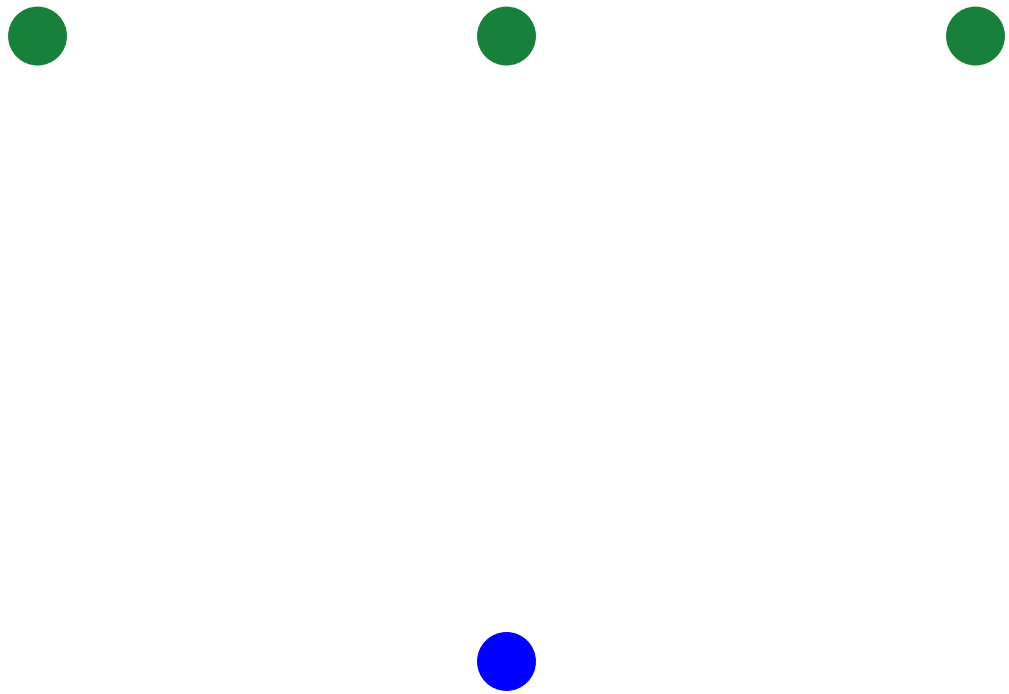
Reasoning with inference rules



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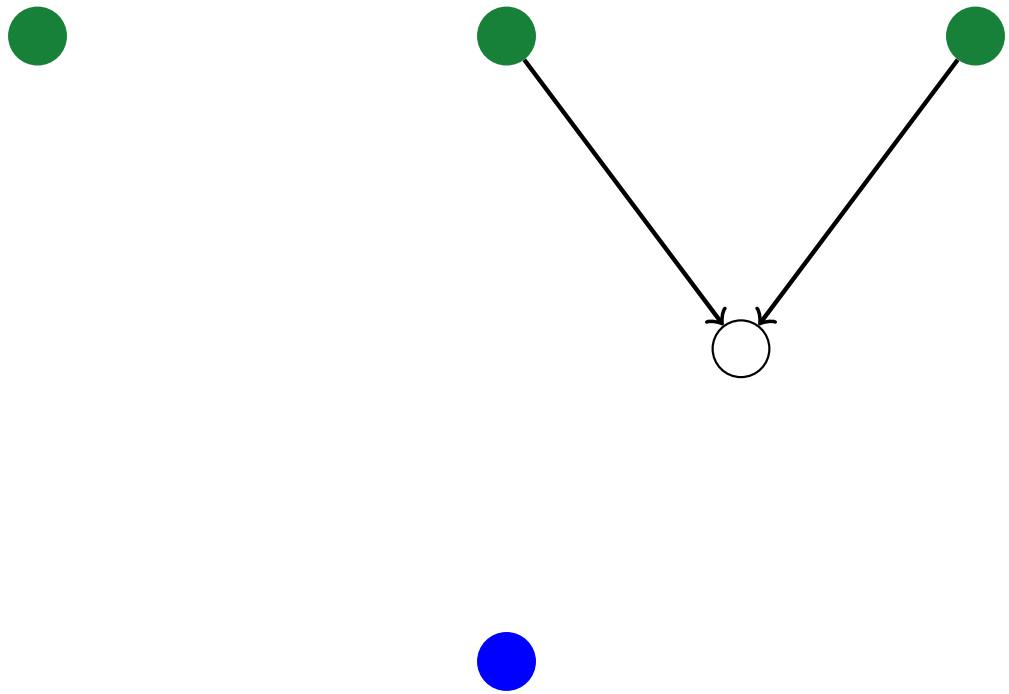

