Deep Network Guided Proof Search

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Progress in ITP and ATP

Large Formalizations

- AFP: 64K lemmas, 593K LoC [*Nipkow+2015*]
- seL4: 49K lemmas, 400K LoC [*Klein+2014*]
- Flyspeck: 27K lemmas, 2B intermediate steps [*Hales+2016*]

Problems handled by ATPs

- Avatar [*Voronkow 2015*]
- E-prover history mining [*Schulz 2016*]
- SAT traces are big data

Little use of machine learning

Fast progress in machine learning

Tasks involving logical inference

- Natural language question answering [*Sukhbaatar+2015*]
- Knowledge base completion [*Socher+2013*]
- Automated translation [*Wu+2016*]

Games

AlphaGo problems similar to proving [*Silver+2016*]

- Node evaluation
- Policy decisions

Computer Vision

Better than human performance on some tasks [*Russakovsky+2015*]

Machine Learning in Theorem Proving so far

Predict Statement Dependencies

- Premise selection and relevance in ATPs
- Heuristics, learning and deep learning useful

Estimate Statement Usefulness

- Heuristics and simple learning methods

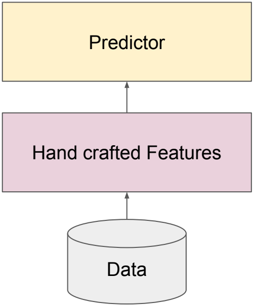
Propose Useful Conjectures

Supervised Learning Task

Assume $G : D \to P$	Ground truth G
$f : D \times M \to P$	Model architecture f
$\sigma : P \times P \to \mathbb{R}$	Prediction Metric σ
$S \subset D \times P$	Training Samples S

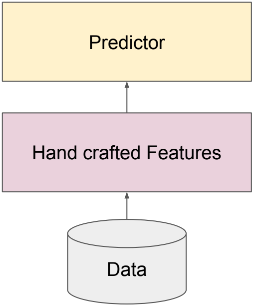
Find model parameters $m \in M$ such that the expected $\mathbb{E}(\sigma(f(d, m), G(d)))$ is minimized.

Deep Learning vs Shallow Learning

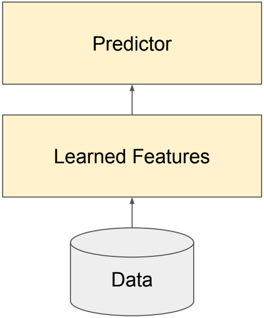


Traditional machine learning

Deep Learning vs Shallow Learning

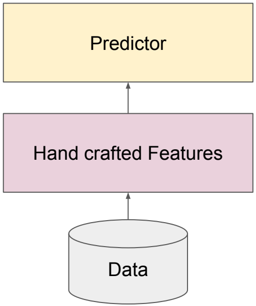


Traditional machine learning



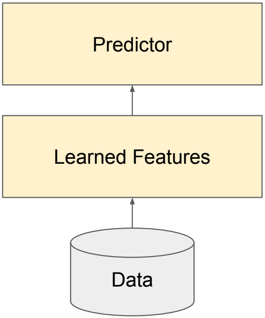
Deep Learning

Deep Learning vs Shallow Learning



Traditional machine learning

- Mostly convex, provably tractable
- Special purpose solvers
- Non-layered architectures



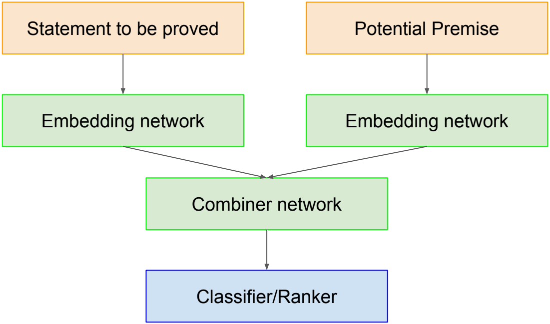
Deep Learning

- Mostly NP-Hard
- General purpose solvers
- Hierarchical models

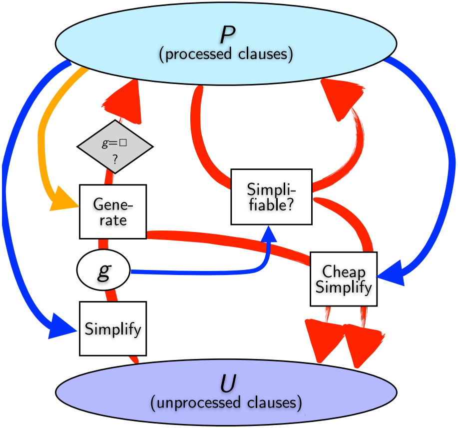
Deep Learning for Mizar Lemma Selection

[Alemi+2016]

