A Nyström method for Fredholm integral equations with exponential weights on \((0, +\infty)\)

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Abstract

We propose a Nyström-type method to approximate the solution of integral equations of the form

\[
f(x) - \mu \int_0^{+\infty} k(x,y)f(y)w(y)\,dy = g(x), \quad x \in (0, +\infty),
\]

where \(\mu\) is a real parameter,

\[
w(y) = e^{-y^{-\alpha}-\beta y}, \quad \alpha > 0, \beta > 1,
\]

the given functions \(k\) and \(g\) can grow exponentially with respect to their arguments, when they approach to \(0^+\) and/or \(+\infty\) [5].

Since the solution of this kind of equations can increase exponentially for \(x \to 0^+\), the methods based on the weighted polynomial approximation with Laguerre-type weights are not suitable in this case. So, a first difficulty is to choose proper function spaces where these equations can be studied. To this aim, we introduce another exponential weight \(u\) and new function spaces \(C_u\) with weighted uniform metric. We prove that the proposed method is stable and convergent in this metric, using our recent results on polynomial approximation with the weight \(u\) [1–3] and Gaussian rules with the weight \(w\) [4].

Finally, we give a priori error estimates and show some numerical examples, including a comparison with other Nyström methods.

Keywords: Fredholm integral equation, Nyström method, Gaussian rule, exponential weights, real semiaxis.
References