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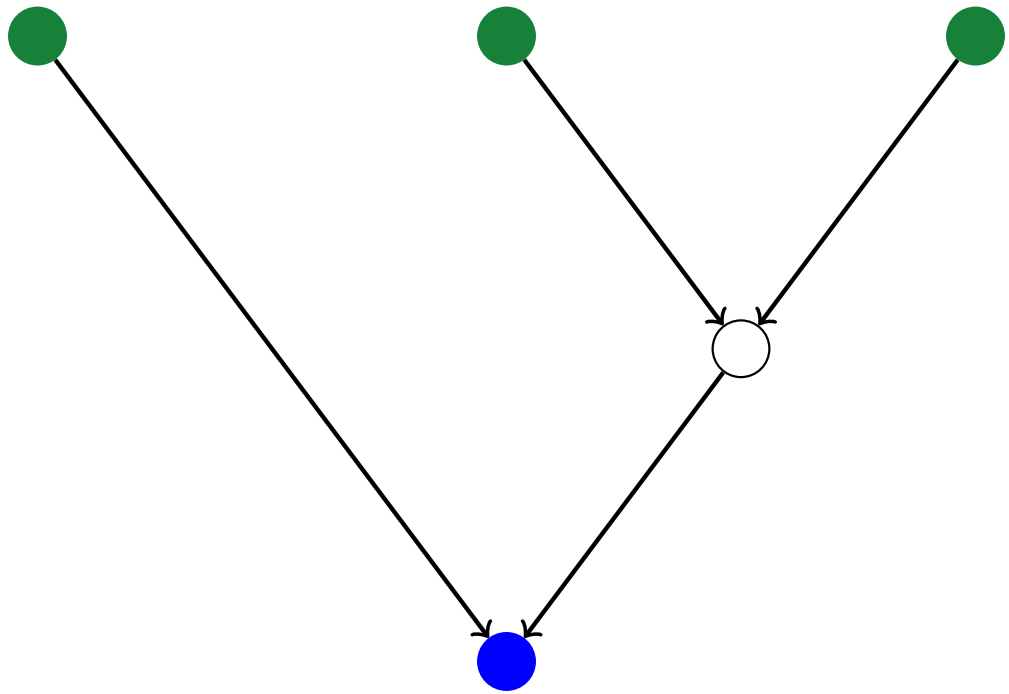
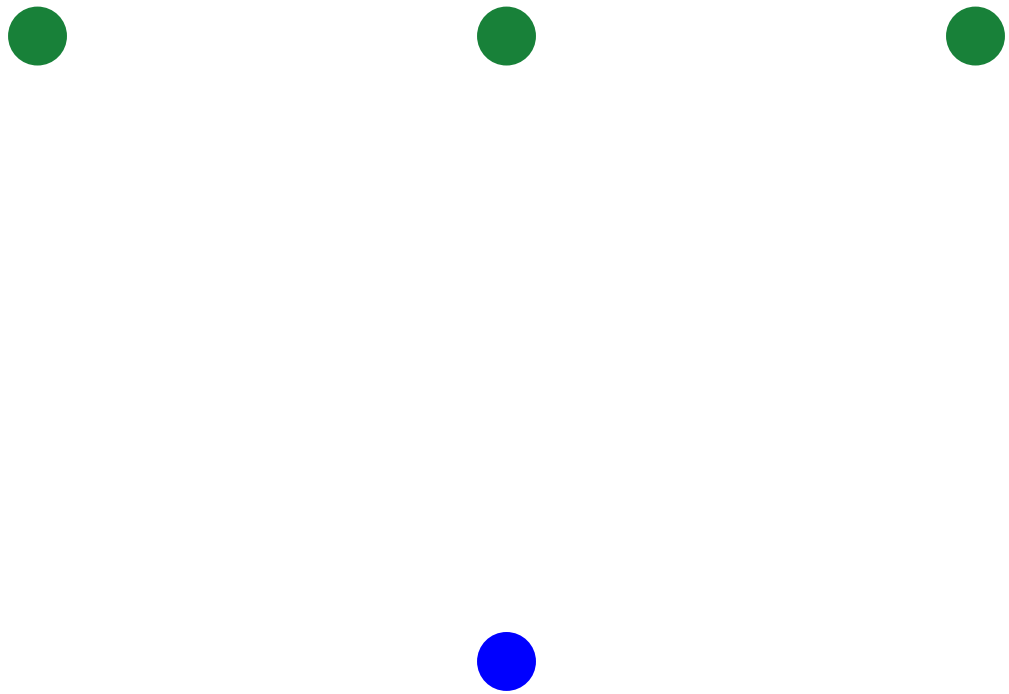