- Embed all lemmas into $\mathbb{R}^n$ using an LSTM
- Embed conjecture into $\mathbb{R}^n$ using an LSTM
- Simple classifier on top of concatenated embeddings
- Trained to estimate usefulness on positive and negative examples



E-Prover given-clause loop



Most important choice: unprocessed clause selection [Schulz 2015]

Data Collection

Mizar top-level theorems

[*Urban 2006*]

- Encoded in FOF

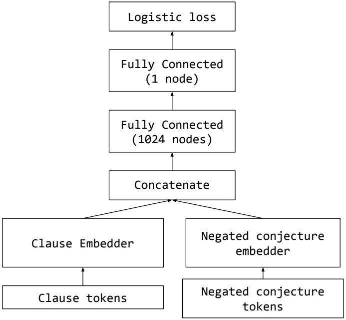
32,521 Mizar theorems with ≥ 1 proof

- training-validation split (90%-10%)
- replay with one strategy

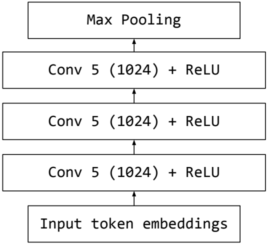
Collect all CNF intermediate steps

- and unprocessed clauses when proof is found

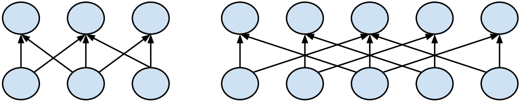
Deep Network Architectures



Overall network



Convolutional Embedding



Non-dilated and dilated convolutions

Recursive Neural Networks

- Curried representation of first-order statements
- Separate nodes for `apply`, `or`, `and`, `not`
- Layer weights learned jointly for the same formula
- Embeddings of symbols learned with rest of network
- Tree-RNN and Tree-LSTM models

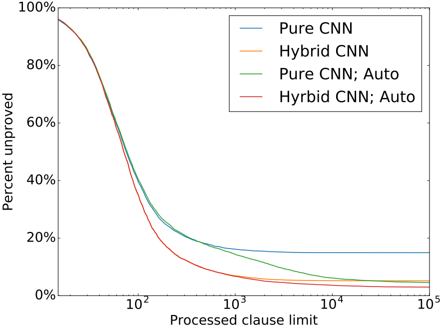
Model accuracy

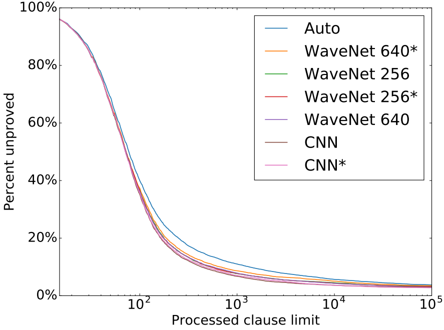
Model	Embedding Size	Accuracy on 50-50% split
Tree-RNN-256×2	256	77.5%
Tree-RNN-512×1	256	78.1%
Tree-LSTM-256×2	256	77.0%
Tree-LSTM-256×3	256	77.0%
Tree-LSTM-512×2	256	77.9%
CNN-1024×3	256	80.3%
⋆CNN-1024×3	256	78.7%
CNN-1024×3	512	79.7%
CNN-1024×3	1024	79.8%
WaveNet-256×3×7	256	79.9%
⋆WaveNet-256×3×7	256	79.9%
WaveNet-1024×3×7	1024	81.0%
WaveNet-640×3×7(20%)	640	**81.5**%
⋆WaveNet-640×3×7(20%)	640	79.9%

⋆ = train on unprocessed clauses as negative examples

Hybrid Heuristic

Already on proved statements performance requires modifications:





Harder Mizar top-level statements

Model	DeepMath 1	DeepMath 2	Union of 1 and 2
Auto	578	581	674
⋆WaveNet 640	644	612	767
⋆WaveNet 256	692	712	864
WaveNet 640	629	685	997
⋆CNN	905	812	1,057
CNN	839	935	1,101
Total (unique)	1,451	1,458	1,712

Overall proved 7.4% of the harder statements

Summary

Guiding superposition proof

- Deep network clause ranking

Performance

- Batching (evaluate clauses together)
- Hybrid heuristic
- Specialized hardware could help?

Deep network models

- Accuracy

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