Reasoning with tactics



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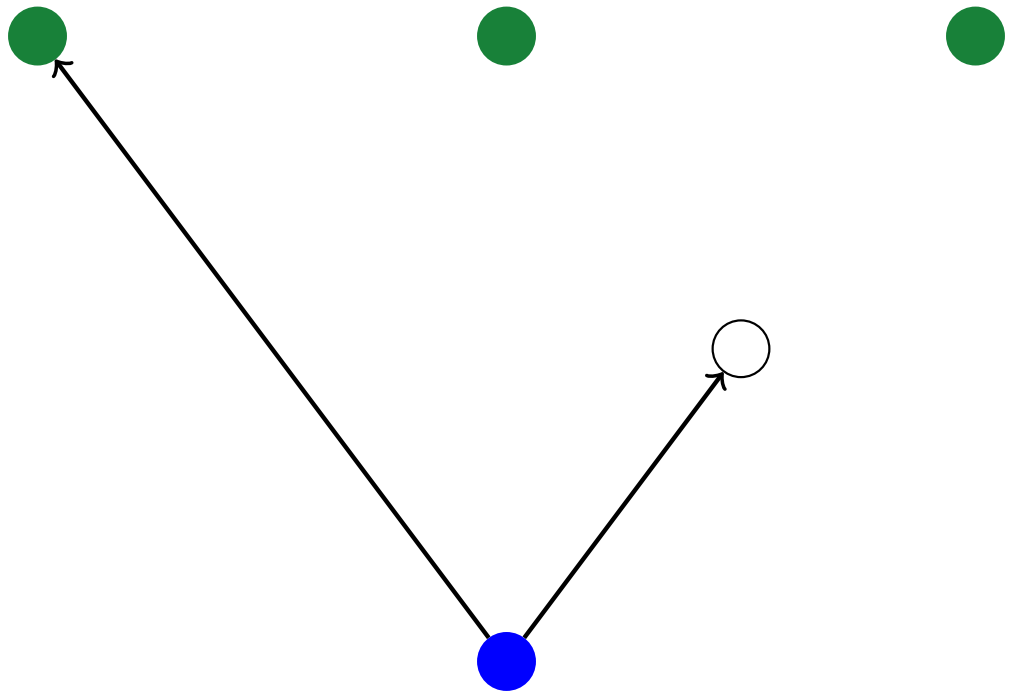

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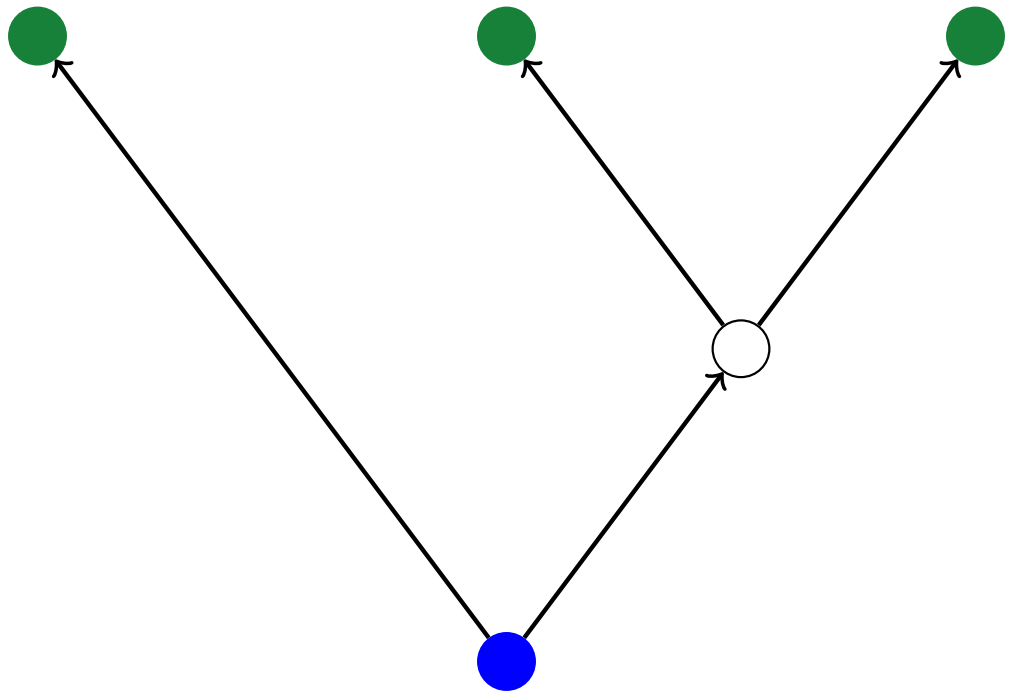

Common tactics

- `REWRITE_TAC`
- `INDUCT_TAC`
- `METIS_TAC`

Composing tactics

`THENL` tactical composes the effect of tactics.

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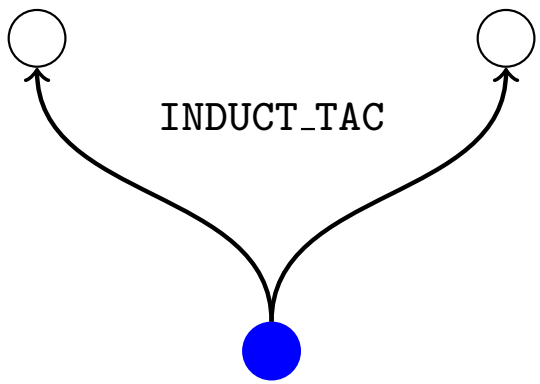

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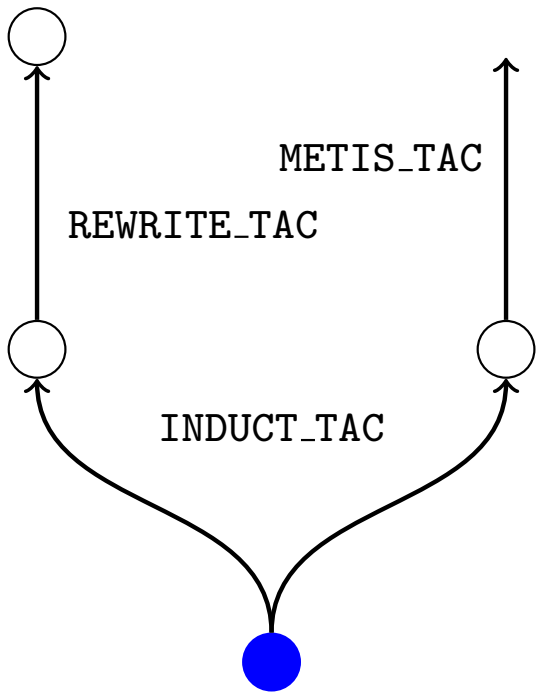

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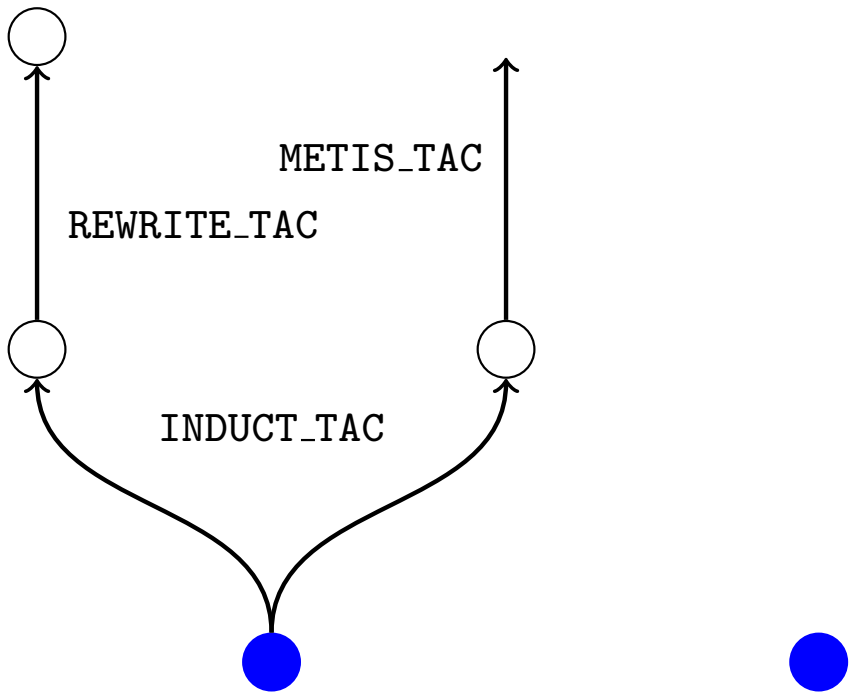

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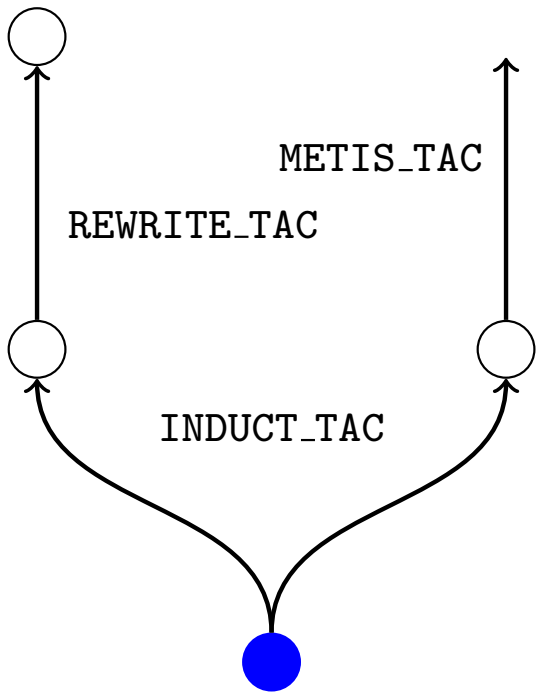


```
INDUCT_TAC
  THENL
[REWRITE_TAC,METIS_TAC]
```

Tactic selection

Was the tactic sucessful **before** on **similar** goals?

Before: Recording tactics

- Globalizing:
  - Local values
    `let val x = 5 in NTAC x INDUCT_TAC end`
  - Modules
    `Ho_rewrite, Rewrite`
- Wrapping:
  `R INDUCT_TAC THENL [R REWRITE_TAC, R METIS_TAC]`
- Database:

```
INDUCT_TAC    x + x >= x
INDUCT_TAC    x * x >= x
REWRITE_TAC   1 + 1 = 2
```

Similarity

Features: constants, subterms, names of variables, ...

```
INDUCT_TAC    [+,>=,...]
INDUCT_TAC    [*,>=,...]
REWRITE_TAC   [1,2,+,=,...]
```

Best predicted tactic for `2 >= 1`?

How to search for a proof?

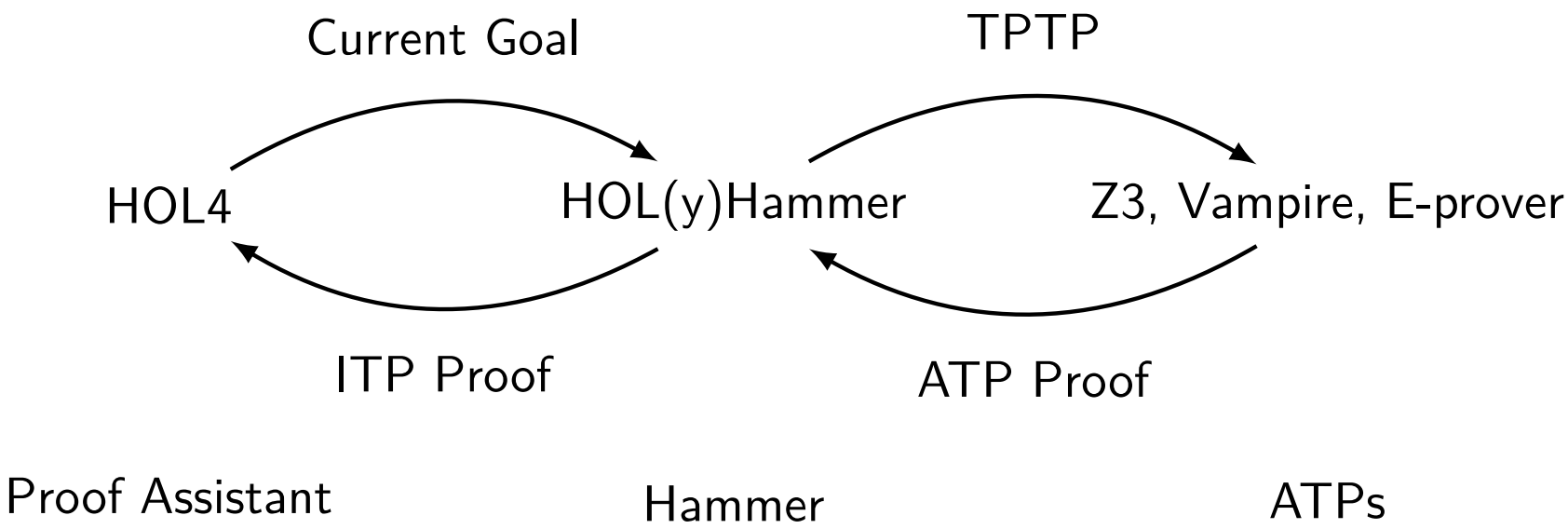
Depth first search:

- Start with the conjecture
- Apply best predicted tactic
- Repeat on the new goals

A*-search:

- Cost: length of the proofs needed to create the goal
- Heuristic: evaluation of the length of the remaining proof

HOL(y)Hammer


Current Goal
TPTP
HOL4
HOL(y)Hammer
Z3, Vampire, E-prover
ITP Proof
ATP Proof
Proof Assistant
Hammer
ATPs

General results

ID		7902 theorems
TacticToe		29.73
TacticToe*	"little hammer"	39.42
HolyHammer	E knn 128 blistr	32.35

Results by theories

	arith	real	compl	meas
TacticToe	37.3	19.7	42.6	19.6
TacticToe*	60.1	46.1	63.7	22.1
HolyHammer	51.9	66.8	72.3	13.1

	proba	list	sort	f_map
TacTicToe	25.3	48.1	32.7	53.4
TacTicToe*	25.3	51.9	34.7	55.5
HolyHammer	25.3	23.3	16.4	18.1

Example in gcdTheory: GCD_ADD_L

$$\forall a\ b.\ gcd\ (a + b)\ a = gcd\ a\ b$$

```
Human proof:  PROVE_TAC [GCD_SYM,GCD_ADD_R]

TacticToe proof:
ARW_TAC
THEN MATCH_MP_TAC (SPECL [a, a + b] IS_GCD_UNIQUE)
THEN ARW [...] IS_GCD_MINUS_R
THEN PROVE_TAC [GCD_IS_GCD, IS_GCD_UNIQUE, IS_GCD_SYM]

HolyHammer proof:  METIS_TAC [GCD_SYM,GCD_ADD_R]
```

Example in listTheory: DROP_NIL

$$\forall ls\ n.\ (DROP\ n\ ls = [\,]) \Leftrightarrow n \geq LENGTH\ ls$$

```
Human proof:  Induct THEN SRW_TAC [ ] [ ] THEN DECIDE_TAC

TacticToe proof:
INDUCT_THEN list_INDUCT ASSUME_TAC
THENL [SRW_TAC [] [], SRW_TAC [ARITH_ss] []]
```

Conclusion

TacticToe combines previous human proofs to solve new goals.

- Induction principle
- Simplification sets
- User-defined domain specific automation

The proofs produced are efficient HOL4 proof scripts.

Future works

- More **features** for goals:
  - Tactic arguments relation to the goal
  - Time to solve, number of tactics necessary
- Extending the **policy**: tactic argument selection
- Better **evaluation** of the difficulty of the goal